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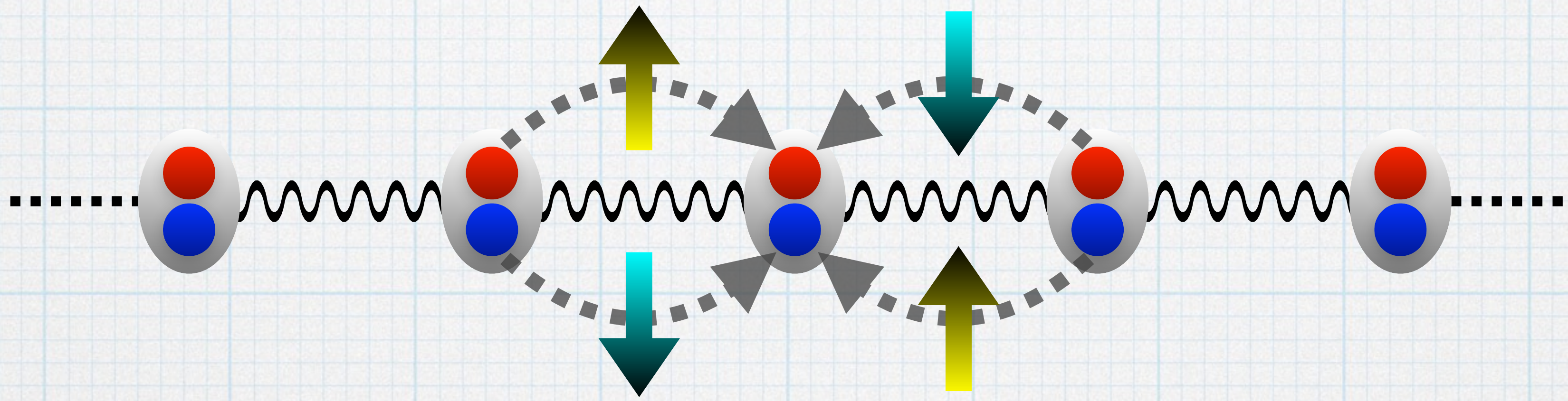
W KRAKOWIE

QUANTERA

Quantum simulations of particle physics



Phys. Rev. Lett. 124, 180602 (2020)

In collaboration with: **Jakub Zakrzewski, Maciej Lewenstein, Luca Tagliacozzo**

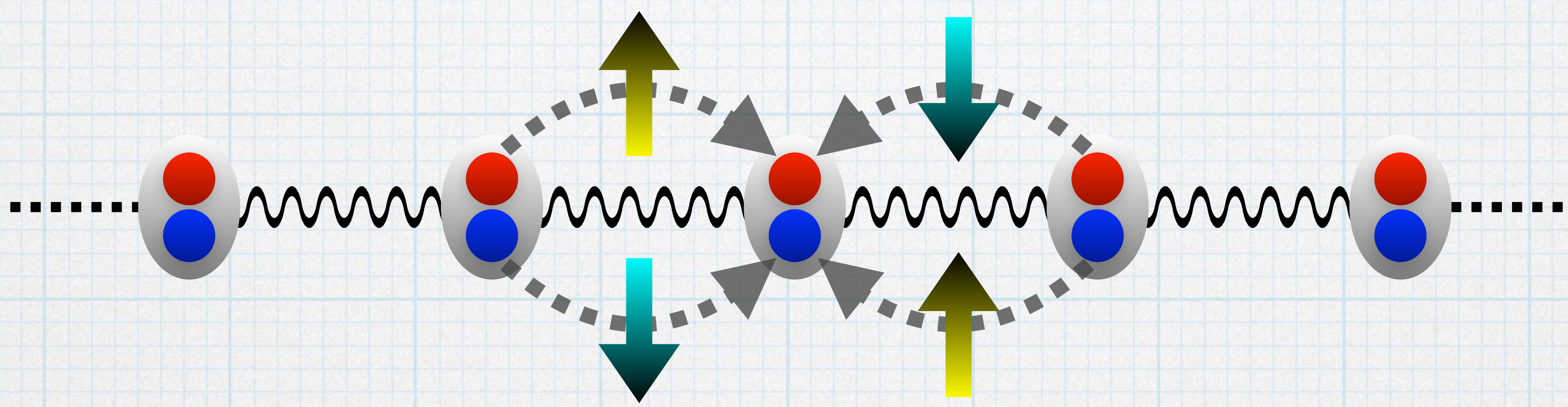
Titas Chanda
Uniwersytet Jagielloński, Kraków, Poland

NARODOWE CENTRUM NAUKI

UNIWERSYTET JAGIELLOŃSKI
W KRAKOWIE

QUANTERA

Lattice gauge theories in the age of quantum technologies: Bosonic Schwinger model out of equilibrium



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NARODOWE CENTRUM NAUKI

Outline

In two parts…

1. Lattice gauge theories in the age of quantum technologies

   Introduction to the subject

2. Bosonic Schwinger model out of equilibrium

   Our work… Phys. Rev. Lett. 124, 180602 (2020)

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In two parts…

1. Lattice gauge theories in the age of quantum technologies

Quantum simulation: proposed by Yuri Manin and Richard Feynman

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Unitaries are simulated using
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in a quantum circuit

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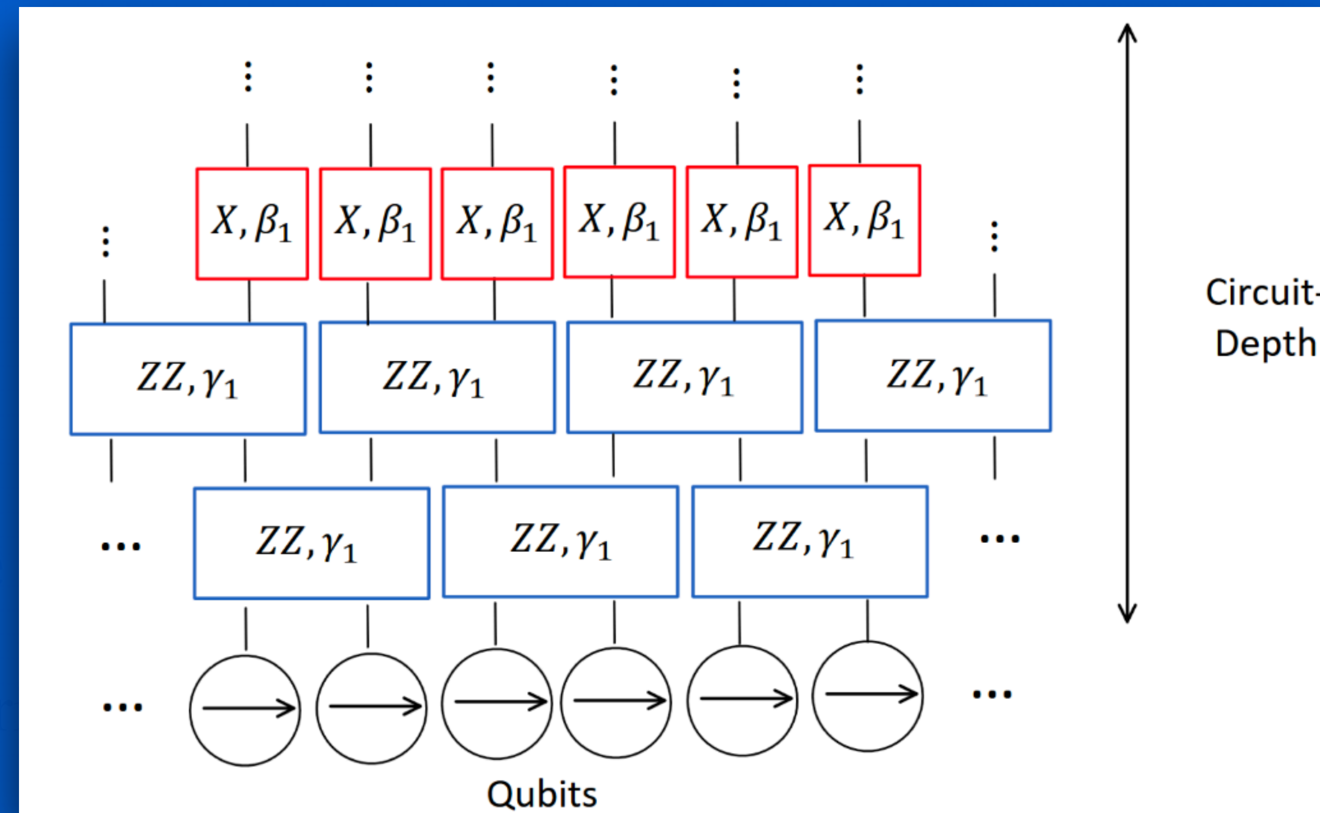

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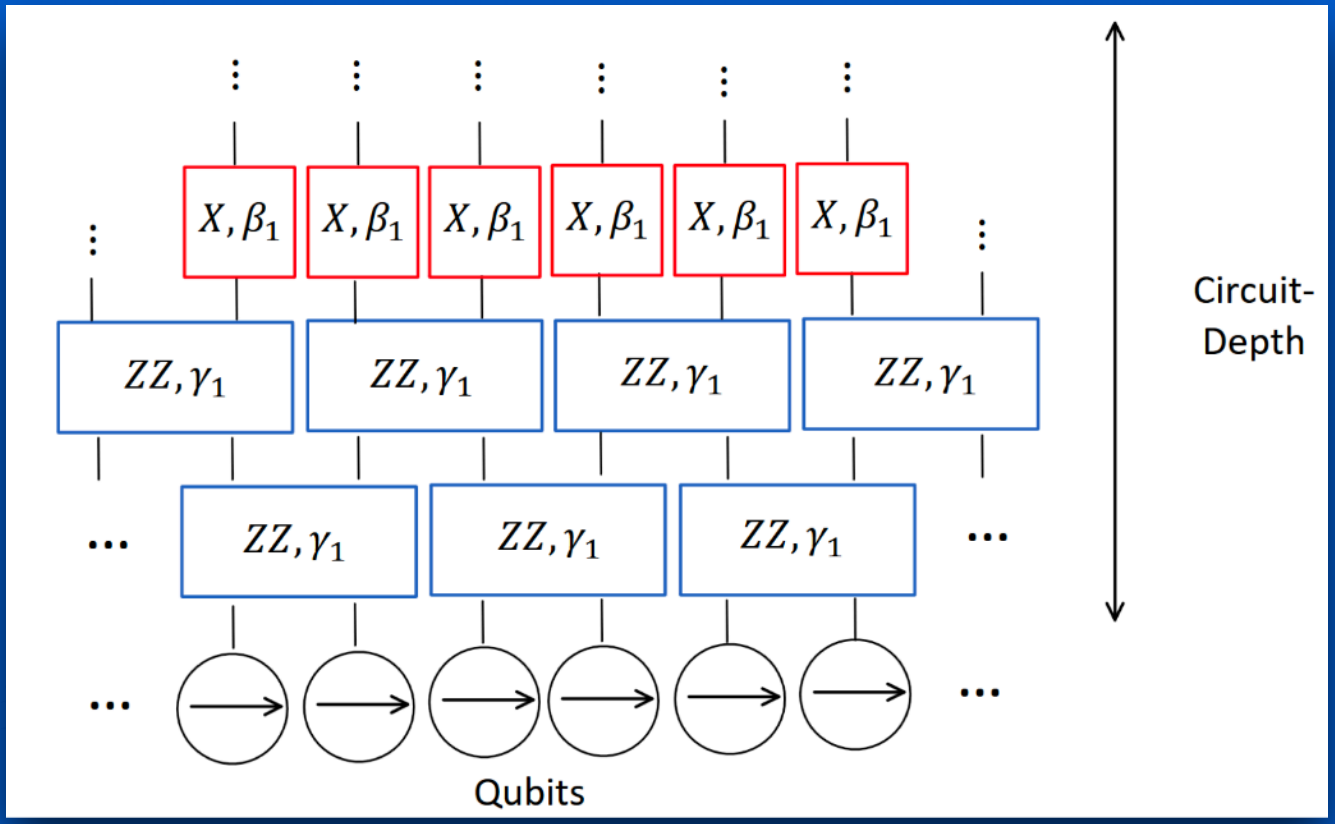

"IBM quantum experience"
https://quantum-computing.ibm.com/

Online q. computing service
Over 20 devices on the service
6 are freely available

Anyone can design and perform digital q. simulations

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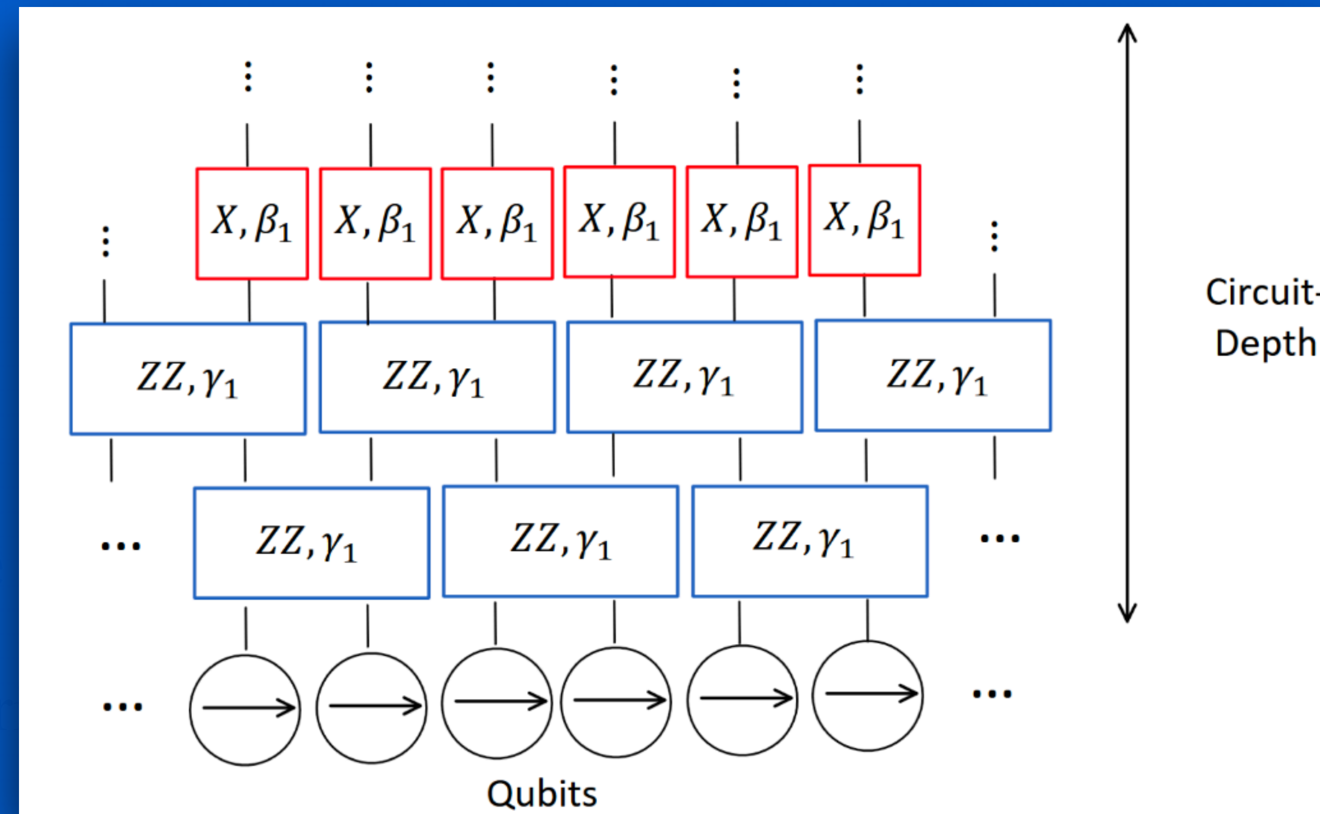

