Statistical hadronization model for heavy-ion collisions in the few-GeV energy regime

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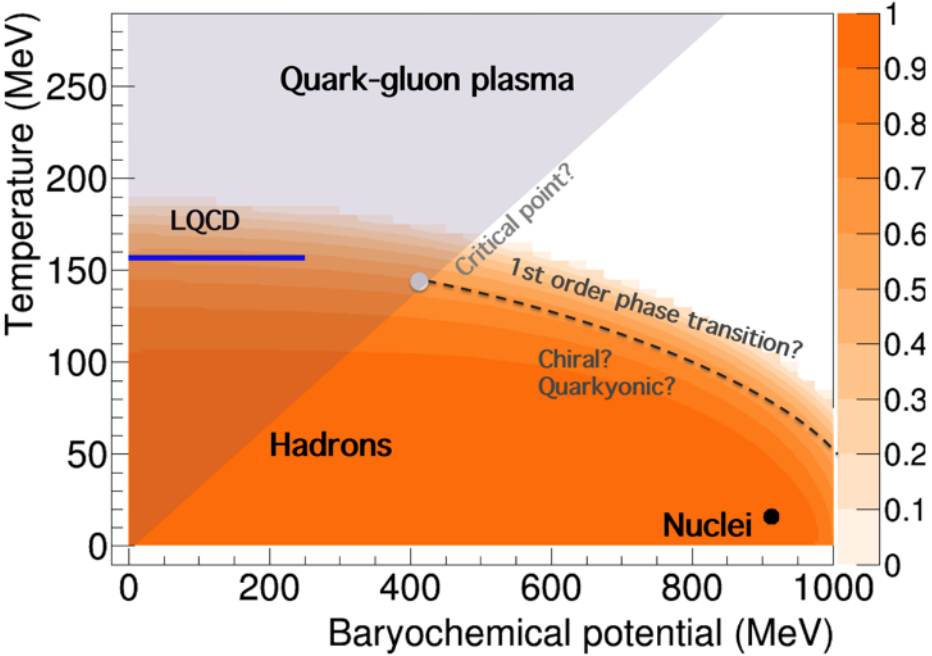
Goethe-University Frankfurt / GSI

based on:

PRC 102 (2020) 5, 054903, arXiv: 2003.12992 [nucl-th]

What is the QCD phase structure?



[P. Steinbrecher (HotQCD), arXiv:1807.05607
Condensate: B.J. Schaefer and J. Wambach]

Vanishing μ_B, high T (lattice QCD)

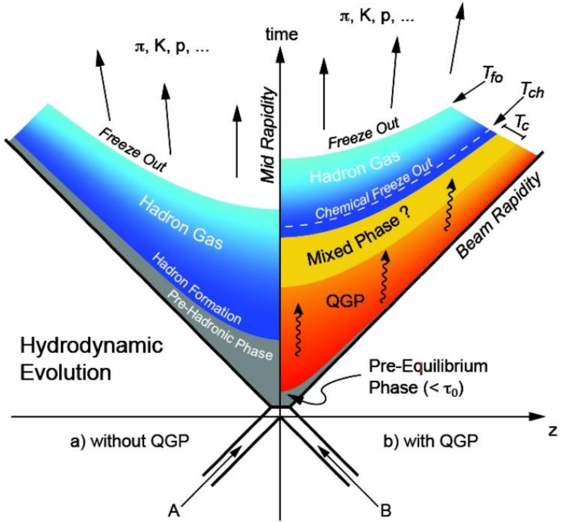
- Crossover, universality
- no CP indicated by lattice QCD at μ_B < 400 MeV, T >140 MeV

Large μ_B moderate T (QCD inspired models)

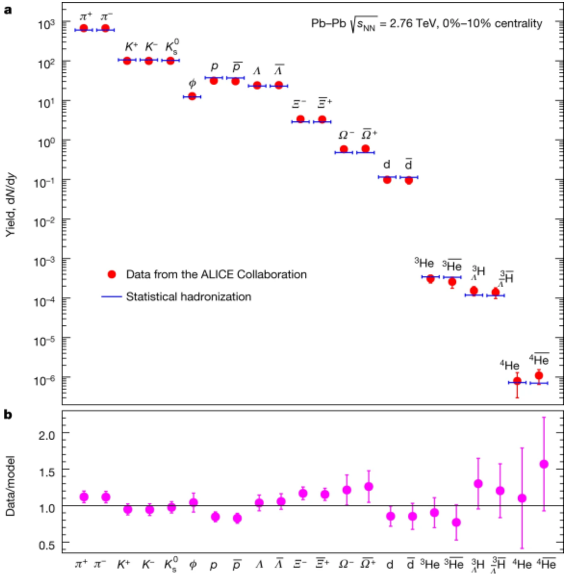
- Thermal equilibrium?
- 1st order transition?
- QCD critical point?
- Melting of the condensate?

$2 < \sqrt{s_{NN}} < 8$ GeV
Large discovery potential!

