Positronium and precision QED

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- Our most accurately tested theory

  - works to (at least) one part in 10^{12}

- Electrons, protons with electric charge interact through massless photon exchange
  - Our first local gauge interaction,
  - Long range interaction with massless photons
- QED binds electrons and nuclei to form atoms
  - What does it say about positronium bound states of e^- and e^+

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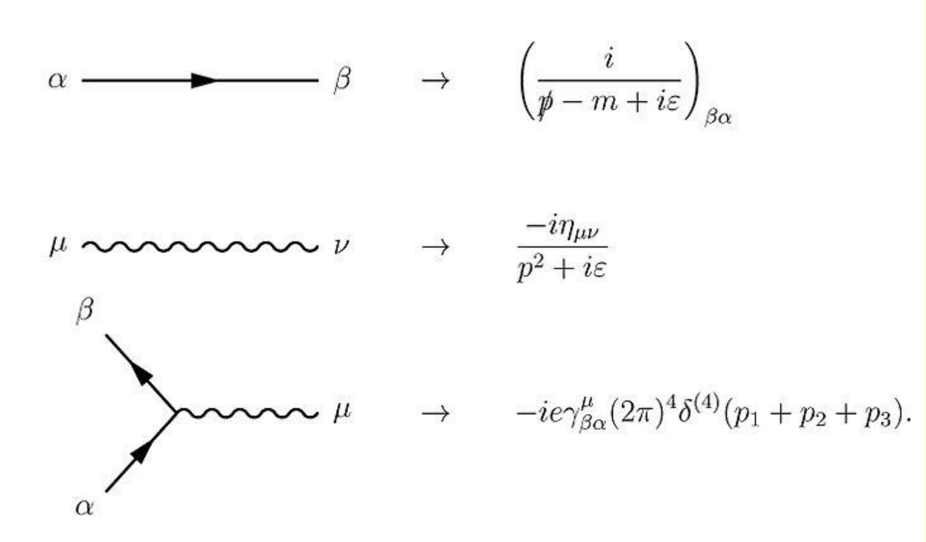
QED Feynman diagrams

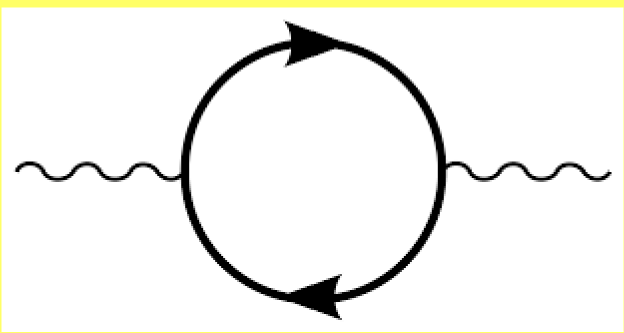
- Propagators and vertices

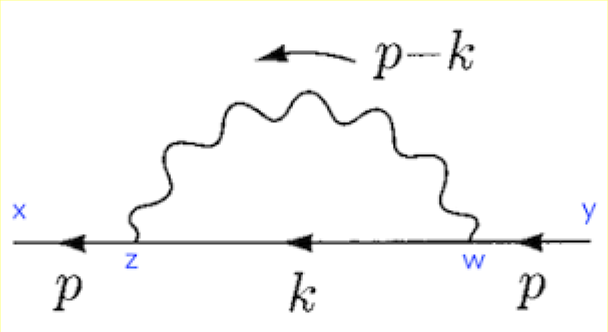
$$\mathcal{L} = \bar{\psi} i \gamma^{\mu} D_{\mu} \psi - m \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

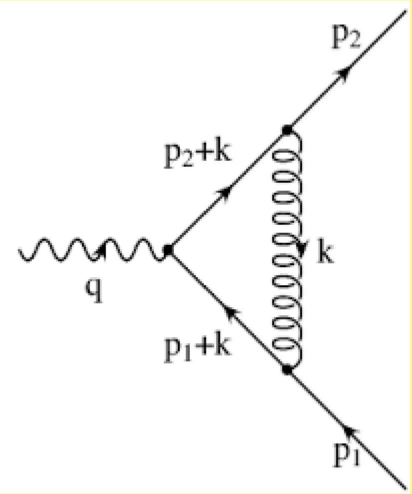
$$\partial_{\mu} \mapsto D_{\mu} = \partial_{\mu} + ieA_{\mu}$$



- Loop diagrams and vacuum polarization







$$\alpha = e^2 / 4\pi.$$

$$\alpha(m^2) = \frac{\alpha(\lambda^2)}{1 + \frac{\alpha(\lambda^2)}{3\pi} \ln \frac{\lambda^2}{m^2}}.$$

Positronium: e^+ e^- bound states

- Ortho-positronium and para-positronium systems
- QED as bound states of an elctron and positron
- p-Ps spin zero, lifetime 125 ps
- o-Ps spin one, lifetime 142 ns

$$E_B = -m_e\alpha^2/4 = -6.8 \text{ eV},$$

- Decays dominated by QED processes

  Rare and exotic decays - evidence for anything else ?