Limitations:

(1) Local Hilbert space dim is restricted to 2

(2) Not scalable in space. Hard to maintain large number of qubits — loss of quantum coherence

(3) Not scalable in time — Trotter errors — loss of quantum coherence

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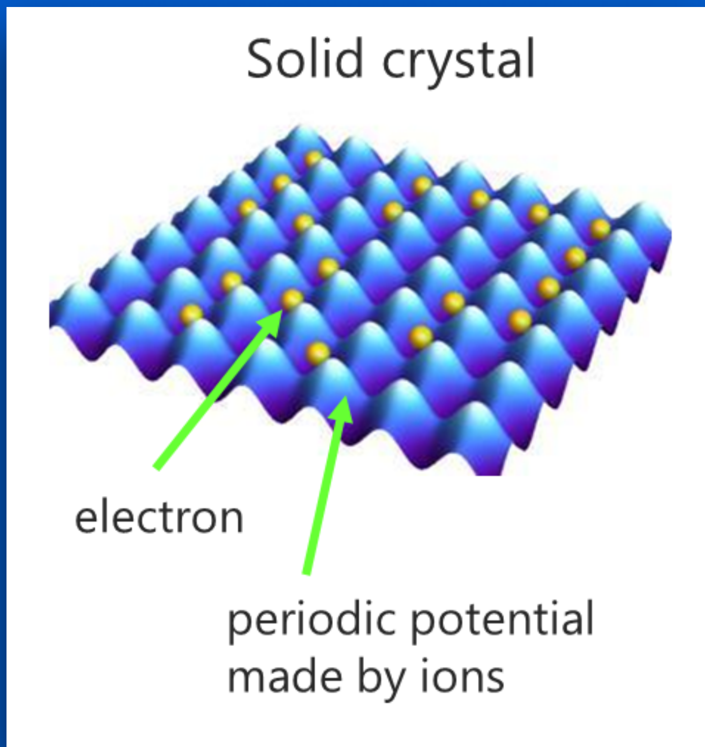
Analog simulation
Interactions of a ‘source system’
(cold atoms, ion-trap etc.) are tuned
to mimic the physics of a ‘target system’

Outline

Very successful in simulating solid state physics



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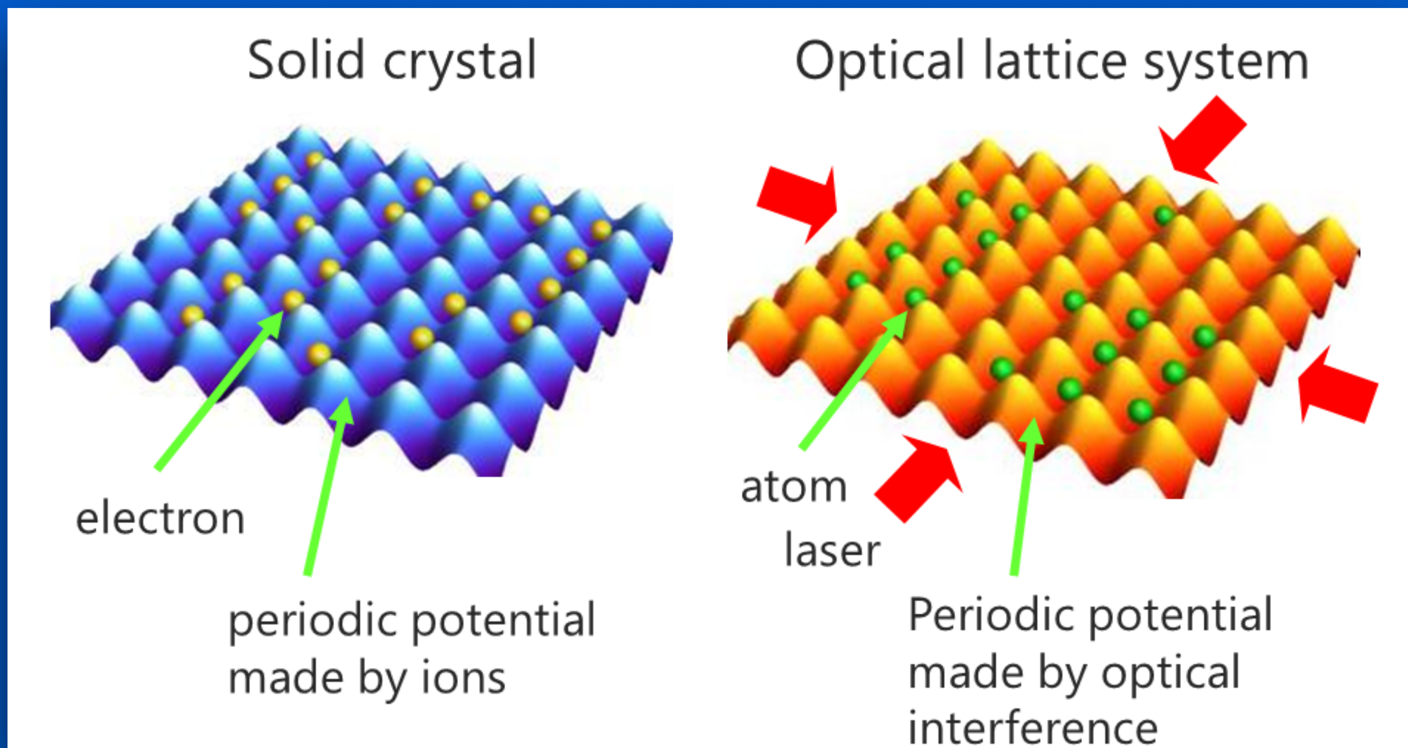

uantum technologies

nd Richard Feynman

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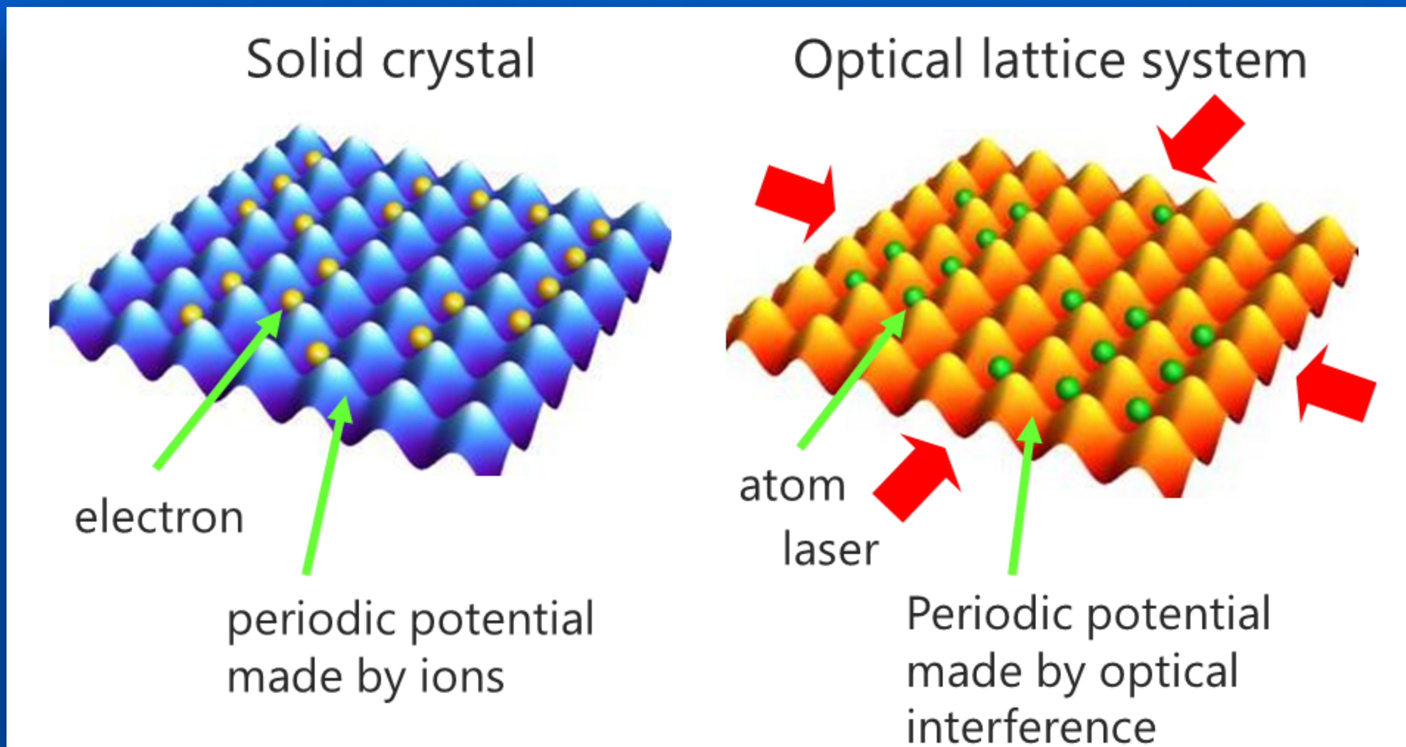

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Analog simulation

Interactions of a ‘source system’
(cold atoms, ion-trap etc.) are tuned
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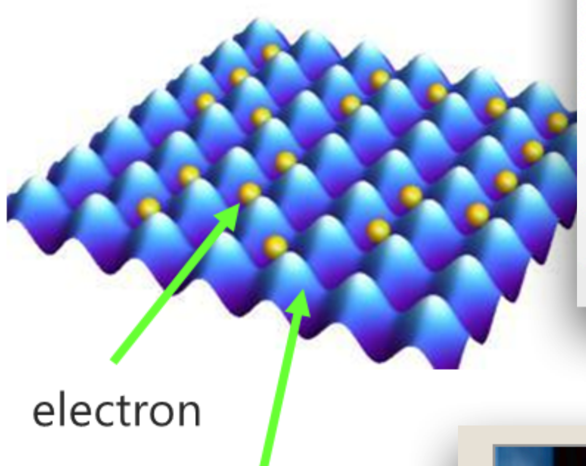
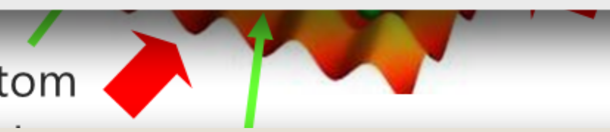
Successful in simulating theoretical models, like

1. Bose-Hubbard model
2. Fermi-Hubbard model
3. Isotropic Heisenberg model
4. Ising model
5. And very recently, XXZ model

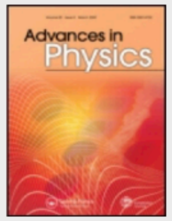
Very successful in si

Solid crystal





Richard Feynman



Advances in Physics ›
Volume 56, 2007 - Issue 2

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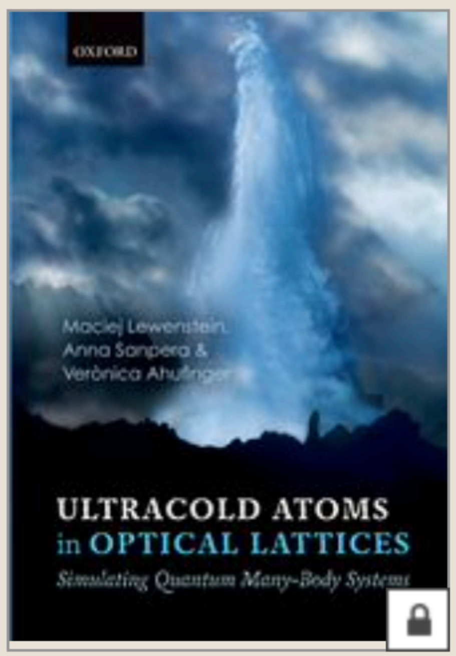
Original Articles

Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond

Maciej Lewenstein, Anna Sanpera, Veronica Ahufinger, Bogdan Damski, Aditi Sen(De) & Ujjwal Sen

Pages 243-379 | Received 31 May 2006, Accepted 11 Jan 2007, Published online: 04 May 2007
Download citation https://doi.org/10.1080/00018730701223200



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Ultracold Atoms in Optical Lattices: Simulating quantum many-body systems

Maciej Lewenstein, Anna Sanpera, and Verònica Ahufinger

ABSTRACT

Quantum computers, although not yet available on the market, will revolutionise the future of information processing. Already now, quantum computers of special purpose, i.e., quantum simulators, are within reach. The physics of ultracold atoms, ions, and molecules offers unprecedented possibilities of control of quantum many systems, and novel possibilities of applications for quantum information and quantum metrology. Particularly fascinating is the possibility of using ultracold atoms in lattices to simulate condensed matter or even high energy physics. This book provides a comprehensive ove ... More

Keywords: ultracold atomic gases, molecular gases, quantum simulators, optical lattices, atomic systems, many-body physics

BIBLIOGRAPHIC INFORMATION

Print publication date: 2012
Published to Oxford Scholarship Online: December 2013
Print ISBN-13: 9780199573127
DOI:10.1093/acprof:oso/9780199573127.001.0001

Successful in si

1. Bose-Hubba
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5. And very re

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Analog simulation
Interactions of a 'source system'
(cold atoms, ion-trap etc.) are tuned
so that it can mimic the physics of a 'target system'

Natural question: Can we simulate gauge theories that describe high-energy physics?

Introduction

Symmetries

Transformation under some **Group**
Physics (the **action**) remains invariant

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Transformation under some **Group**
Physics (the **action**) remains invariant

- Global symmetries

e.g.