Heavy-ion collisions as a tool to study QCD

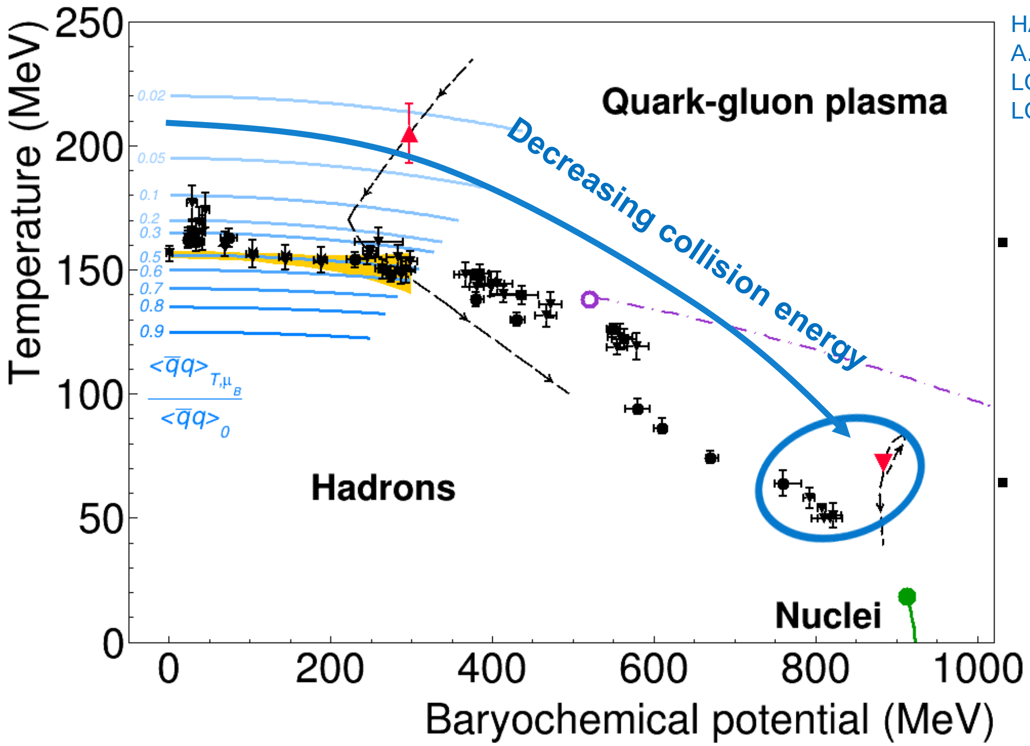


- In chemical equilibrium density of particle *i* can be written as:
  $n_i = \frac{g_s}{2\pi^2} \Upsilon T m^2 K_2\left(\frac{m}{T}\right)$
  Statistical Hadronization Model (SHM)
- One can fit the ratios of measured particle yields and extract free parameters
  - Location in the phase diagram



A. Andronic et al., Nature 561 (2018) 7723

Mapping the phase diagram with the Statistical Hadronization Model



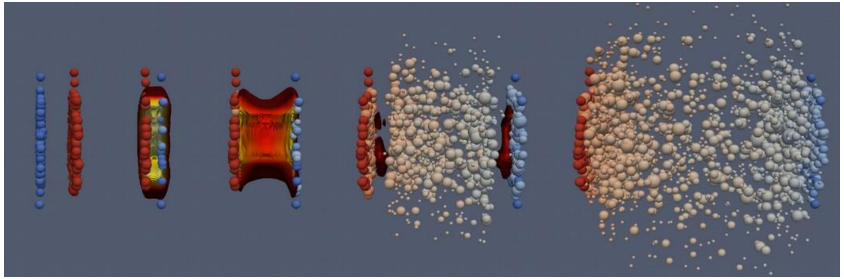
HADES, Nature Phys. 15 (2019) 10, 1040-1045
A. Andronic *et al.*, Nature 561 (2018) no.7723
LQCD: S. Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009 (2010) 073
LQCD: A. Bazavov *et al.*, PLB 795 (2019) 15-21

- Is it valid at all to use equilibrium methods at low energies?
  - Particles with strange quarks produced deep below the NN threshold
  - Low number of newly produced particles in the interaction zone: ~40 in central events (mainly pions)
- On the other hand:
  - Original nucleons stopped in the interaction zone (~300 particles in central events)
  - Longer life-time of the system – enough to thermalize

Dynamic description of heavy-ion collisions

Standard prescription at high beam energies (RHIC/LHC):

- Non-equilibrium initial conditions
- Viscous hydrodynamic evolution
  - Equilibrium
  - People often assume: fluid = QGP
- Hadronic final-state rescattering



Standard prescriptipn at "low" beam energies (GSI/FAIR/NICA/…):

- Hadronic transport
- Importance of:
  - Resonance dynamics
  - Nuclear potentials

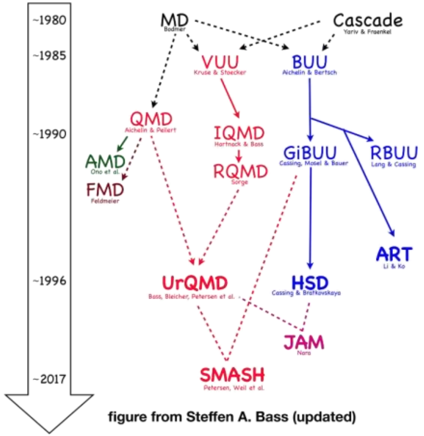
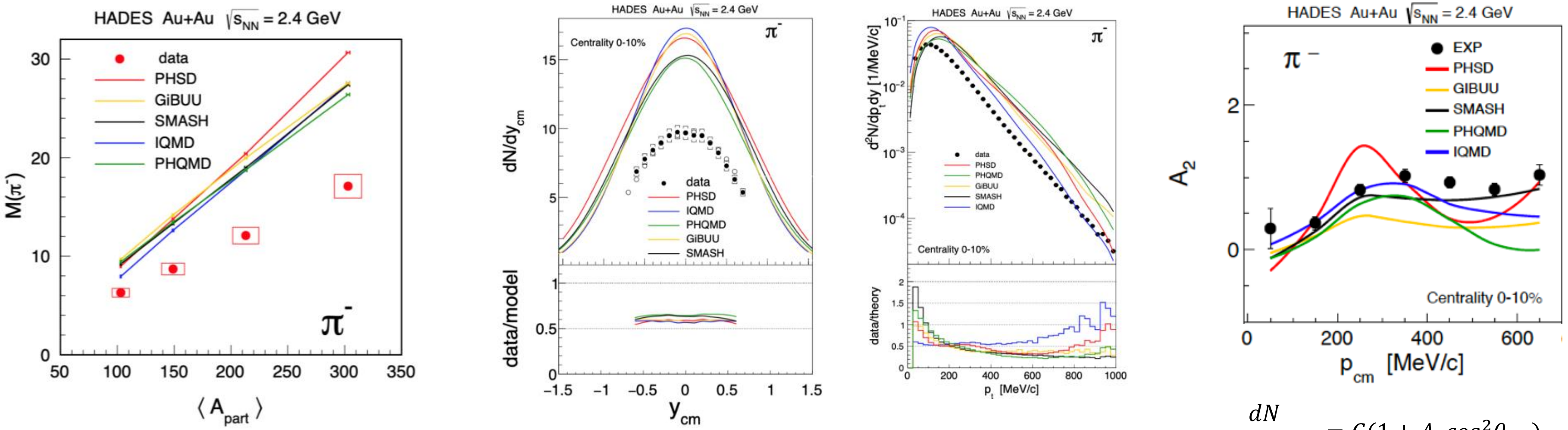


figure from Steffen A. Bass (updated)

Not everything is known yet about few-GeV HIC

Example for π^-, same holds for π^+

HADES Collaboration, EPJA 56 (2020) 10, 259
https://www.hepdata.net/record/ins1796710



$$\frac{dN}{d(cos\theta_{cm})} = C(1 + A_2 cos^2\theta_{cm})$$

- Only width of the rapidity distributiion is correctly described by the models
- Is there something fundamentally missing?