  - searching for extra forces
- Constraints from other processes: QED and Standard Model work very well !

QED as precision science

- Measuring the fine structure constant α

- Most accurate determinations:
  - Indirect through electron magnetic moment, g-2

    [D Hanneke, et al PRL 100 (2018) 120801]

$$a_e^{\mathrm{exp}} = 0.00115965218073(28)$$

$$1/\alpha|_{a_e^{\mathrm{SM}}} = 137.0359991491(331).$$

Can be calculated in precision QED+Standard Model in terms of α

  - Direct in atom interferometry

    Comparison is a precision test of QED

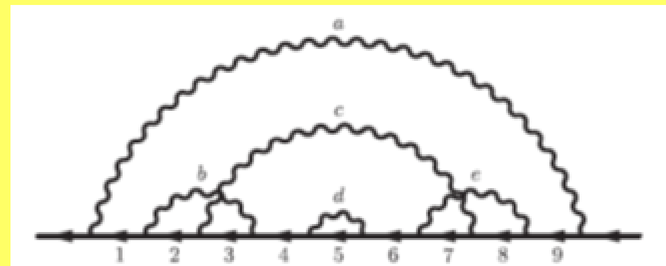
$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_{\mathrm{At}}}{m_{\mathrm{e}}}\frac{h}{m_{\mathrm{At}}}$$

Precision QED: Electron g-2

- Anomalous magnetic moment measures a particle's internal structure

$$H_{\text{int}}^{(M)} = \frac{d_M}{2}\,\overline{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$$

$d_M = (1+a_e)\ e\hbar/2m,$ $\qquad a_e=(g-2)/2$



- g=2 for a pointlike electron
- Variation from g=2 tells us about structure and vacuum polarization

- Very accurate QED and Standard Model calculations of g-2.
  Precision experiments – can we find a deviation ?

QED as precision science

- Electron $a_e = (g-2)/2$

$$a_e^{\rm exp} = 0.00115965218073(28)$$

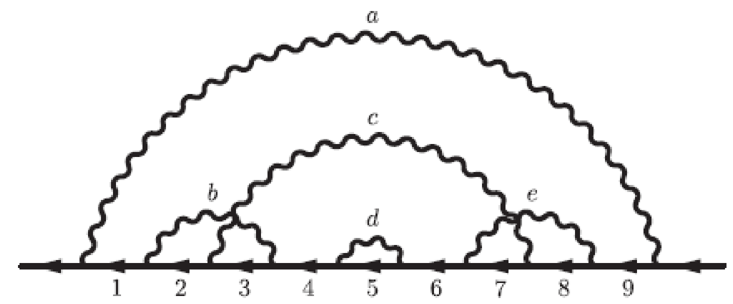
PHYSICAL REVIEW D **97**, 036001 (2018)

Revised and improved value of the QED tenth-order electron anomalous magnetic moment

Tatsumi Aoyama,[1,2] Toichiro Kinoshita,[3,4] and Makiko Nio[2]

- Calculated up to 5-loop Feynman diagrams

$$a_e^{\rm QED,e} = \frac{\alpha}{2\pi} - 0.328478965579193...\left(\frac{\alpha}{\pi}\right)^2 + 1.181241456587...\left(\frac{\alpha}{\pi}\right)^3 - 1.912245764...\left(\frac{\alpha}{\pi}\right)^4 + 6.675(192)\left(\frac{\alpha}{\pi}\right)^5 + ...$$



$$a_e({\rm QED}:\mu,\tau) = 2.7475719(13) \times 10^{-12}$$

$$a_e^{\rm SM} = a_e^{\rm QED} + 0.03053(23) \times 10^{-12}\ ({\rm weak}) + 1.6927(120) \times 10^{-12}\ ({\rm hadronic})$$

Precision determinations of α

- From atom interferometry experiments

$$1/\alpha|_{\mathrm{Cs}} = 137.035999046(27)$$

$$1/\alpha|_{\mathrm{Rb}} = 137.035999206(11).$$

- From a_e

$$1/\alpha|_{a_e^{\mathrm{SM}}} = 137.0359991491(331).$$

Precision Electrodynamics

- Substituting the most accurate direct Caesium and Rb measurement into the theoretical expression for a_e, one finds