Noether's (first) theorem

1. Translational symmetry: $\psi(x) \to \psi(x+a)$ $\longrightarrow$ Momentum conserved
2. Phase symmetry in QM: $\psi(x) \to e^{i\alpha}\psi(x)$ $\longrightarrow$ Total probability conserved, continuity eq

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- Local (gauge) symmetries

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$$\partial_\mu \epsilon^{\mu\nu\delta\eta} F_{\delta\eta} = 0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Big\} \text{ Invariant under } A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

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- Global symmetries → Physics remain unchanged after transformation

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'Technically' not a symmetry
Just a redundancy in our description
Can't be broken spontaneously (Elitzur's theorem)
Noether's (first) theorem not applicable
Instead we get Gauss law

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Instead we get Gauss law

Then why not??

$$\nabla \, . \, \mathbf{E} = \rho$$

$$\nabla \, . \, \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$$

→ Turns out to be very difficult to work with, especially in QED

Introduction

Gauge theories

(Classical) Gauge theories existed since mid-19th century…

$$\partial_\mu F^{\mu\nu} = J^\nu$$

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… $U(1)$ gauge theory

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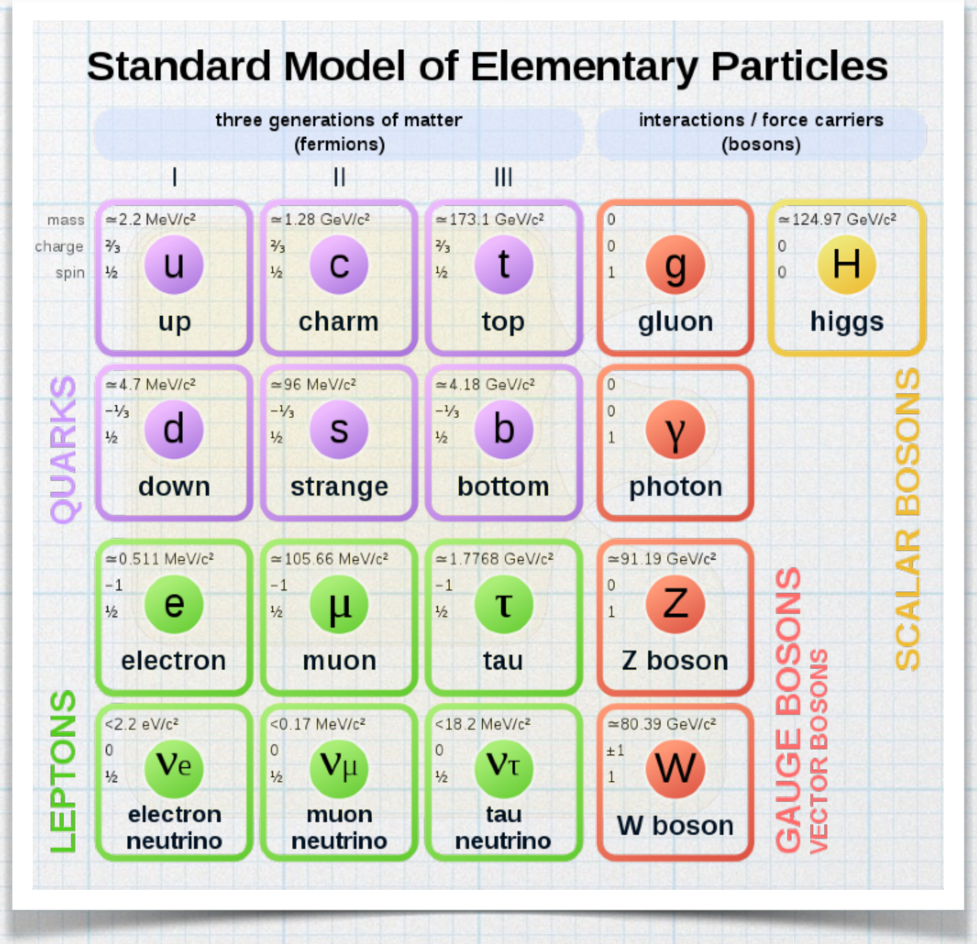
(Quantum) Gauge theories came in the form of quantum electrodynamics, non-Abelian Yang-Mills theories etc.

Standard model of particle physics is a non-Abelian gauge theory with the symmetry group $U(1) \times SU(2) \times SU(3)$.



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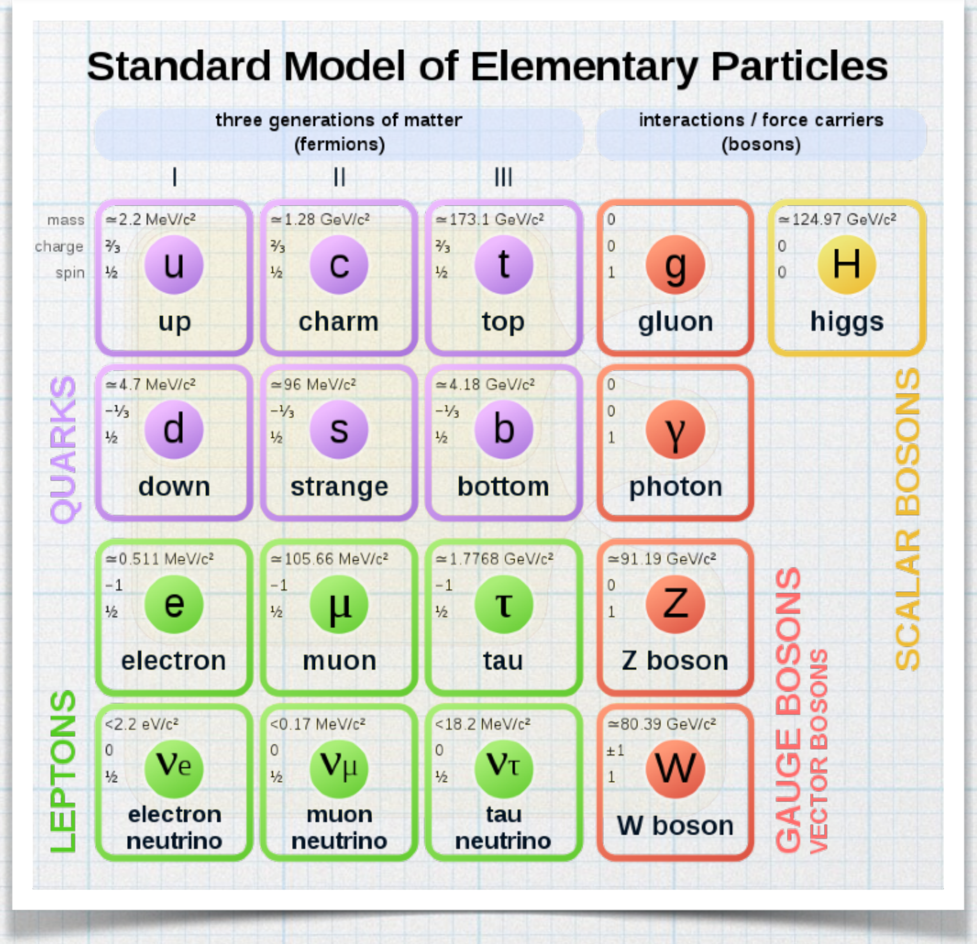

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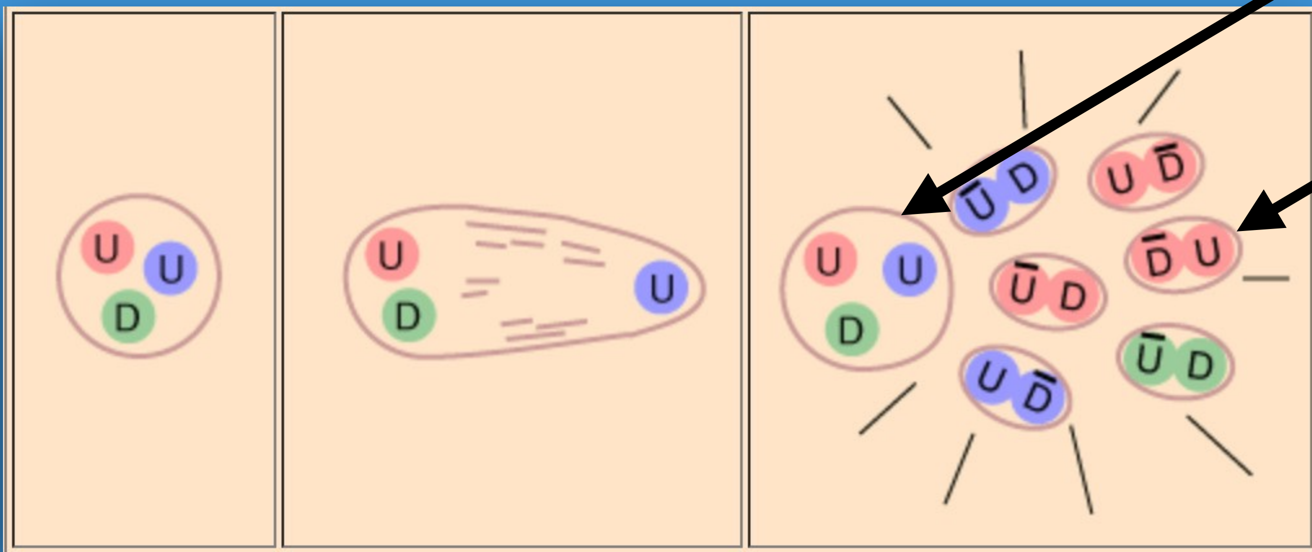
Free quarks are not seen naturally…



Hadrons

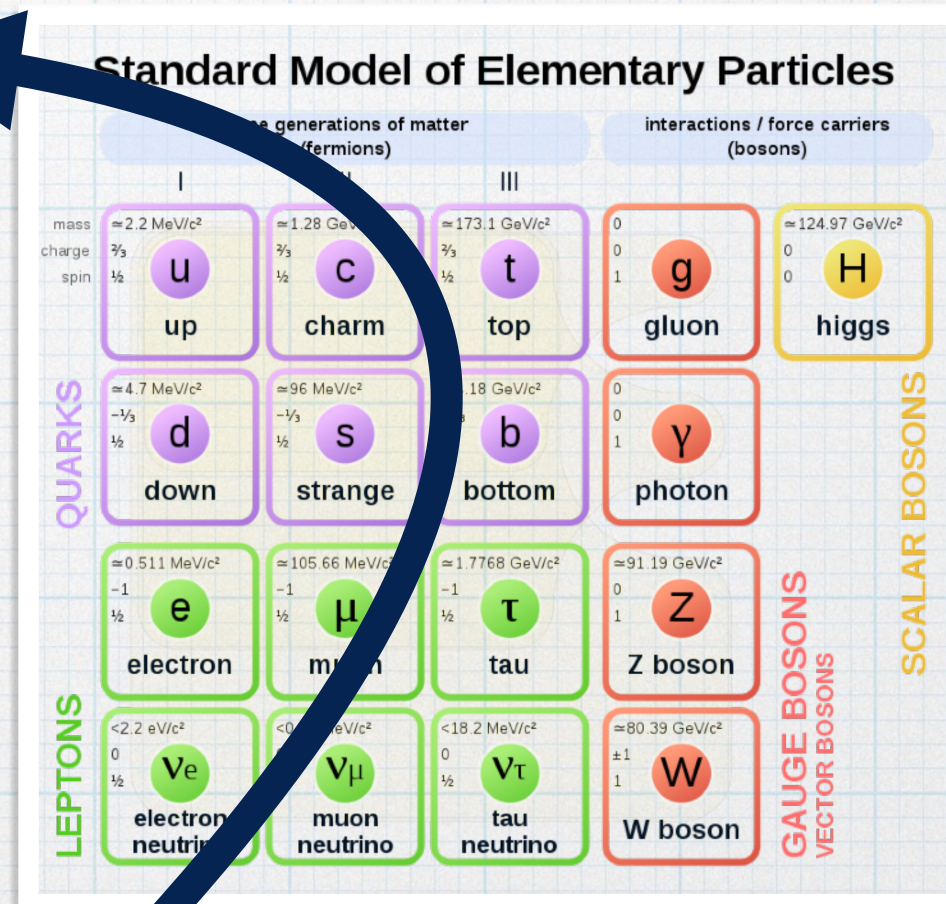
Mesons

ntury…

theory

ntum
etc.

gauge
(3).



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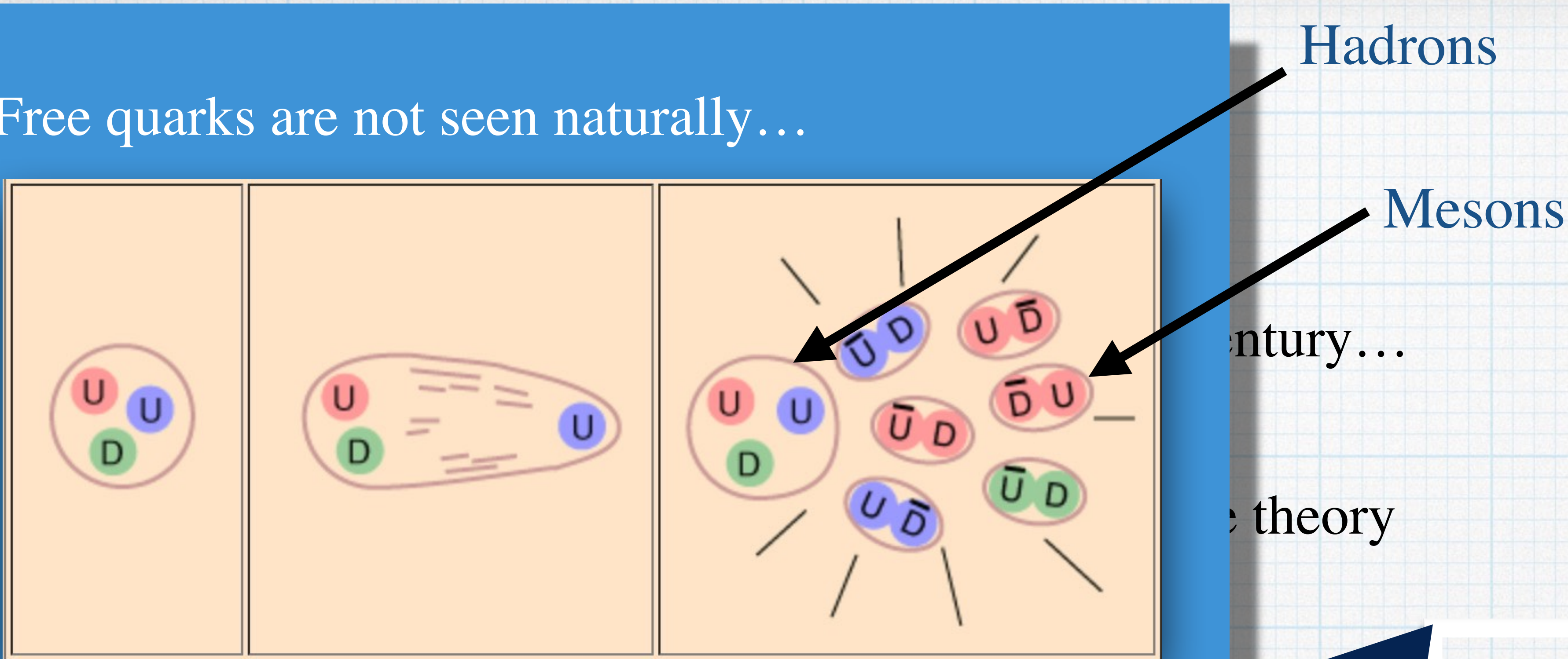

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…gauge
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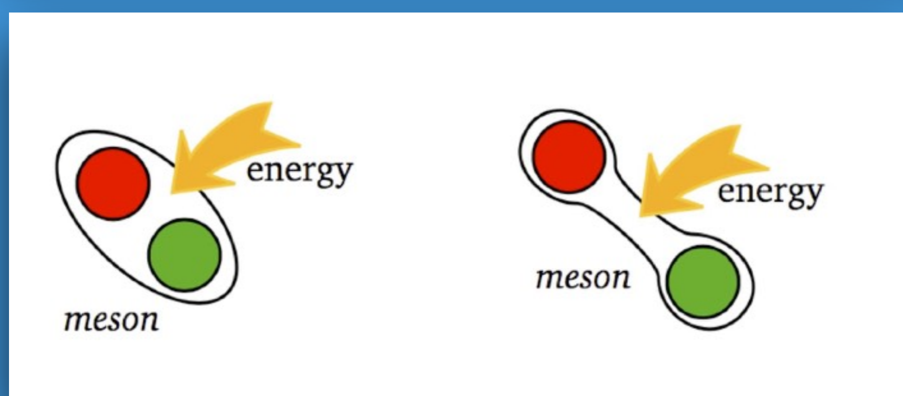