Pion and Proton "Temperatures" in HIC
R. Brockmann *et al.*, PRL 1984

HADES data vs. other experiments

HADES Collaboration, EPJA 56 (2020) 10, 259
https://www.hepdata.net/record/ins1796710

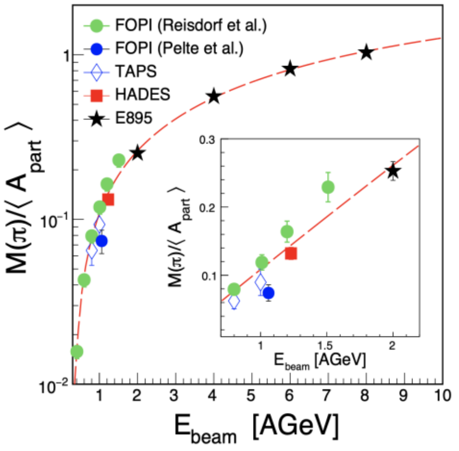


Fig. 8 Pion multiplicity $M(\pi)$ per mean number of participating nucleon $\langle A_{part} \rangle$ as a function of the kinetic beam energy E_{beam}. The dashed curve is a fit to the data points except for the one labeled "FOPI (Pelte et al.)", as suggested in [4]. The inset magnifies the energy region around the HADES point.

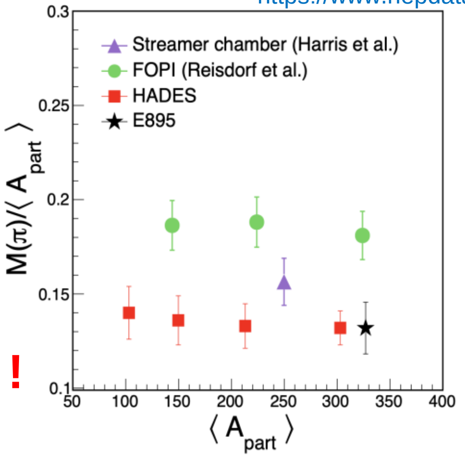


Fig. 9 Comparison of the centrality dependence of $M(\pi)/\langle A_{part} \rangle$ in Au+Au collisions to earlier measurements at similar energies. The results from FOPI, E895, and from the BEVALAC Streamer Chamber group (the latter for $La + La$ collisions) have been scaled to 1.23 A GeV; note the suppressed zero on the ordinate.

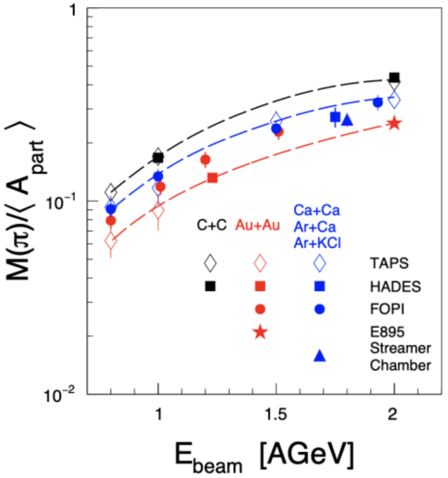


Fig. 10 Pion multiplicity per participating nucleon as a function of beam energy for three different systems: C+C (black) [7, 22, 39], Ar+KCl (blue) [4, 7–9, 40] and Au+Au (red) [4, 6, 7, 11]. The curves are polynomial fits to these data used to interpolate the multiplicities as a function of bombarding energy for corresponding systems.

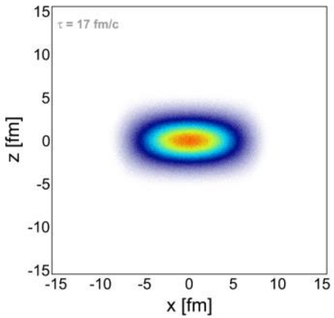
HADES results are consistent with the trends established by previous experiments at similar beam energies

Hydro-inspired models

of particle production at the freeze-out

- **First idea:**
  P. J. Siemens and J. O. Rasmussen, PRL 42 (1979) 880
  - Used for Ne+NaF at $E_{kin}/A = 0.8$ GeV!
  - Thermal source of spherical geometry and spherically symmetric expansion
  - Constant radial velocity (non-physical for $r = 0$?)

Guidance from dynamic models



- Density evolution in Au+Au at $E_{kin}/A = 1.23$ GeV
- Coarse-grained hadronic transport
  T. Galatyuk *et al.*, EPJA 52 (2016) 5, 131
- Spherical symmetry clearly more realistic than boost invariance

- **Modification:**
  E. Schnedermann, J. Sollfrank, U. W. Heinz, PRC 48 (1993) 2462
  - Appropriate for higher-energy collisions (originally S+S at $E_{kin}/A = 200$ GeV)
  - Cylindrically-symmetric geometry and expansion
  - Boost invariance in Z direction – "Bjorken scaling"
  - Velocity profile: $\beta(r) = \beta_{\max}(r/\mathrm{r}_{\max})^n$