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{th}}|_{\mathrm{Cs}} = (-88 \pm 36) \times 10^{-14}$$

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{th}}|_{\mathrm{Rb}} = (+44 \pm 30) \times 10^{-14}$$

- QED is working to (at least) one part in 10^{12}

- Also, CPT test

$$g(e^-)/g(e^+) = 1 + (0.5 \pm 2.1) \times 10^{-12}$$

QED Pauli term

- Additional Pauli term (dimension 5) treating QED as an effective theory gives a finite value of a without Landau Pole or triviality issues (Meissner et al, 2018)

$$i\frac{e}{2M}\,\bar{\psi}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\psi\;F_{\mu\nu}$$

- M here has the dimension of a mass term.
- Gives contribution to electron magnetic moment of $4e/M.$
- Experiment constrains M > 3 x 10^{10} GeV.

The electron EDM – is the electron round?

$$H_{\rm int}^{(E)} = -\frac{d_E}{2}\,\overline{\psi}\sigma_{\mu\nu}\psi\tilde{F}^{\mu\nu} = i\frac{d_E}{2}\,\overline{\psi}\gamma_5\sigma_{\mu\nu}\psi F^{\mu\nu}$$

- EDM probes possible new sources of CP violation coupled to the electron

$$H_{\rm int}^{(E)} = -\boldsymbol{d}_E \cdot \boldsymbol{E}$$

- Electron is round to very high accuracy ☺
- Gabrielse et al, Nature 2018

$$|d_e| < 1.1 \times 10^{-29} e\ \mathrm{cm}$$

$$\frac{d_e}{e} \approx \kappa \left(\frac{\alpha_{\rm eff}}{2\pi}\right)^n \left(\frac{m_e c^2}{\Lambda^2}\right) \sin(\phi_{\rm T})\,(\hbar c)$$

- New physics scale (CP violation) > 3 TeV (2 loops), 30 TeV (1 loop)

Decays and Spectroscopy

- Experiments are compared with NRQED precision calculations
  - Bound state calculations from pure QED hard (intractable)
  - Instead, work with NRQED approximations
    - Schrödinger equation works well for spectrum in leading approx.
    - „Relativistic effects“ in interactions are defined via cut-off and introduced via extra operators, supplementing NR terms.
    - Effective theory approach with expansion in $v/c \sim \alpha \ll 1$, analogous to ChPT for low-energy QCD

If fails: look to possible enhanced relativistic effects in wavefunction and or Ps resonances in intermediate state of calculation...

Decays and Spectroscopy

- Hyperfine discrepancy in $2^3S_1 \rightarrow 2^3P_0$ transition at order 10^{-4}, 4.5σ effect [Cassidy et al, 2020]

- Decay rates for o-Ps agree with NRQED TH to order ~ 10^{-4},
  - Experimental error 100 x NRQED theoretical error.
  - What will next generation experiments give with improved precision?

- Underlying QED is working, e.g. Electron g-2 and eEDM,
  - Testing NRQED bound state theory at this level (presently do-able experiments) and not searching for new interactions.

Discrete symmetries

- CPT Symmetry follows in any local Quantum Field Theory with Lorentz invariance, Hermitian Hamiltonian and Spin-Statistics
- Symmetries of positronium bound states should inherit the symmetries of their constituents
  - (unless e.g. some SSB like phenomena through strong coupling effects – e.g. Chiral Symmetry in QCD)
- Not eigenstate of T through decays (Ps mass bigger than massless photons, so spontaneous decays). FSI can mimic CP and CPT violation through radiative corrections at ~ 10^{-9} [Bernreuther+Nachtmann]
- CPT in o-Ps should be working to this order, constrained by (g-2)
- CP violations only allowed within present experimental reach if some phase cancellation in e-EDM, but with what interactions (e.g., with constraint from g-2) ??

New interactions searches

- In decays constrained by g-2 as well as axion experiment constraints for possible pseudoscalars in the final state, with BRs << 10^{-9} or so

- Invisibles od most experimental interest: search for mirror matter with interaction term

$$\mathcal{L} = \epsilon\, F^{\mu\nu} F^{m}_{\mu\nu}.$$

- ETHZ epxeriments give

$$BR(\text{o-Ps} \to \text{invisible}) < 3 \times 10^{-5}, \quad 90\%\ \text{C.L.}$$

corresponding to $\varepsilon < 5 \times 10^{-8}$

- Compares with astrophysics constraints $\varepsilon < 3 \times 10^{-8}$ (primordial He^4 abundance from Standard Model).
  - » getting close to interesting parameter region ☺

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