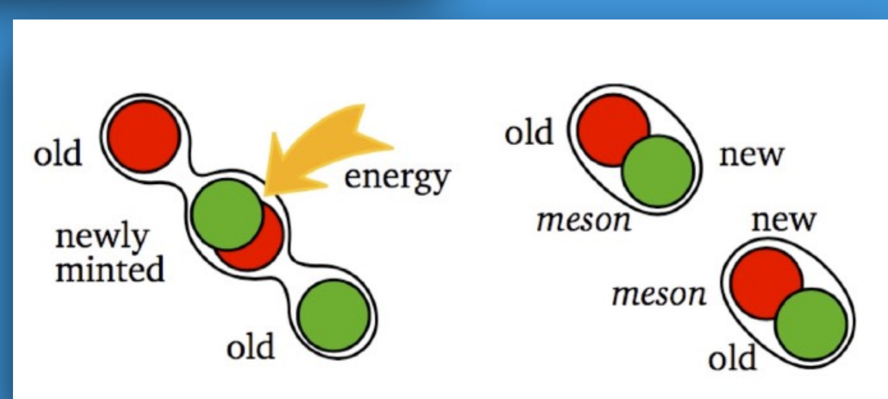

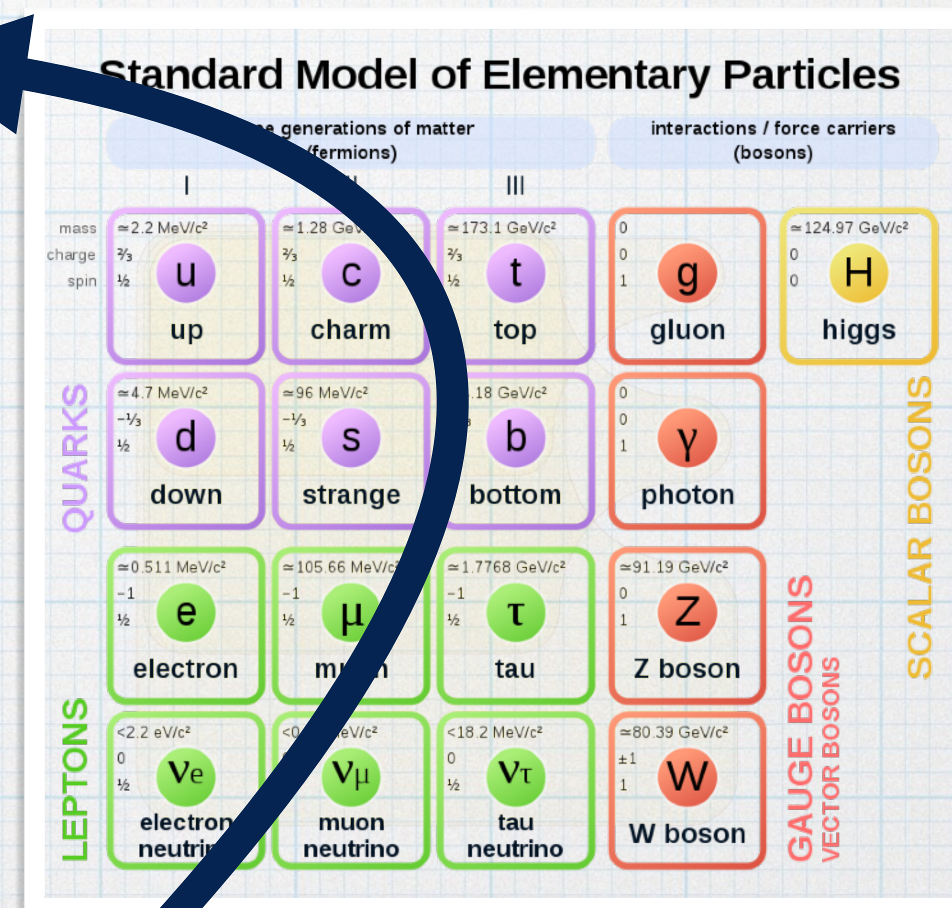

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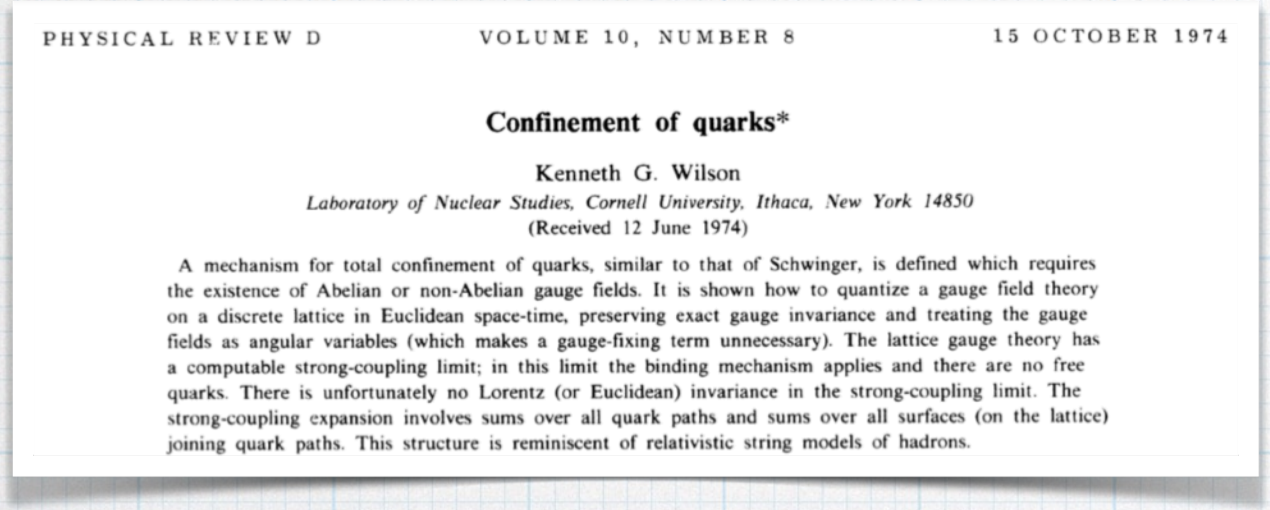
Gauge theories on lattice

Lattice gauge theory (LGT) on Euclidean space-time



PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

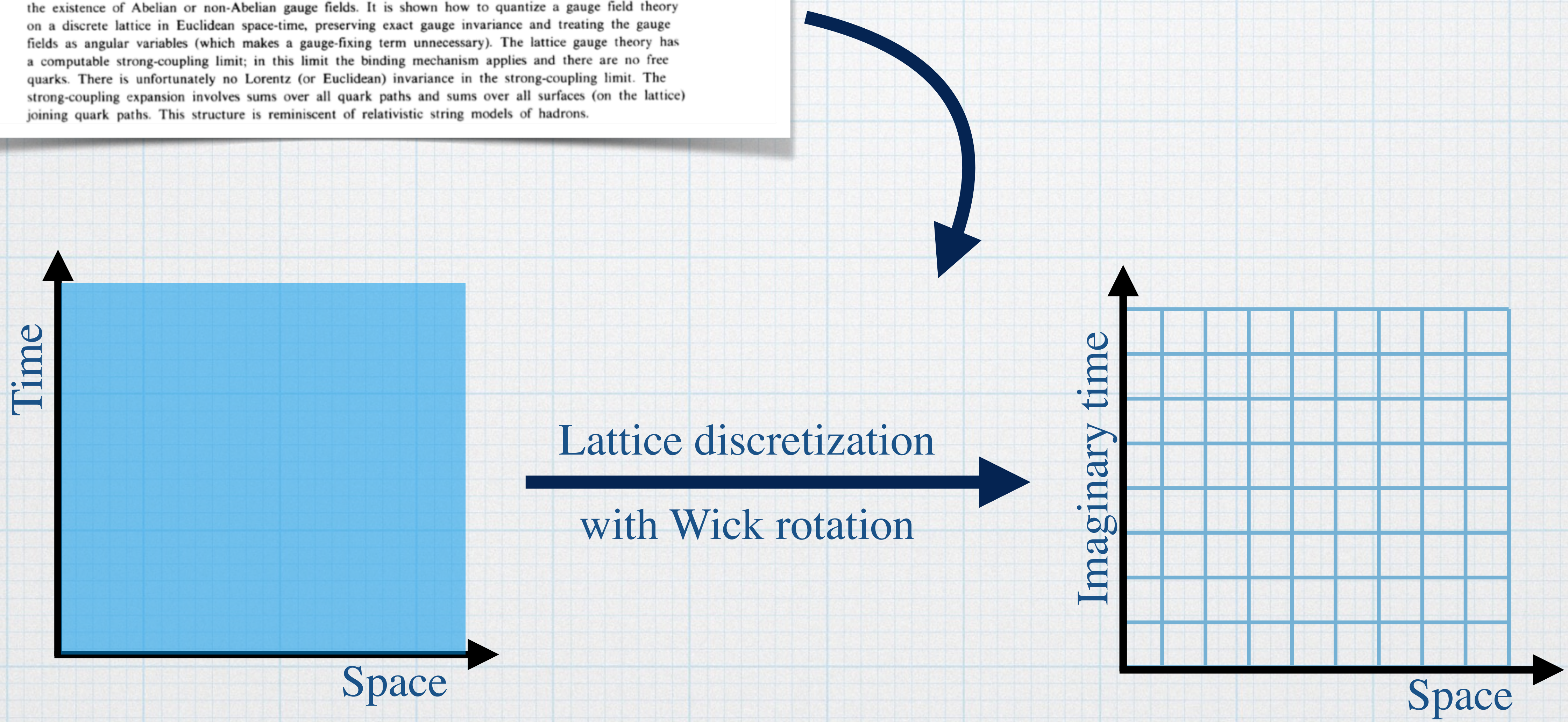
Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.



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Hamiltonian formulation of LGT

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind[†]
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
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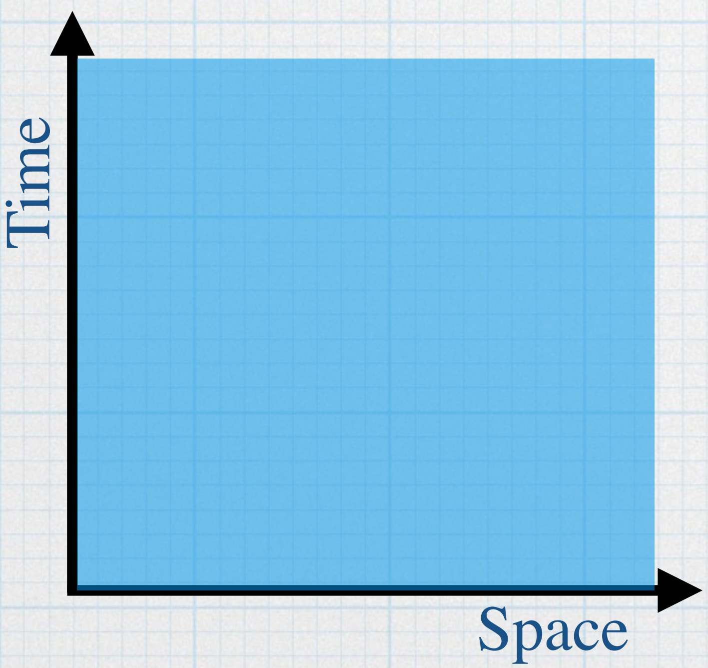
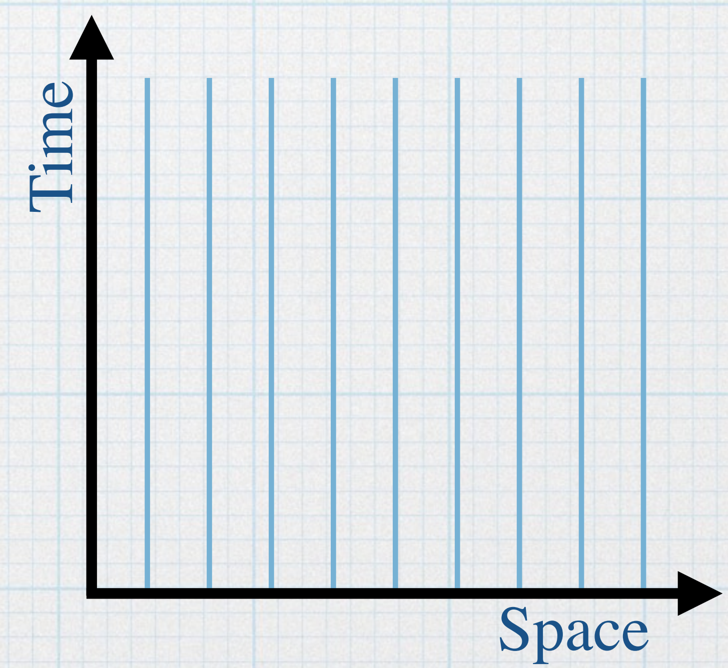
Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.



Space discretization



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Opened up new possibilities to approach non-perturbative limits…

Since then Monte-Carlo simulations have been used to study various facets of high energy physics
on lattice…

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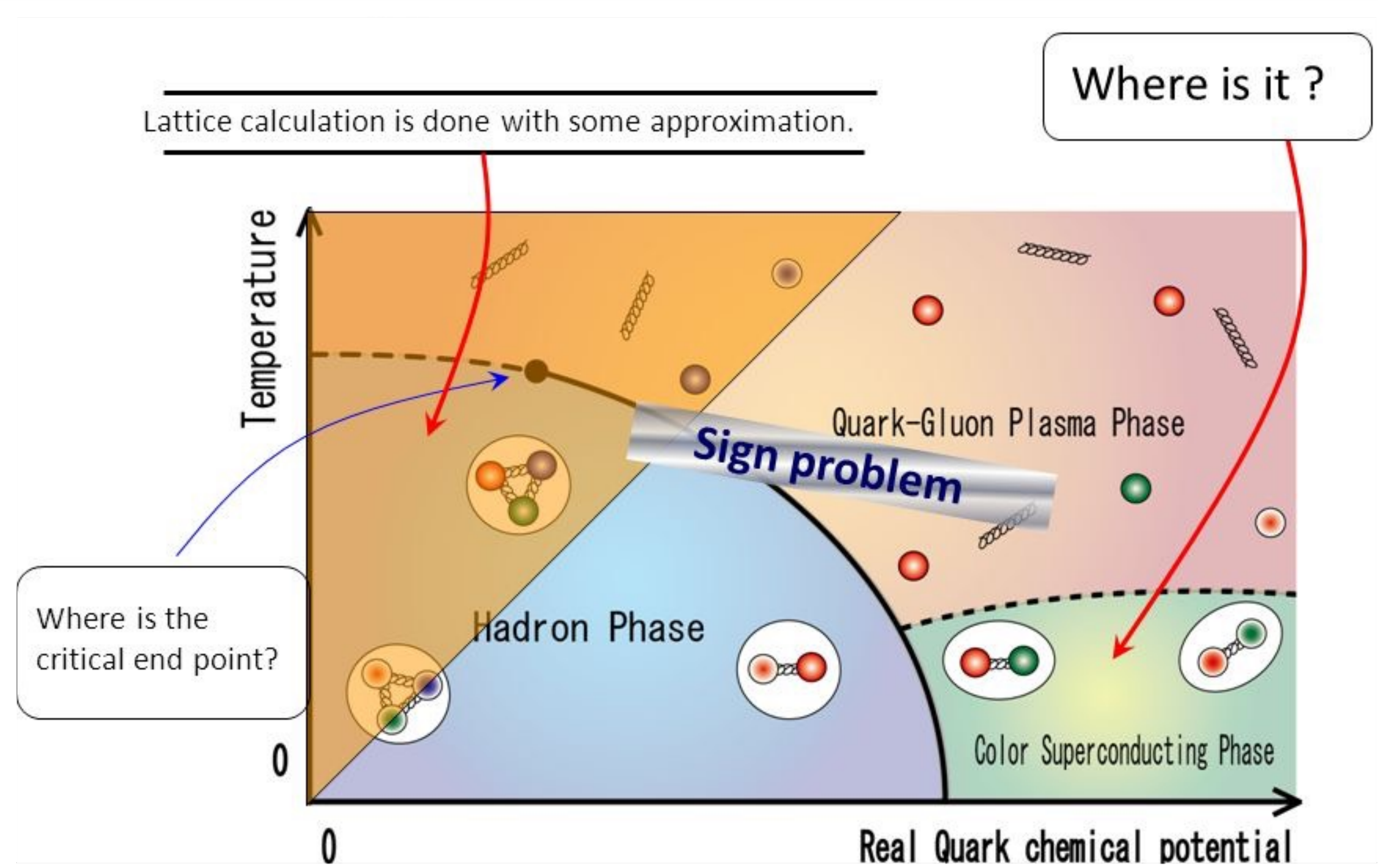