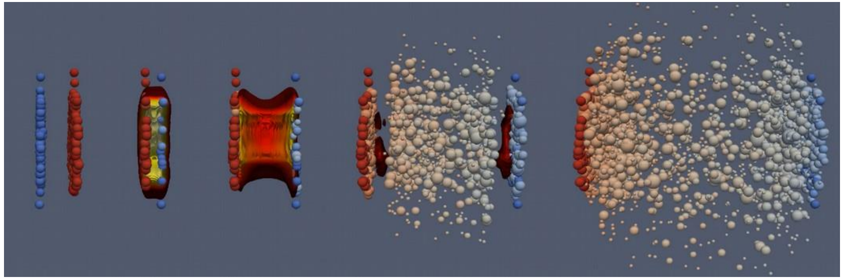
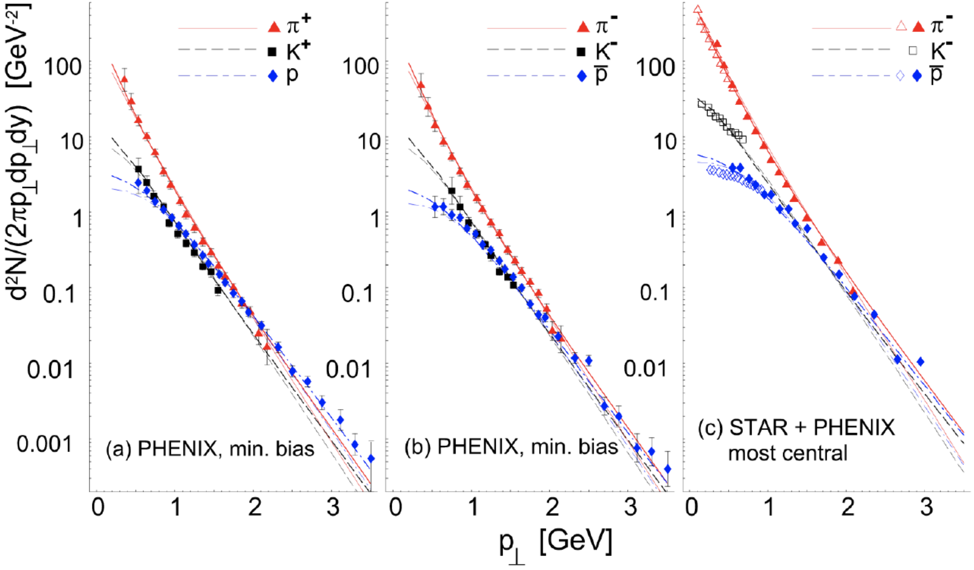


Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard

Single freeze-out scenario

W. Broniowski and W. Florkowski, PRL 87 (2001) 272302

- Chemical freeze-out coincides with kinetic freeze-out
- Hadron yields are given by the integrals of hadron spectra
- Feed-down from resonance decays included
- Successful at RHIC, does it work at SIS18 energies?
- Idea is implemented in the Thermal Event Generator (Therminator 2)



Cooper-Frye formula

F. Cooper and G. Frye, PRD 10 (1974) 186

"Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production"

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu\, f(x,p)$$

- Spherically symmetric system:
$$x^\mu = (t(r), r\mathbf{e_r})$$
- Spherical expansion of the "fluid":
$$u^\mu = \frac{1}{\sqrt{1-v^2(r)}}(1, v(r)\mathbf{e_r})$$
- Sudden freeze-out in the "lab" frame ($t = \mathrm{const}(r)$):
$$d^3\Sigma_\mu \equiv \varepsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial \zeta}\frac{\partial x^\beta}{\partial \phi}\frac{\partial x^\gamma}{\partial \theta} d\zeta\, d\phi\, d\theta$$
$$= (r^2 \sin\theta\, d\theta\, d\phi\, dr, 0, 0, 0)$$

Parameter of $\zeta \to (t(\zeta), r(\zeta))$

Local thermodynamic equilibrium

$$f(x,p) = \frac{g_s}{2\pi}\left[\Upsilon^{-1} \exp\left(\frac{p_\mu u^\mu}{T}\right) \pm 1\right]^{-1}$$

Fugacity factor:

$$\Upsilon \equiv \gamma_q^{N_q+N_{\bar{q}}} \gamma_s^{N_s+N_{\bar{s}}} \exp\left(\frac{\mu_B B + \mu_S S + \mu_{I_e} I_3}{T}\right)$$

(in this work we assume $\gamma_q = 1$)

- Integrating over the freeze-out hypersurface and phase-space gives back particle multiplicity
- Right sets of assumptions recover the original Siemens-Rasmussen and Schnedermann-Sollfrank-Heinz formulas
- But we assume Hubble-like expansion:
$$v(r) = \tanh(Hr)$$

Resonance treatment

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718
W. Weinhold, and B. Friman, Phys. Lett. B 433 (1998) 236
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

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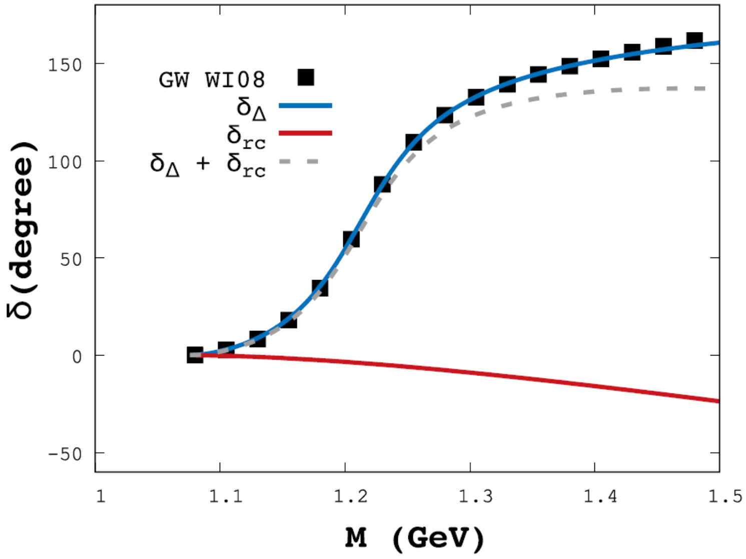
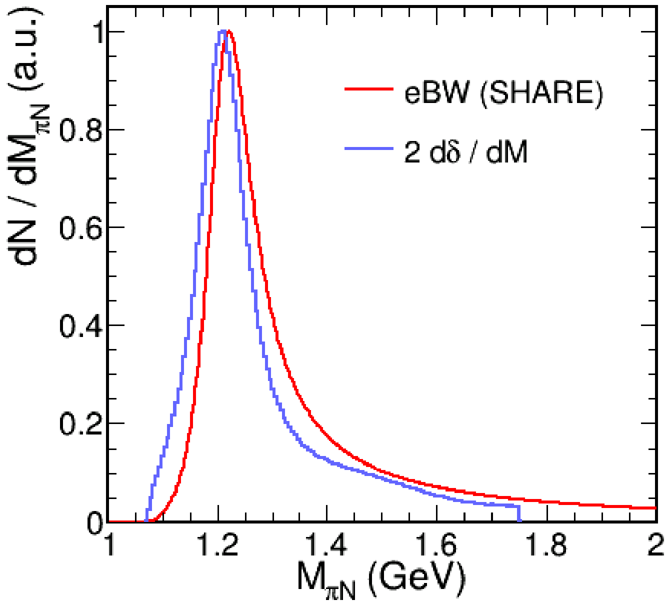
πN phase shift in the P_{33} channel

Pok Man Lo *et al.*, PRC 96, 015207,
GW WI08: R.L. Workman *et al.* PRC 86, 035202



Spectral function: $B_l(M) = 2\frac{d}{dM}\delta_l$



Thermal Event Generator (Therminator 2)

HADES data:
M. Szala, Proceedings of SQM 2019
EPJA 56 (2020) 10, 259
PLB 778 (2018) 403-407
PLB 793 (2019) 457-463

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Ingredients of the method: M. Chojnacki *et al.*, Comput. Phys. Comm. 103 (2012) 746-773
SH, W. Florkowski, T. Galatyuk *et al.*, PRC 102 (2020) 5, 054903

- Single (chemical and kinetic) freeze-out on a **spherically symmetric hypersurface** (Siemens-Rasmussen blast-wave model)
- Fix thermodynamic paremeters with multiplicities of particles:
  — Solve numerically 6 equations for 6 parameters:

protons (incl. those bound in light nuclei):	124.1
π^+:	9.3
π^-:	17.1
K^+:	0.0598
K^-:	0.00056
Λ:	0.0822

T = 49.6 MeV

μ_B = 776 MeV

μ_{I3} = -14.1 MeV

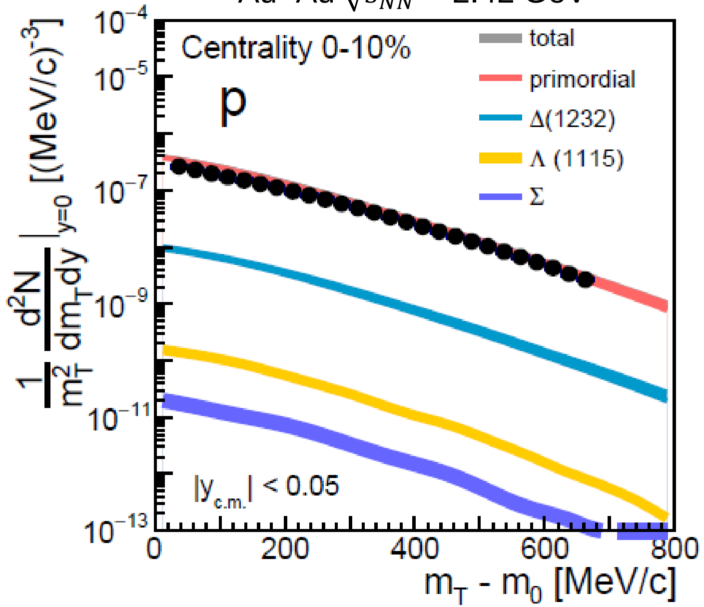
μ_S = 123.4 MeV

γ_S = 0.16

R = 16.02 fm

- Proton m_t spectrum at mid-y is fitted to get the expansion velocity profile: $v = \tanh(Hr)$ with **$H = 0.04\ \mathrm{fm}^{-1}$**
- Δ spectral function from πN phase shift

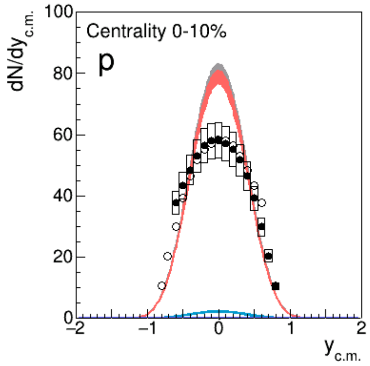
Au+Au $\sqrt{s_{NN}}$ = 2.42 GeV



GSI HADES

Spectra of bulk particles

Au+Au $\sqrt{s_{NN}}$ = 2.42 GeV, 0-10%



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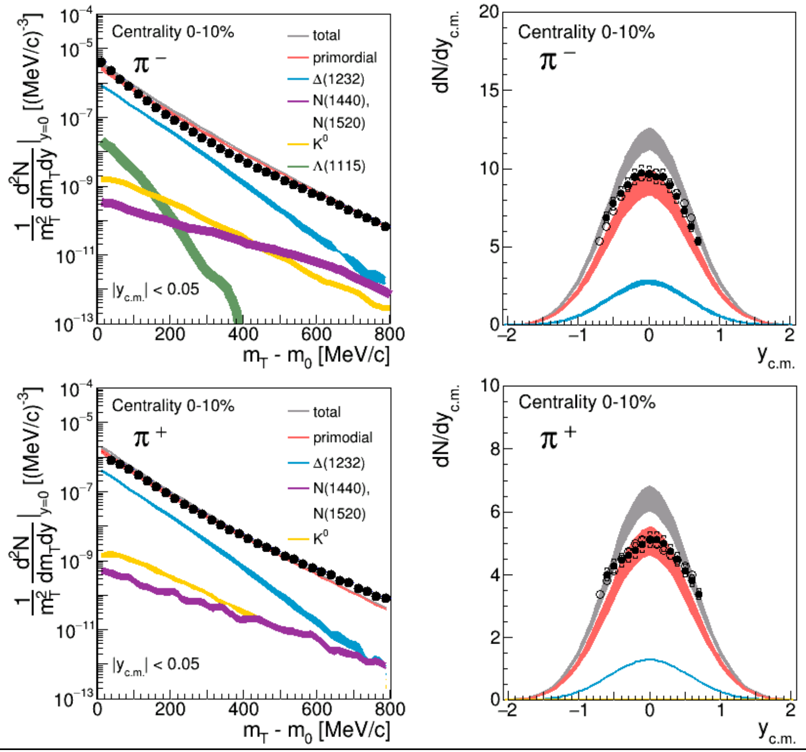
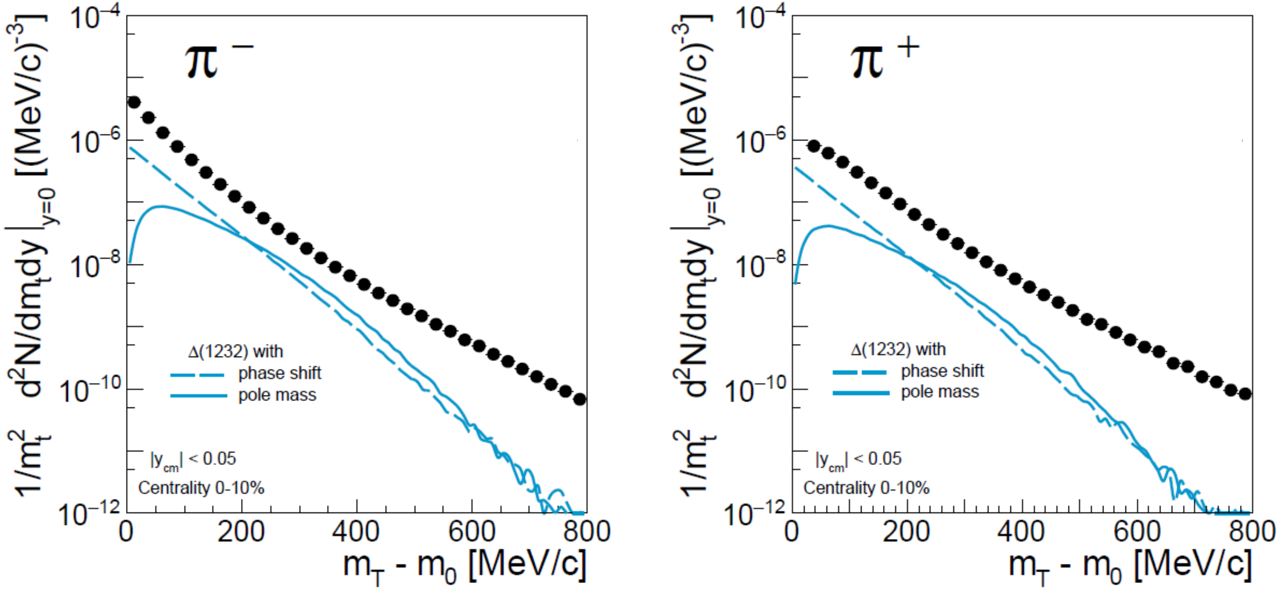