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Gauge theories on lattice

How to gauge a system?
Ans: via ‘minimal coupling’

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Gauge theories on lattice

How to gauge a system?
Ans: via 'minimal coupling'

In continuum…

replace $\partial_\mu :\to \partial_\mu + iqA^a_\mu \tau^a = D_\mu$ in Lagrangian

Gauge bosons Generators of symmetry group

Eg. Dirac Lagrangian

$$\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi :\to \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

+ gauge-invariant interactions among gauge bosons

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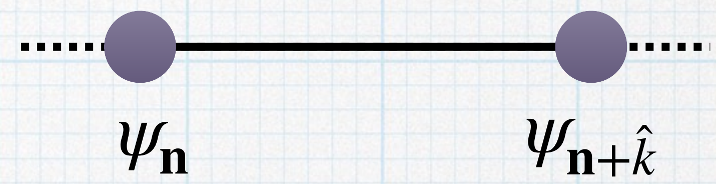
On lattice… Hamiltonian picture…
(tunneling is only n.n.)



Under gauge transformations: $\psi_{\mathbf{n}} \to V_{\mathbf{n}}\psi_{\mathbf{n}}$

$$\psi^\dagger_{\mathbf{n}}\psi_{\mathbf{n}+\hat{k}} \to \psi^\dagger_{\mathbf{n}} V^\dagger_{\mathbf{n}} V_{\mathbf{n}+\hat{k}} \psi_{\mathbf{n}+\hat{k}}$$

Introduction

Gauge theories on lattice

How to gauge a system?
Ans: via ‘minimal coupling’

In continuum…

replace $\partial_\mu :\rightarrow \partial_\mu + iqA^a_\mu \tau^a = D_\mu$ in Lagrangian

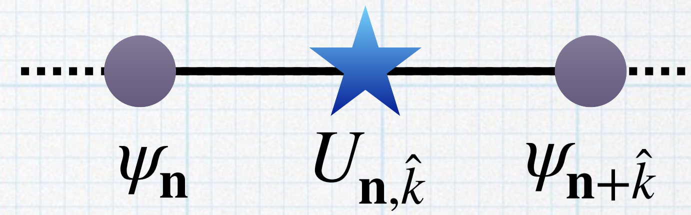
Gauge bosons   Generators of symmetry group

Eg. Dirac Lagrangian

$$\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi :\rightarrow \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

+ gauge-invariant interactions among gauge bosons

On lattice… Hamiltonian picture…
(tunneling is only n.n.)



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Minimal coupling…

$$\psi^\dagger_{\mathbf{n}}\psi_{\mathbf{n}+\hat{k}} :\rightarrow \psi^\dagger_{\mathbf{n}} U_{\mathbf{n},\hat{k}}\psi_{\mathbf{n}+\hat{k}}$$

s.t. under gauge transformations…

$$U_{\mathbf{n},\hat{k}} \rightarrow V_{\mathbf{n}} U_{\mathbf{n},\hat{k}} V^\dagger_{\mathbf{n}+\hat{k}}$$

As a result…

$$\psi^\dagger_{\mathbf{n}} U_{\mathbf{n},\hat{k}}\psi_{\mathbf{n}+\hat{k}} \rightarrow \psi^\dagger_{\mathbf{n}}(V^\dagger_{\mathbf{n}} V_{\mathbf{n}}) U_{\mathbf{n},\hat{k}} (V^\dagger_{\mathbf{n}+\hat{k}} V_{\mathbf{n}+\hat{k}})\psi_{\mathbf{n}+\hat{k}}$$
$$= \psi^\dagger_{\mathbf{n}} U_{\mathbf{n},\hat{k}}\psi_{\mathbf{n}+\hat{k}}$$

+ gauge-invariant kinetic and interaction terms for gauge fields

Introduction

Gauge theories on lattice

Standard Bose-Hubbard model in 1D…

$$\hat{H} = -t\sum_j (\hat{a}_j^\dagger \hat{a}_{j+1} + h.c.) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)$$

On lattice… Hamiltonian picture…
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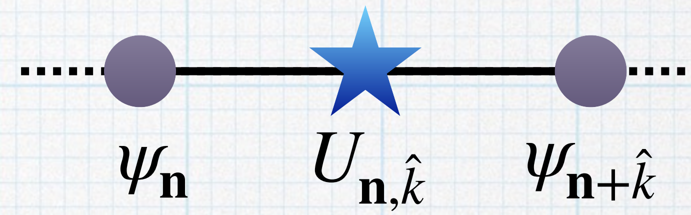

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Making it a gauge theory…

$$\hat{H} = -t\sum_j (\hat{a}_j^\dagger \ \hat{U}_{(j,j+1)} \ \hat{a}_{j+1} + h.c.) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)$$

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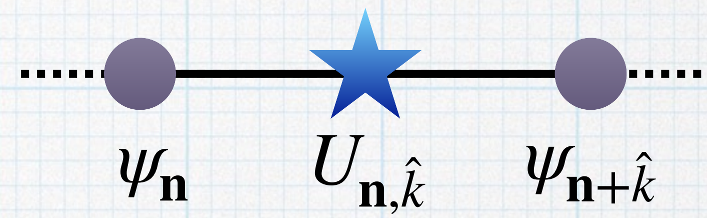

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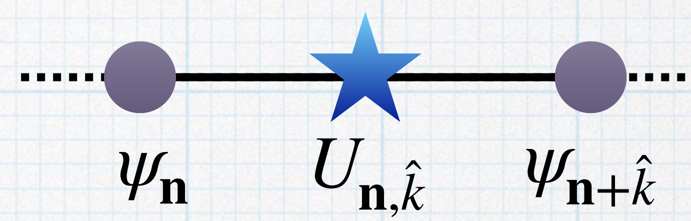
where…

$[\hat{L}, \hat{U}] = -\hat{U}$

$[\hat{L}, \hat{U}^\dagger] = +\hat{U}^\dagger$

On lattice… Hamiltonian picture…
(tunneling is only n.n.)



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Gauge theories on lattice

In present days… form low-energy perspective…

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In present days… form low-energy perspective…

Advancements in quantum simulation
(digital + analog)

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Advancements in quantum simulation
(digital + analog)

nature

First proof of concept

Letter | Published: 22 June 2016

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

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Schwinger mechanism "observed"
for the first time in Lab

Vacuum polarization under **STRONG** E.M. field

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Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

A. Mezzacapo, E. Rico, C. Sabín, I. L. Egusquiza, L. Lamata, and E. Solano
Phys. Rev. Lett. **115**, 240502 – Published 9 December 2015

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Atomic Quantum Simulation of $\mathbf{U}(N)$ and $\mathrm{SU}(N)$ Non-Abelian Lattice Gauge Theories

D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller
Phys. Rev. Lett. **110**, 125303 – Published 21 March 2013

Article | Published: 16 September 2019

Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch & Monika Aidelsburger

Nature Physics **15**, 1168–1173(2019) | Cite this article

4664 Accesses | **4** Altmetric | Metrics

New experimental results and propositions are coming

REPORT

A scalable realization of local U(1) gauge invariance in cold atomic mixtures

Alexander Mil[1,*], Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]

[1]Kirchhoff-Institut für Physik, Heidelberg University, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany.
[2]Institut für Theoretische Physik, Heidelberg University, Philosophenweg 16, 69120 Heidelberg, Germany.
[3]INO-CNR BEC Center and Department of Physics, University of Trento, Via Sommarive 14, I-38123 Trento, Italy.
[*]Corresponding author. Email: block@synqs.org
- Hide authors and affiliations

Science 06 Mar 2020:
Vol. 367, Issue 6482, pp. 1128-1130
DOI: 10.1126/science.aaz5312

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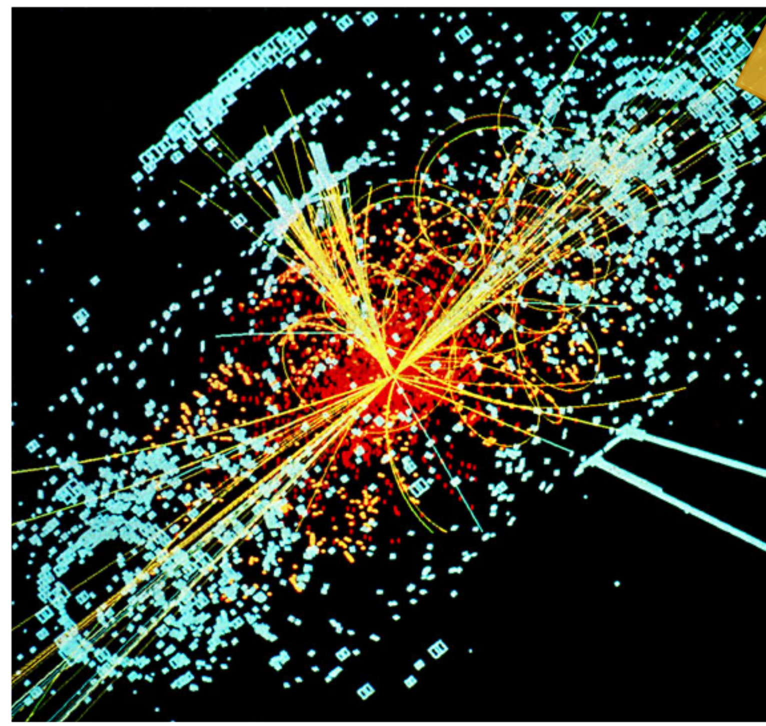
Highlights R

Non-Abeli
Circuits

A. Mezzacap
Phys. Rev. Le



May be in 50 years or so...

REVIEW LETTERS

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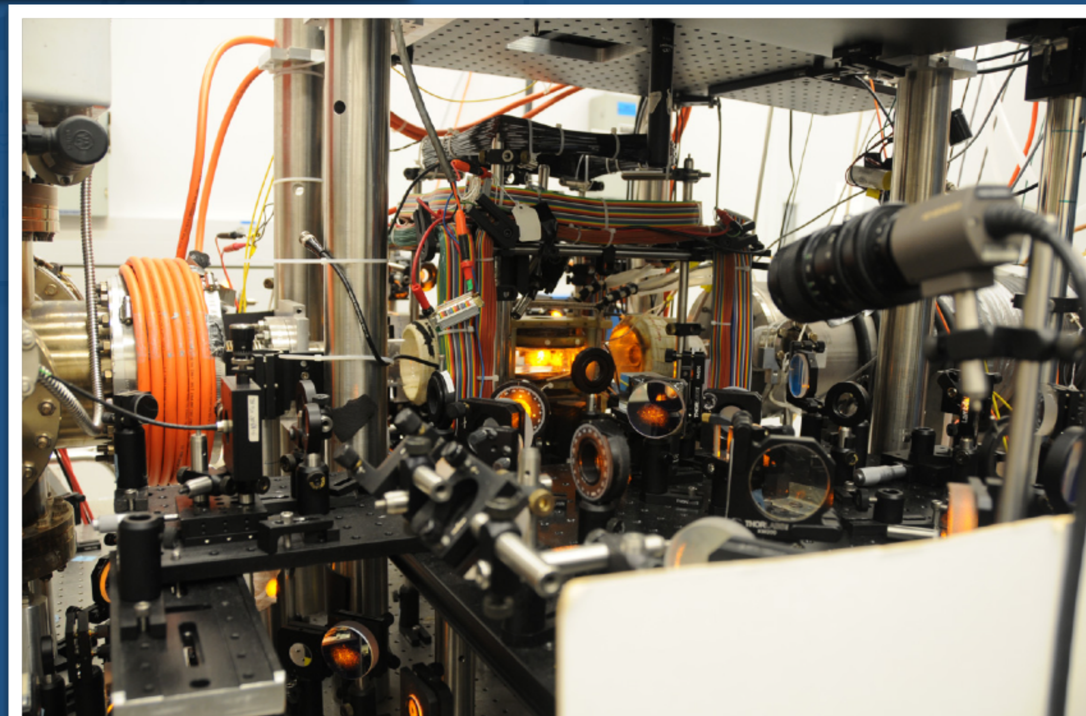

ce in

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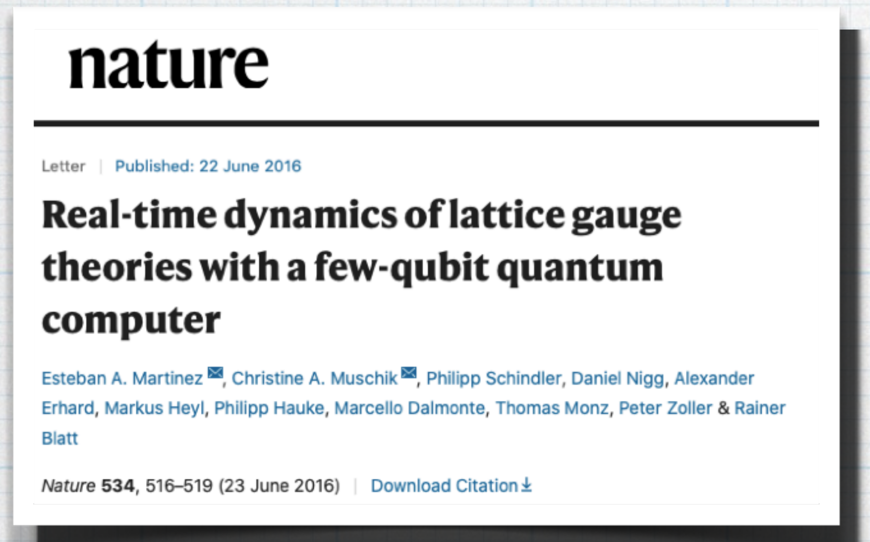