- These spectra are **not fitted**, but **predicted** by the model
- Bands: uncertainty from errors on hadron yields
- Pion slope at high m_t described with T ~ 50 MeV and Hubble
- Rapidity too narrow in the model
  - — Spherical symmetry is not exactly fulfilled
  - — Further improvements are ongoing

Influence of the Δ description on pion spectra



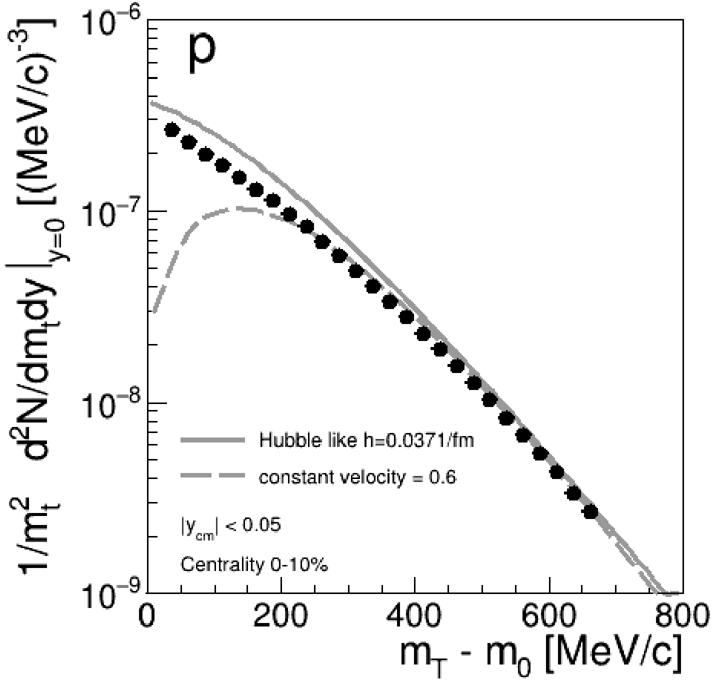
Transverse mass of pions from Δ decay for different spectral functions:

- Δ with fixed mass of 1.232 GeV
- Spectral function from the πN phase shift in the P_{33} channel

Finite Δ width:
→ populate low m_t pions

Influence of the velocity profile

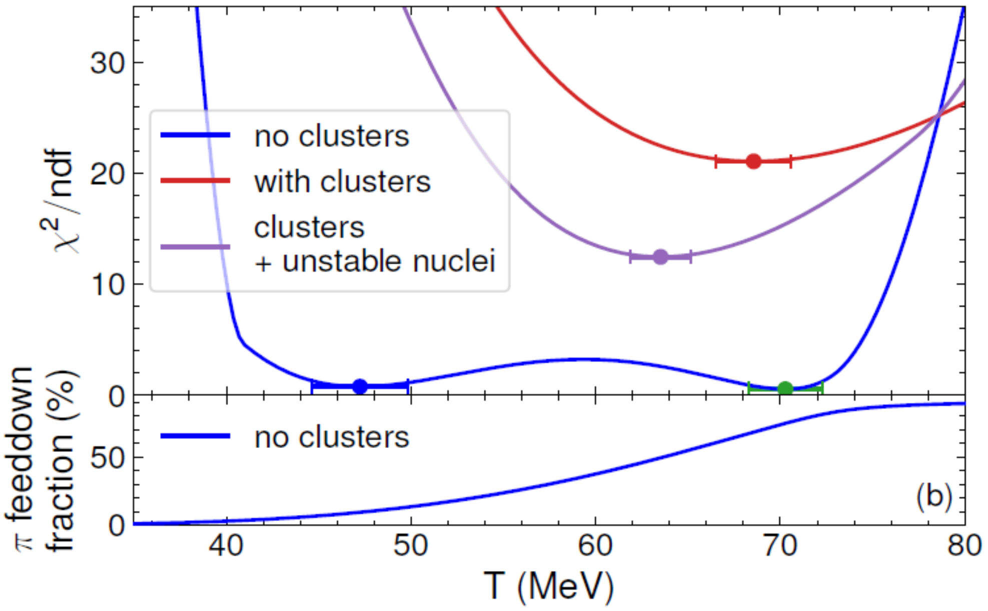
- Hubble-like fireball expansion:
$$v(r) = \tanh(Hr)$$
- The parameter *H* fitted to the proton m_t spectra:
$$H = 0.037\, fm^{-1}$$
- Mean value:
$$\langle v \rangle = \frac{2}{3} HR \left(1 - \frac{1}{5} H^2 R^2\right) \approx 0.4$$
- Best fit with constant velocity
  — Gives $\langle v \rangle = 0.6$
  — Fails to describe the data at low m_t