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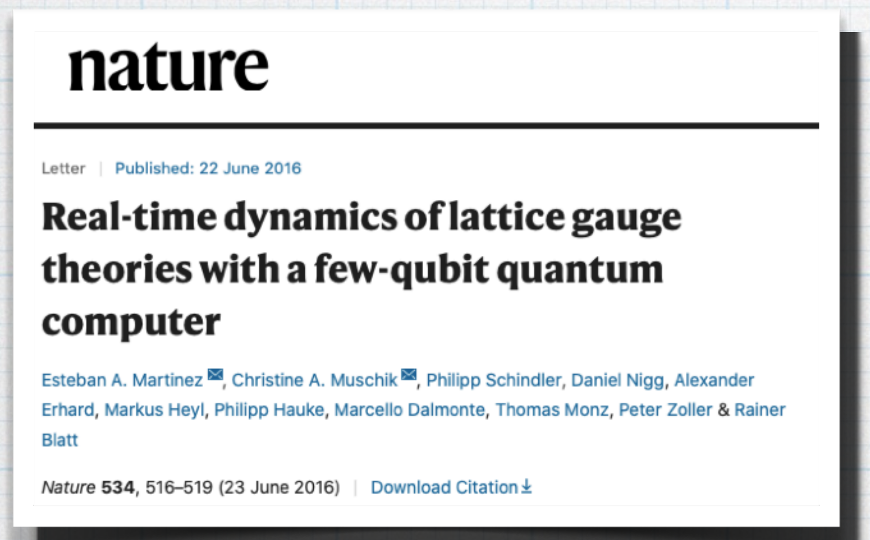
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1. Hamiltonian formulation
2. Access to state or wave-function
3. Entanglement entropy becomes almost free
4. **No sign problem**
5. **Real-time dynamics**

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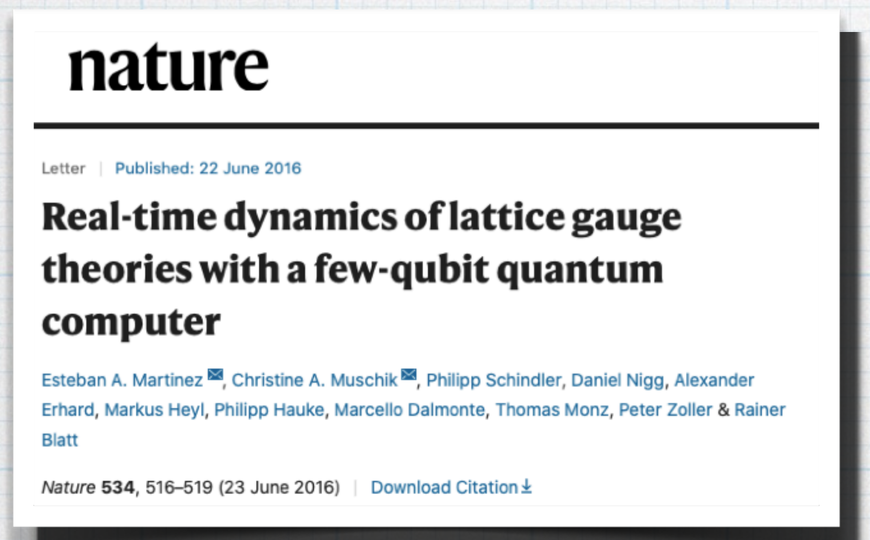
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For fermionic Schwinger model (QED in 1+1 D)…

1. J. High Energy Phys. **11**, 158 (2013)
2. Phys. Rev. A **90**, 042305 (2014)
3. Phys. Rev. Lett. **113**, 091601 (2014)
4. Phys. Rev. D **92**, 034519 (2015)
5. Phys. Rev. D **94**, 085018 (2016)
6. Phys. Rev. X **6**, 011023 (2016)
7. Phys. Rev. X **6**, 041040 (2016)
8. Phys. Rev. D **96**, 114501 (2017)
9. NOT COMPLETE…

Worked perfectly where QMC fails

There are also studies in non-Abelian GT in 1+1

Introduction

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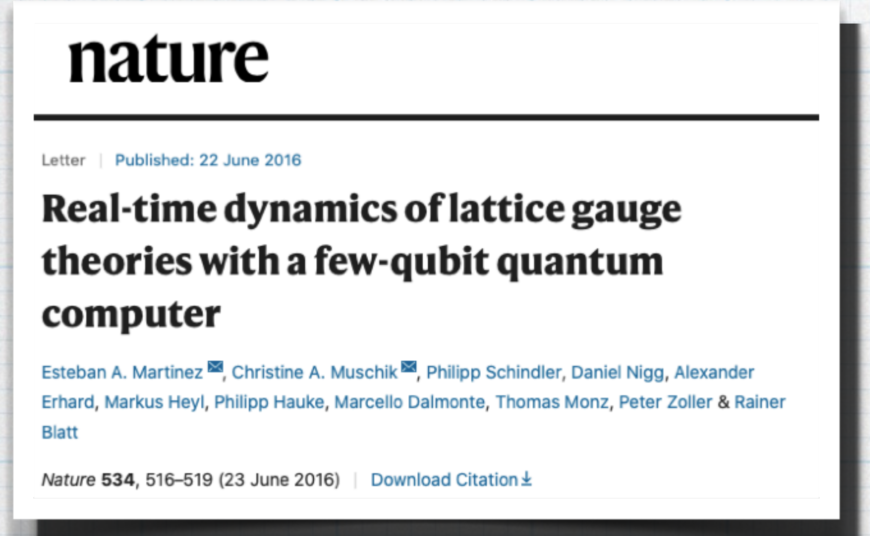
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In 2+1 D…

Some advancement using PEPS

But finite PEPS is computationally very hard

A new way forward ➞ Tensor network + MC

(Zohar, Cirac PRD 2018, and upcoming papers)

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Eur. Phys. J. D (2020) 74: 165
https://doi.org/10.1140/epjd/e2020-100571-8

THE EUROPEAN PHYSICAL JOURNAL D

Colloquium

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

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2 Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 Muenchen, Germany
3 Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21a, 6020 Innsbruck, Austria
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5 LENS and Dip. di Fisica e Astronomia, Università di Firenze, I-50019 Sesto Fiorentino, Italy
6 CNR-INO, S.S. Sesto Fiorentino, I-50019 Sesto Fiorentino, Italy
7 INFN Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, I-50019 Sesto Fiorentino, Italy
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10 Abdus Salam ICTP, Strada Costiera 11, I-34151 Trieste, Italy
11 NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
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13 ICREA, Pg. Lluís Companys 23, 08010 Barcelona, Spain
14 Dipartimento di Fisica e Astronomia "G. Galilei", Università degli Studi di Padova, I-35131 Padova, Italy
15 INFN Istituto Nazionale di Fisica Nucleare, Sezione di Padova, I-35131 Padova, Italy
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19 Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain
20 Department of Physics and Astronomy, Ghent University, Krijgslaan 281, S9, 9000 Gent, Belgium
21 Vienna Center for Quantum Science and Technology, Faculty of Physics, University of Vienna, Boltzmanngaße 5, 1090 Vienna, Austria
22 Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstraße 5, CH-3012 Bern, Switzerland
23 Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK
24 Institute of Theoretical Physics, Jagiellonian University in Krakow, Lojasiewicza 11, 30-348 Kraków, Poland
25 Mark Kac Complex Systems Research Center, Jagiellonian University, Lojasiewicza 11, 30-348 Kraków, Poland

Outline

In two parts…

1. Lattice gauge theories in the age of quantum technologies

   Introduction to the subject

2. **Bosonic Schwinger model out of equilibrium**

   Our work… Phys. Rev. Lett. 124, 180602 (2020)

Tensor network algorithms

1D TN → matrix product states (MPS)

Tensor network algorithms

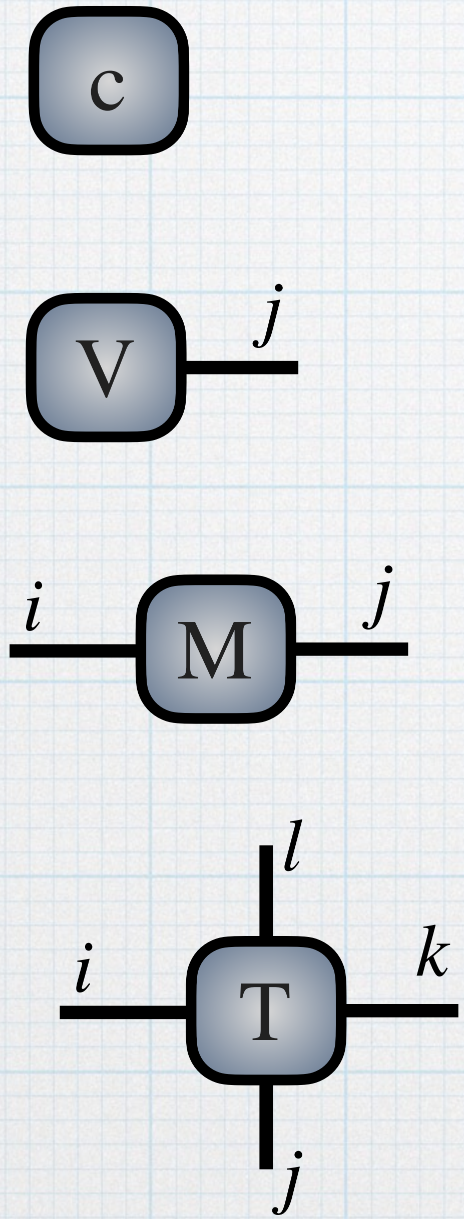
1D TN → matrix product states (MPS)

Scalar, $c\ldots$

Vector, $V_j\ldots$

Matrix, $M_{ij}\ldots$

Tensor, $T_{ijkl}\ldots$



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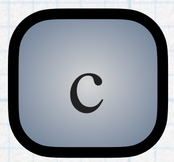
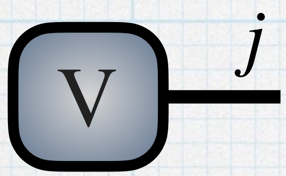
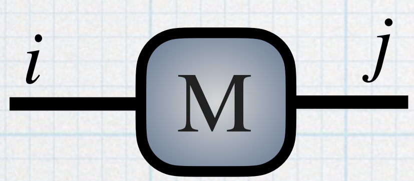


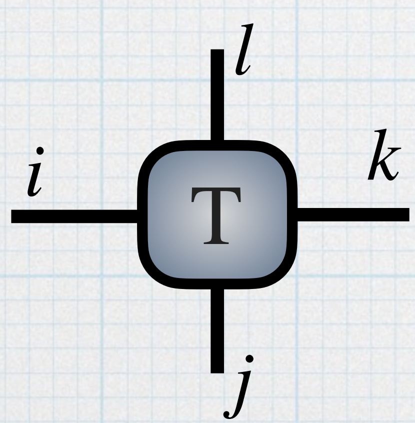
Vector, V_j…



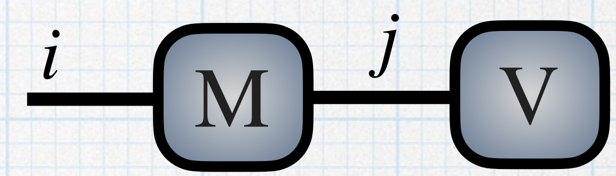
Matrix, M_{ij}…



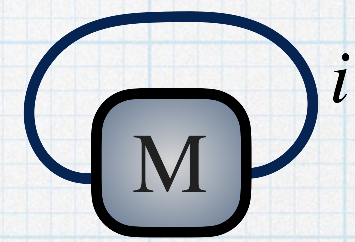
Tensor, T_{ijkl}…



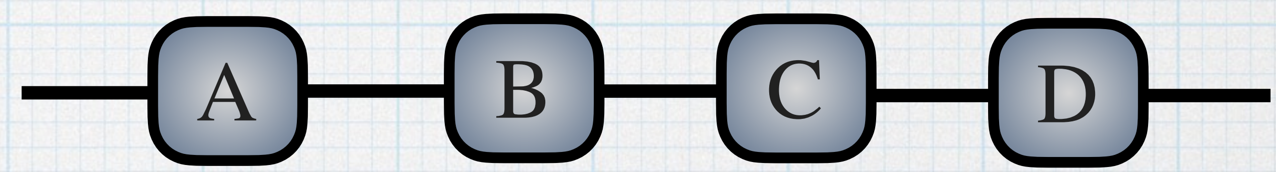
Vector multiplication, $\sum_j M_{ij} V_j$…



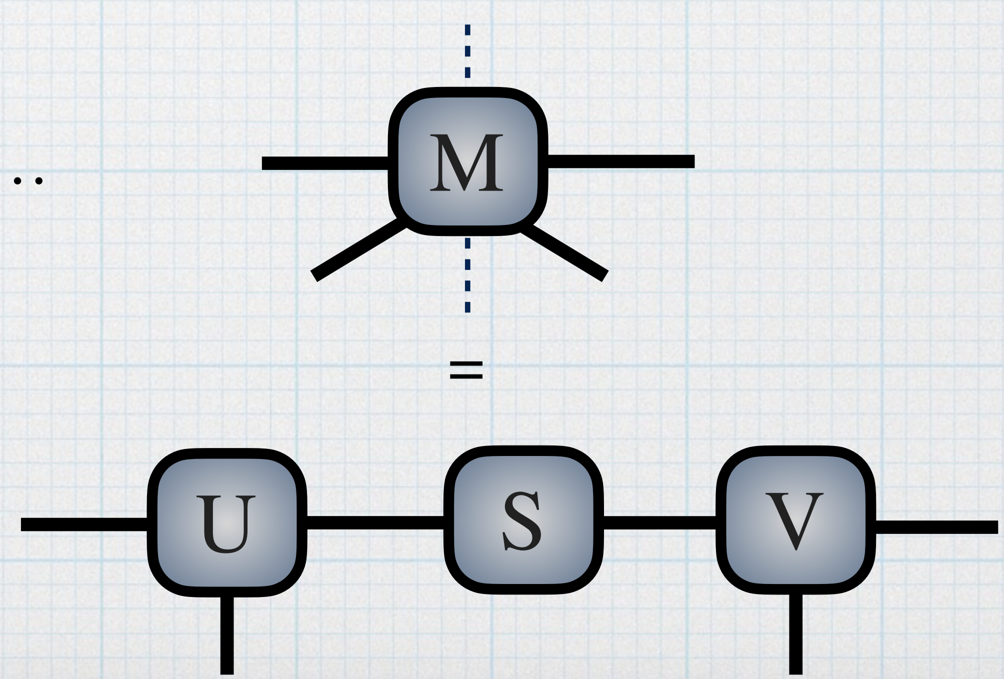
Trace of matrix, $\sum_j M_{ii}$…



$ABCD$…



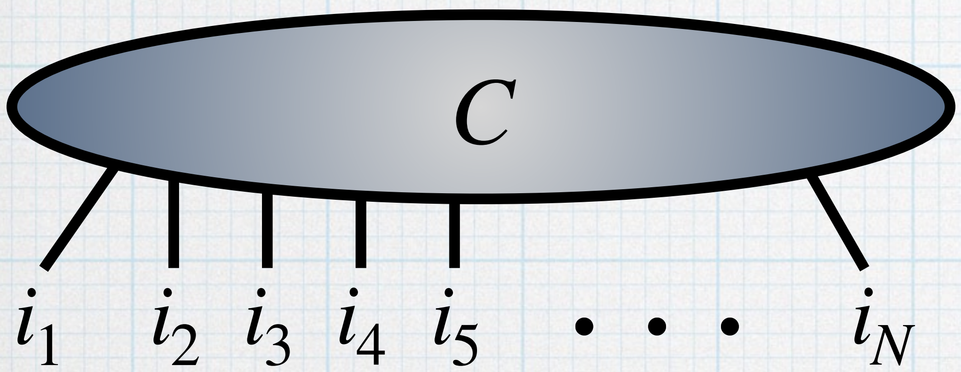
SVD, $M = USV$…



Tensor network algorithms

1D TN → matrix product states (MPS)

$$|\psi\rangle = \sum_{i_1,i_2,i_3..,i_N} C_{i_1 i_2 i_3 \dots i_N} |i_1 i_2 i_3 \dots i_N\rangle$$



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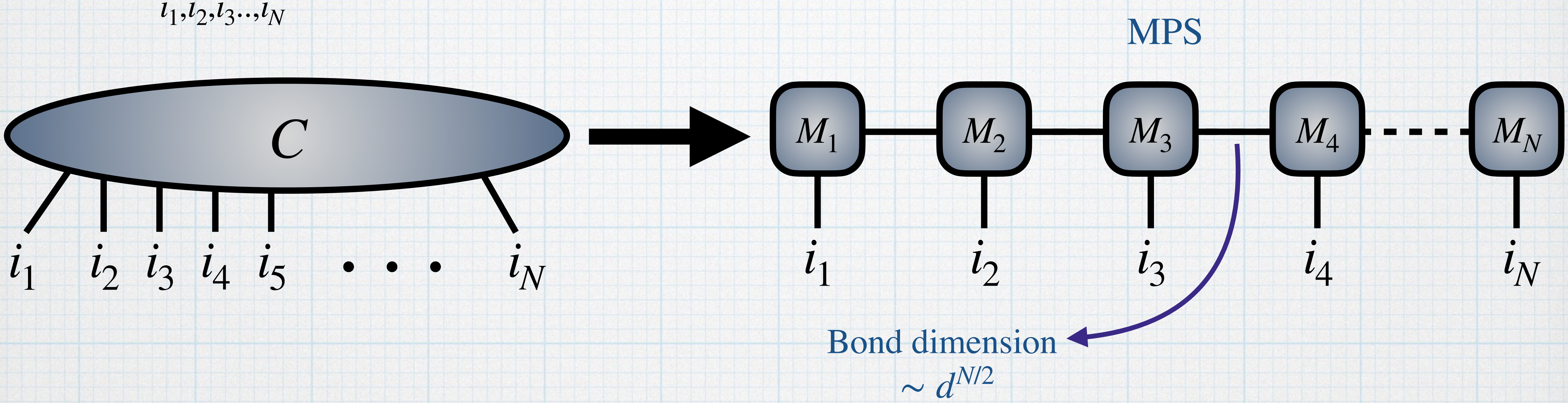

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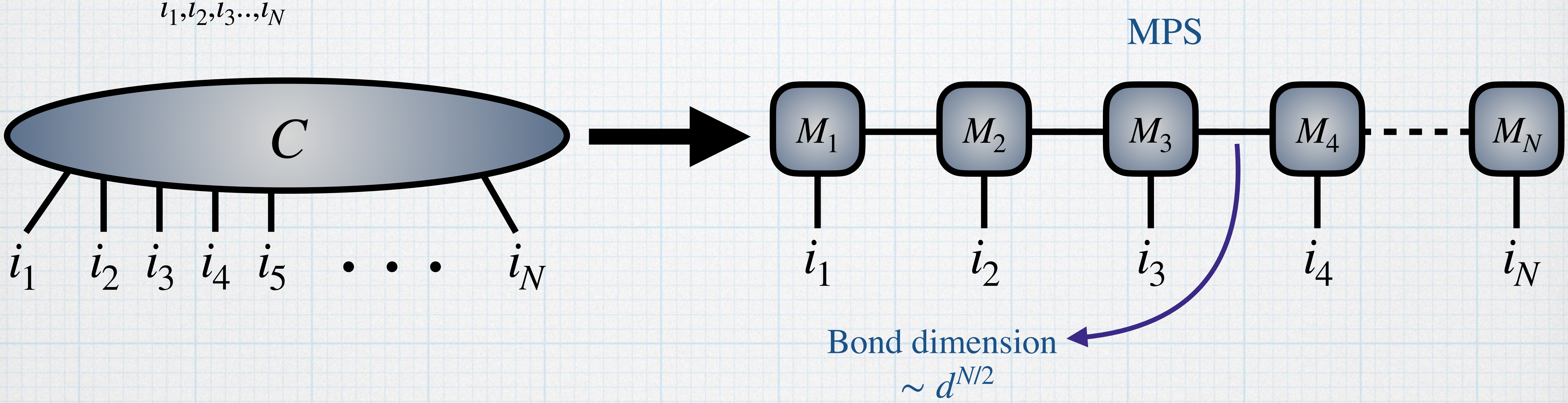

Basic idea: MPS with finite bond dimension as an ansatz for many-body wavefunction

Equilibrium physics

Ground state or low-lying excited states

1. Density matrix renormalization group (DMRG)
2. One-site variational eigenstate search, with or without subspace expansion (colloquially, one-site DMRG)

Out-of-equilibrium dynamics

1. Time-evolving block decimation (TEBD), or tDMRG via Trotter decomposition (~2004)
2. Tangent-space method of time-dependent variational principle (TDVP) (2011 - 2016)

Introduction

The Bosonic Schwinger Model

Our work → Out-of-equilibrium dynamics of bosonic Schwinger model

- Out-of-equilibrium dynamics are hard to simulate numerically. Tensor network shows a way forward.
- Important for understanding of important questions such as the existence of new phases of matter, the presence or absence of thermalization.
- Bosonic Schwinger model: Matter particles are also bo
  1. many works have been done with fermioni
  2. ultra-cold atomic experiments with bosons are easier to handle.

Equilibrium characterization of confinement requires calculation of “Wilson loops”

Not possible in experiments

Goal:

1. Signatures of confinement out-of-equilibrium, easier to experimentally verify confinement.
2. Lack of thermalization and slow dynamics due to confinement.

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  1. many works have been done with fermionic matter but not with bosons
  2. ultra-cold atomic experiments with bosons are easier to handle

Thermalization ⇒ described by only one parameter (temperature, T)
no memory of the initial state

Lack-of-thermalization ⇒ retains memory of the initial state
Very useful in quantum technologies
e.g. engineering quantum memory

Goal:

1. Signatures of confinement out-of-equilibrium, easier to experimentally verify confinement.
2. Lack of thermalization and slow dynamics due to confinement.

The model

The Bosonic Schwinger Model

$$\mathscr{L} = -\left[D_\mu\phi\right]^* D^\mu\phi - m^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu = (\partial_\mu + iqA_\mu)$$

Metric convention → (-1,1,1,1) or (-1,1)

Prescription for discretization:

1. Fix temporal gauge $A_t(x,t) = 0$ in 1+1 dimension
2. Canonical quantization, get the Hamiltonian in continuum
3. Discretize the Hamiltonian on a lattice with spacing a
4. Discretization is such that matter fields sit on lattice sites, gauge fields on bonds
5. Some simplifications

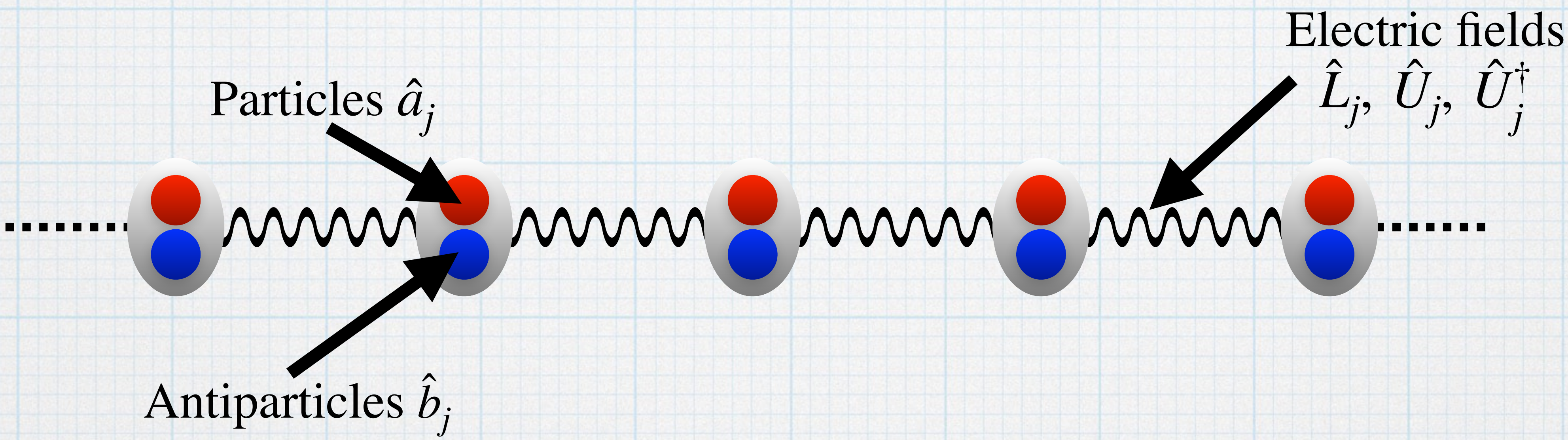
The model

The Bosonic Schwinger Model

$$x = 1/a^2q^2$$

Hamiltonian after discretization…

$$\hat{H} = \sum_j \hat{L}_j^2 + 2\left(x\left((m/q)^2 + 2x\right)\right)^{1/2} \sum_j \left(\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger\right) - \frac{x^{3/2}}{\left((m/q)^2 + 2x\right)^{1/2}} \sum_j \left[\left(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}\right)\hat{U}_j\left(\hat{a}_j + \hat{b}_j^\dagger\right) + \text{h.c.}\right]$$



$\hat{L}_j|l_j\rangle = l_j|l_j\rangle$, with $l_j \in [\ldots, -2, -1, 0, 1, 2, \ldots]$

$\hat{U}_j|l_j\rangle = |l_j - 1\rangle$

$\hat{U}_j^\dagger|l_j\rangle = |l_j + 1\rangle$

$[\hat{L}_j, \hat{U}_j] = -\hat{U}_j$

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The model

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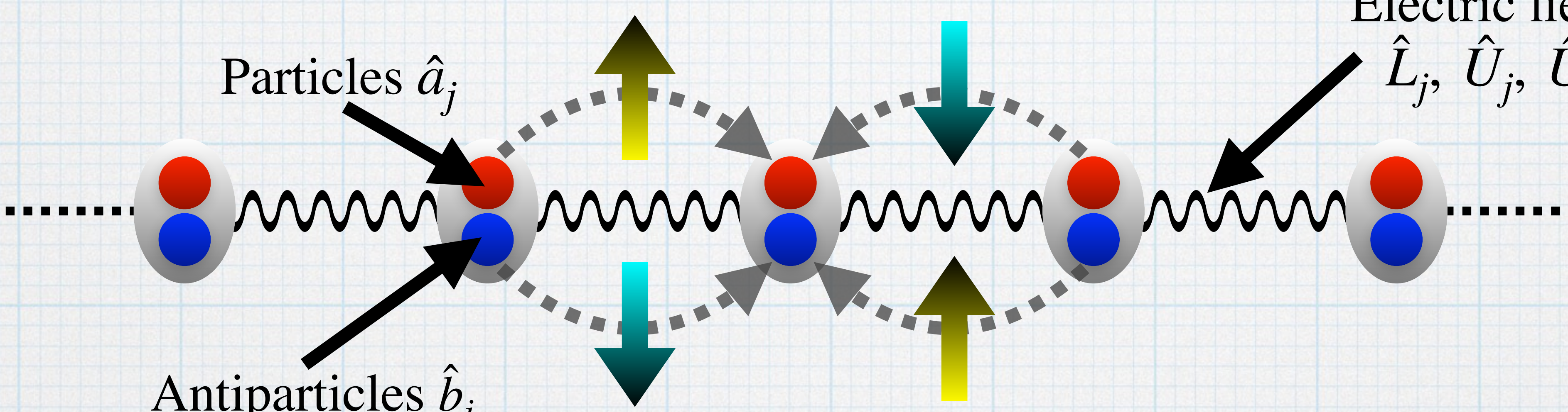

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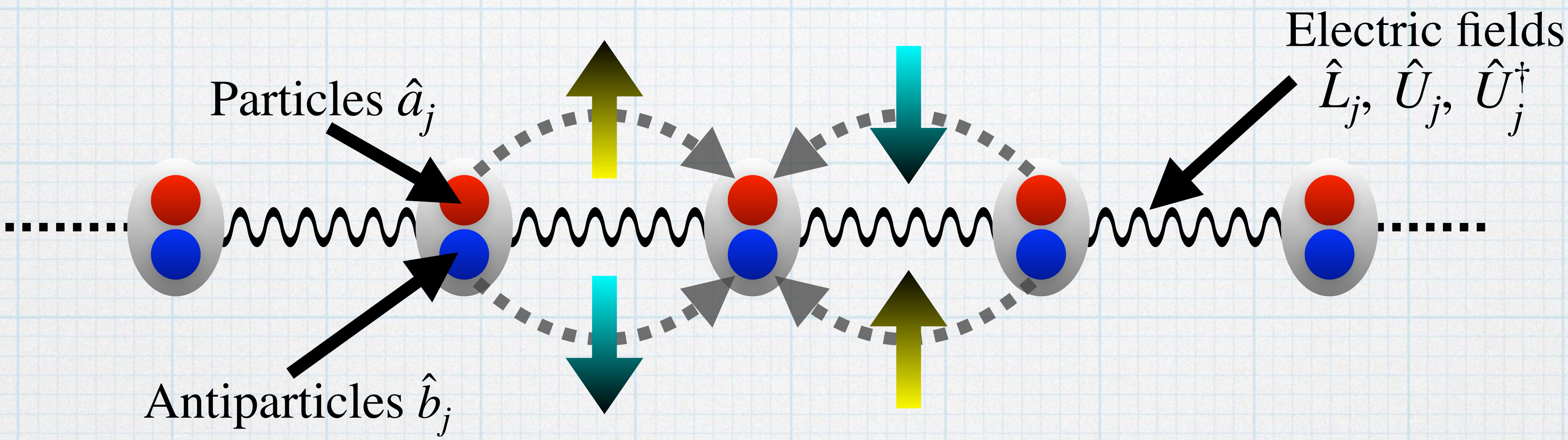

Local $U(1)$ invariance…

$$\hat{a}_j \to e^{i\alpha_j}\ \hat{a}_j$$

$$\hat{b}_j \to e^{-i\alpha_j}\ \hat{b}_j$$

$$\hat{U}_j \to e^{-i\alpha_j}\ \hat{U}_j\ e^{i\alpha_{j+1}}$$

Corresponding Gauss law generators…

$$\hat{G}_j = \hat{L}_j - \hat{L}_{j-1} - \underbrace{\left(\hat{a}_j^\dagger \hat{a}_j - \hat{b}_j^\dagger \hat{b}_j\right)}_{\hat{Q}_j}$$

Dynamical charge: Particle—anti-particle number difference

We restrict ourself to $\hat{G}_j|\psi\rangle = 0$ sector for $\forall j$

The model

Comment on the ground state

Dispersion relation without gauge fields (Klein-Gordon theory)…

Free theory…
Excitations are free 🔴 or 🔵

$$\omega(k) = 2\sqrt{xm^2/q^2 + 2x^2(1 - \cos ka)}$$

$$\lim_{a\to 0} \omega(k) = \sqrt{k^2 + m^2}$$

Gapless in massless scenario

The model

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Confined theory…
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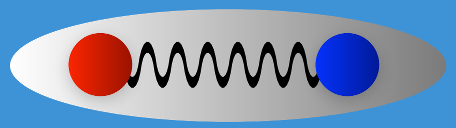

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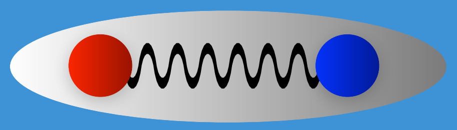

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2. A finite mass-gap is generated due to matter-gauge coupling.
3. Mass-gap, $M/q = (E_1 - E_0)/4\sqrt{x} > m/q$.
4. Extra energy, $E_B/q = M/q - m/q$, arises as binding energy required to tether particle-antiparticle pairs into mesons.