Outlook:

other approaches to thermal parameters

A. Motornenko *et al.*, arXiv:2104.06036 [hep-ph]



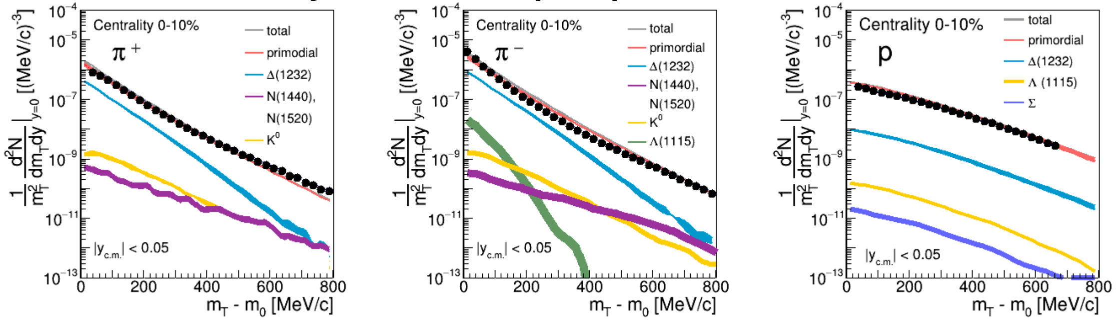
Parameter	Harabasz *et al.* [1]	no clusters low T minimum	no clusters high T minimum	with clusters	with clusters + unstable nuclei
T (MeV)	49.6 ± 1.1	47.2 ± 2.6	70.3 ± 2.0	68.6 ± 2.0	63.5 ± 1.6
R (fm)	16.0	18.9 ± 2.2	6.8 ± 0.9	9.0 ± 0.4	10.4 ± 0.3
μ_B (MeV)	776 ± 3	780.1 ± 3.8	872.1 ± 24.3	786.7 ± 2.9	781.1 ± 3.3
γ_S	0.16 ± 0.02	0.19 ± 0.07	0.05 ± 0.01	0.03 ± 0.01	0.04 ± 0.01
χ^2/N_{df}	$N_{df} = 0$	1.58/2	1.13/2	105.30/5	62.30/5

- In this manuscript Q/B = 0.4 and total S = 0 are kept as constraints
- We recover parameters needed to run Therminator:
  - μ_{I3} = -21.05 MeV
  - μ_S = 198.63 MeV
- We fix the Hubble constant H and readjust R:
  - H = 0.097 1/fm
  - R = 6.1 fm

Outlook:

other approaches to thermal parameters

Parameters from Phys. Rev. C 102 (2020) 5, 054903



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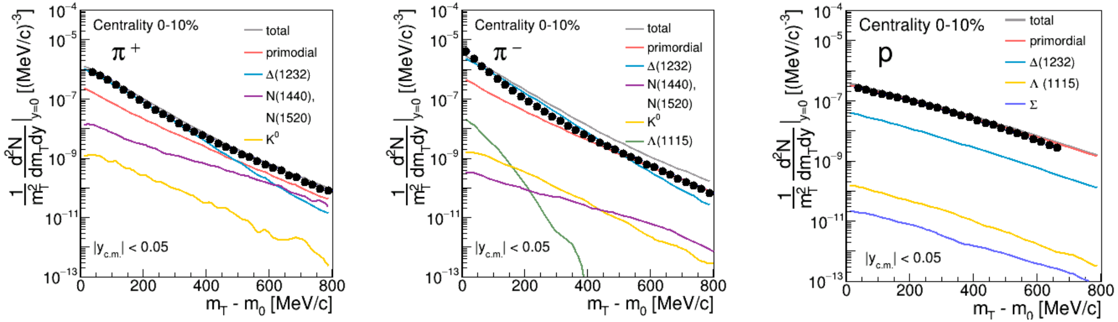


T = 70.3 MeV

μ_B = 876 MeV

μ_{I_3} = -21.5 MeV

μ_S = 198.3 MeV

γ_S = 0.05

R = 6.1 fm

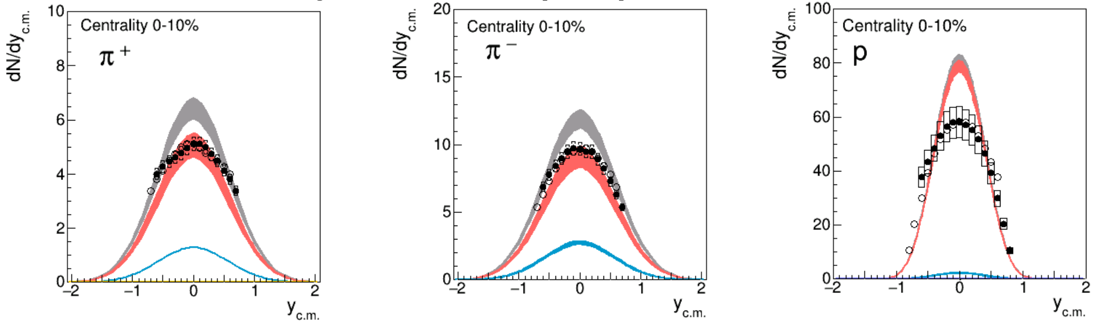
H = 0.097 1/fm

- As expected, stronger contribution of resonance decays in the high-T case
- No grounds to exclude one of the minima by looking qualitatively at the spectra

Outlook:

other approaches to thermal parameters

Parameters from Phys. Rev. C 102 (2020) 5, 054903



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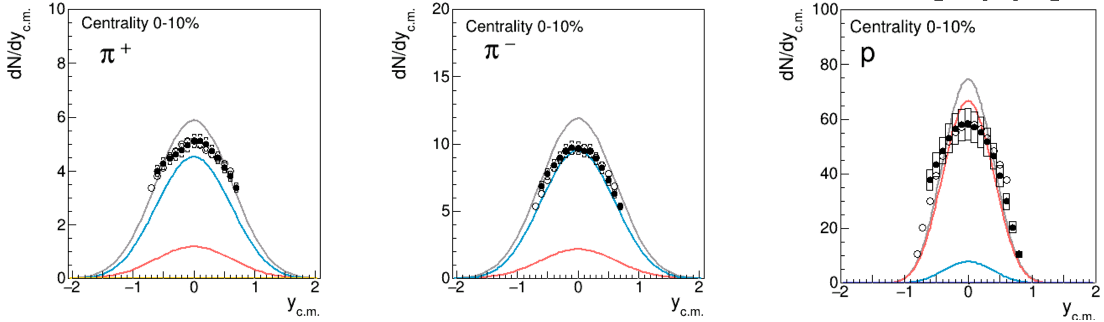

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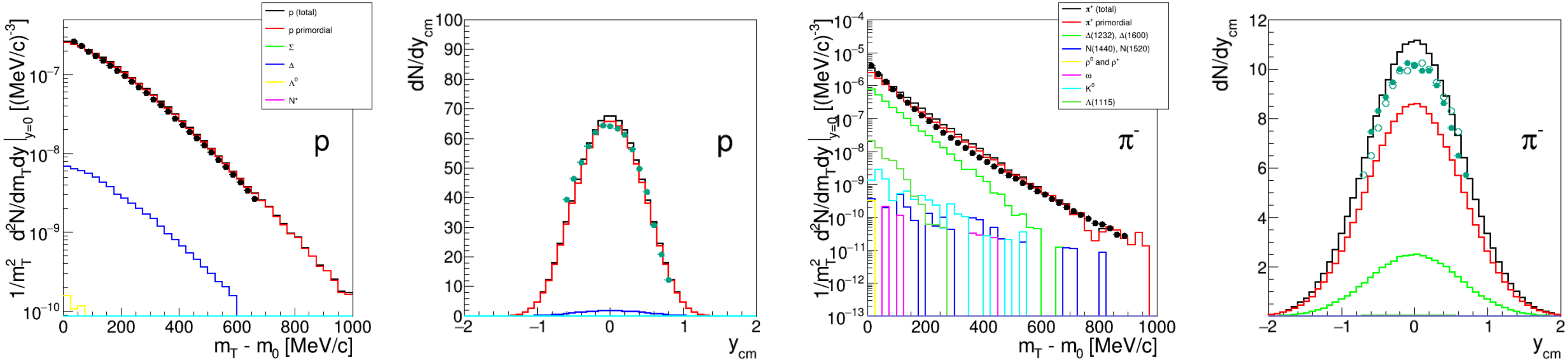
- As expected, stronger contribution of resonance decays in the high-T case
- No grounds to exclude one of the minima by looking qualitatively at the spectra
- No strong influence on the width of *y* spectra
  - — Need to modify the freeze-out hypersurface

Outlook:
moving from spherical to spheroid symmetry

- Transverse momentum spectra are well described, and
- Rapidity spectra are too narrow compared to experiment
  — Expansion in longitudinal direction should be stronger than in transverse direction
- Guidance from dynamic models
  — Freeze-out hypersurface should be narrower in the longitudinal direction

Ongoing work on systematic fitting the shape parameters



Outlook:

Afterburner for final-state EM interaction

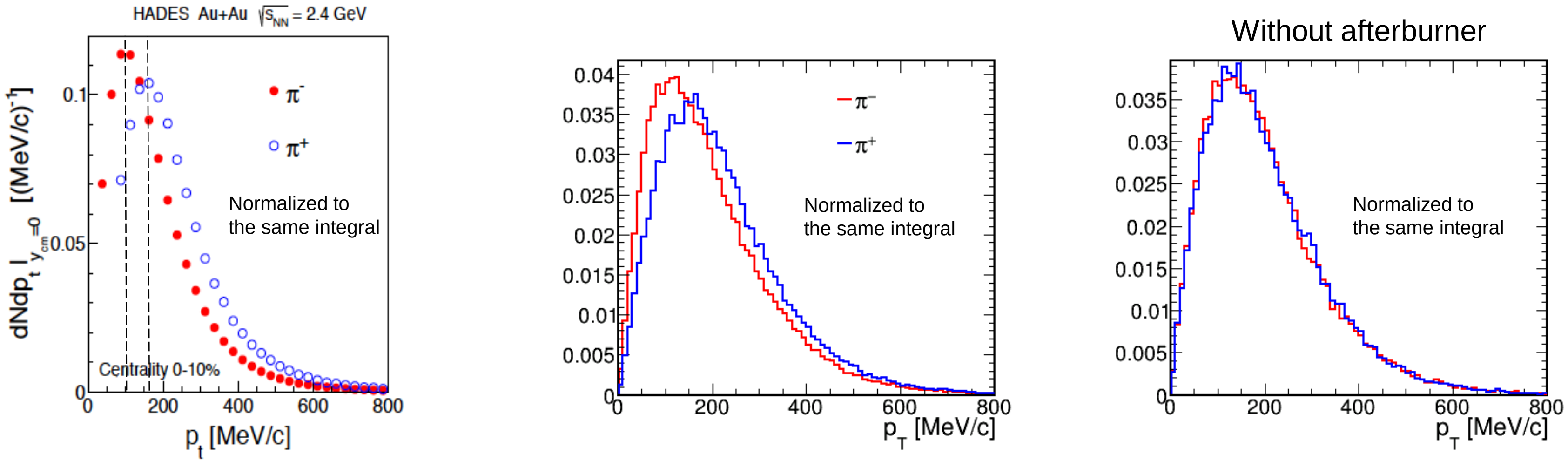
After the freeze-out, particles a propagated according to standard formulas:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{R}{(\mathbf{R}\cdot\mathbf{u})^3}\left[(c^2-v^2)\mathbf{u}+\mathbf{R}\times(\mathbf{u}\times\mathbf{a})\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\widehat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{R}\equiv\mathbf{r}-\mathbf{w}(t_r),\qquad \mathbf{v}\equiv\dot{\mathbf{w}}(t_r)$$

$$\mathbf{u}\equiv c\widehat{\mathbf{R}}-\mathbf{v},\qquad |\mathbf{r}-\mathbf{w}(t_r)| = c(t-t_r)$$



Conclusions

- Statistical hadronization model can describe not only multiplicities, but also spectra of bulk particles produced in heavy-ion collisions in $\sqrt{s_{NN}}$ of few GeV
- Ingredients:
  - Spherical, Siemens-Rasmussen-type fireball expansion
  - Hubble-like velocity profile
  - Sudden freeze-out
  - Careful treatment of baryonic resonances

Outlook:

- Spheroidal instead of spherical symmetry
- Final-state EM interactions
- HBT radii, nucleon coalescence, data from STAR fixed-target, FAIR, NICA…

THANK YOU FOR YOUR ATTENTION