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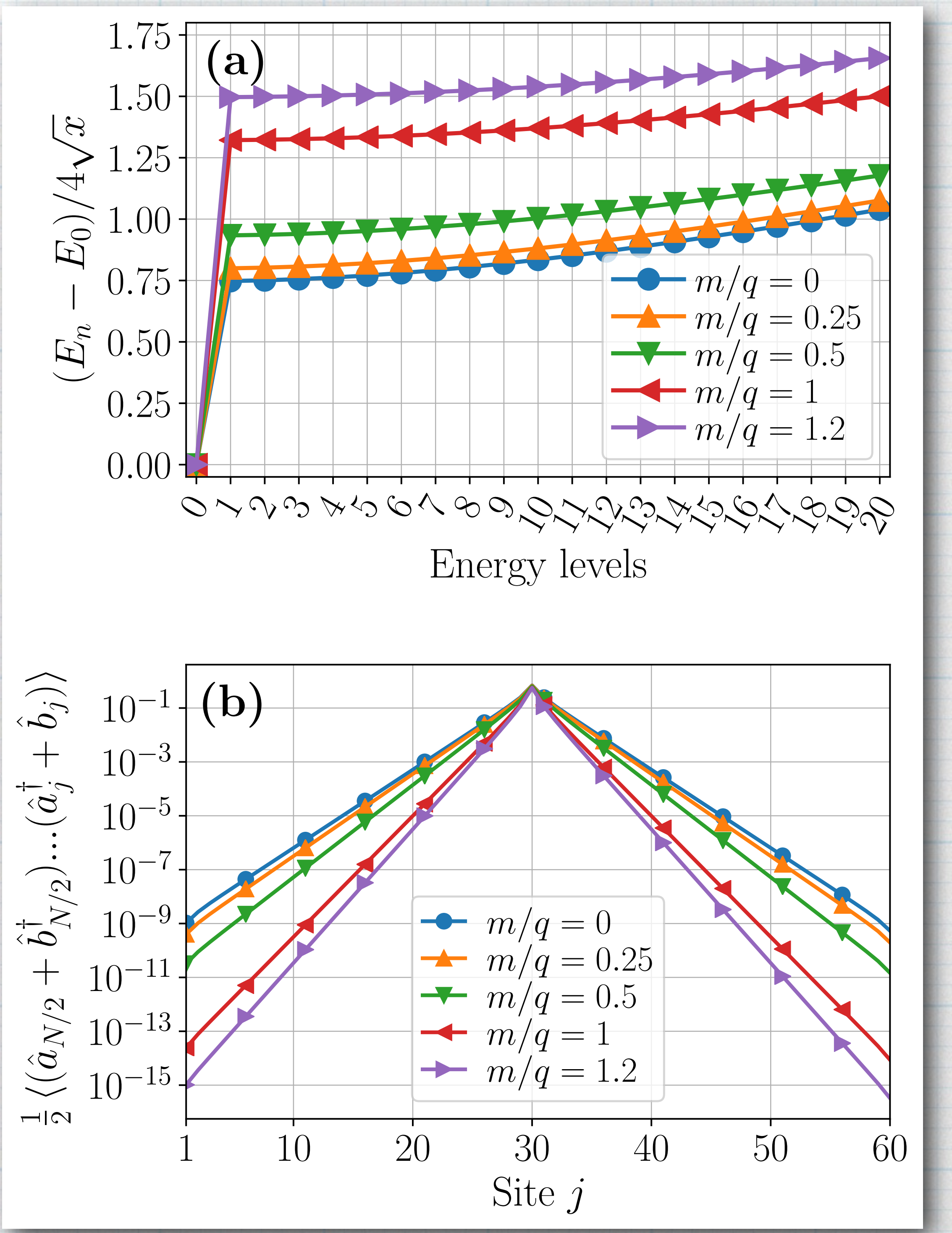

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4. Extra energy, $E_B/q = M/q - m/q$, arises as binding energy required to tether particle-antiparticle pairs into mesons.
5. Ground state is always gapped with finite correlations.

Time evolution

We excite the system out of equilibrium via the non-local operator…

$$\hat{M}_R \equiv \left(\hat{a}^\dagger_{\frac{N}{2}-R} + \hat{b}_{\frac{N}{2}-R}\right)\left[\prod_{j=\frac{N}{2}-R}^{\frac{N}{2}+R} \hat{U}^\dagger_j\right]\left(\hat{a}_{\frac{N}{2}+R+1} + \hat{b}^\dagger_{\frac{N}{2}+R+1}\right)$$

Creates unit opposite charges separated by a distance of $2R$+1 connected by a string of electric field **i.e., an extended meson**

Initial state ➞ $|\psi(t=0)\rangle = \mathcal{N}\hat{M}_R|\Omega\rangle$

with extra energy ➞ $\approx (2R+1) + 4\left(x((m/q)^2 + 2x)\right)^{1/2}$

Time evolution

We excite the system out of equilibrium

$$\hat{M}_R \equiv \Big(\hat{a}^{\dagger}_{\frac{N}{2}-R} + \hat{b}_{\frac{N}{2}}$$

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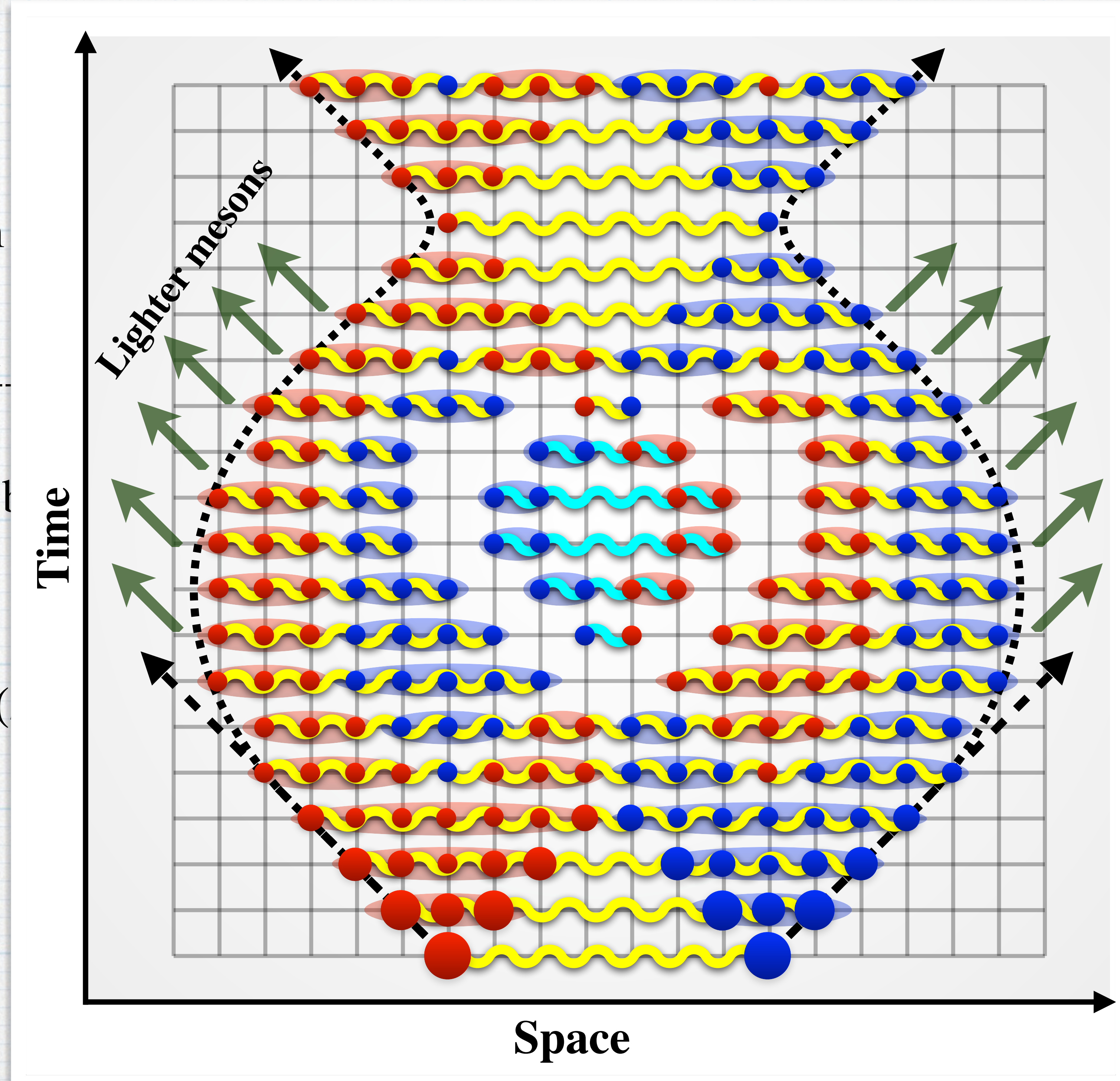
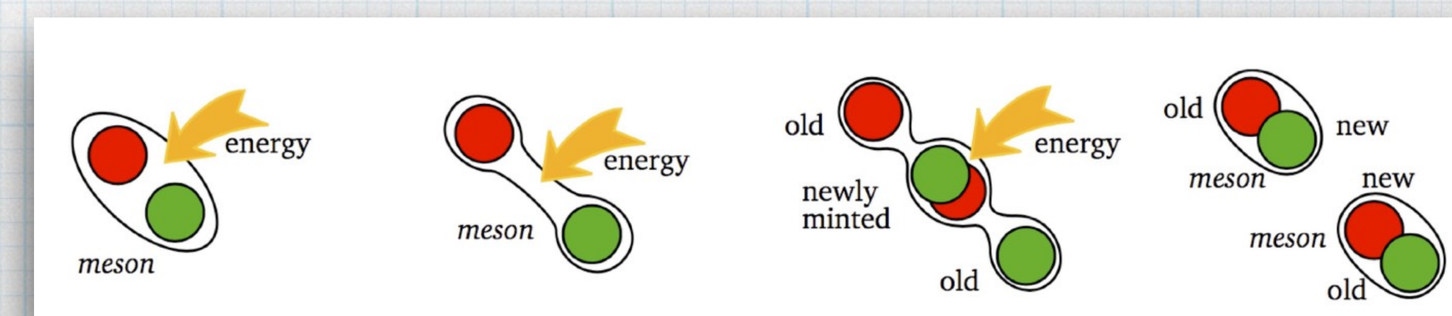
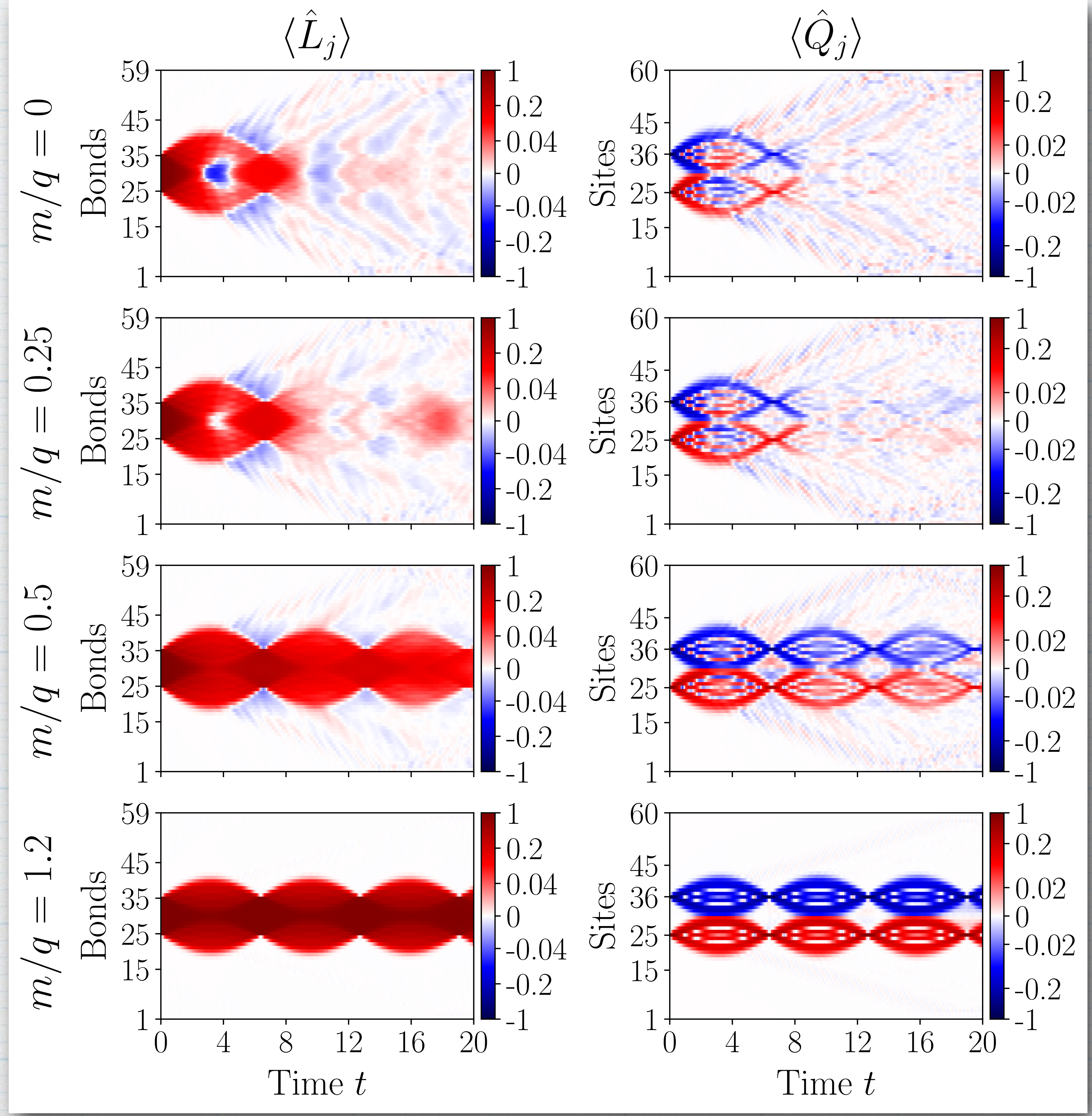


1. **Light-cone bends.**
2. Coherent oscillation of the string.
3. Partial string breaking.
4. String inversion.
5. Radiation of lighter mesons.
6. **Two domains — confined core and deconfined outer region.**
7. Slow depletion of coherent core.



Time evolution

Particle and gauge sector

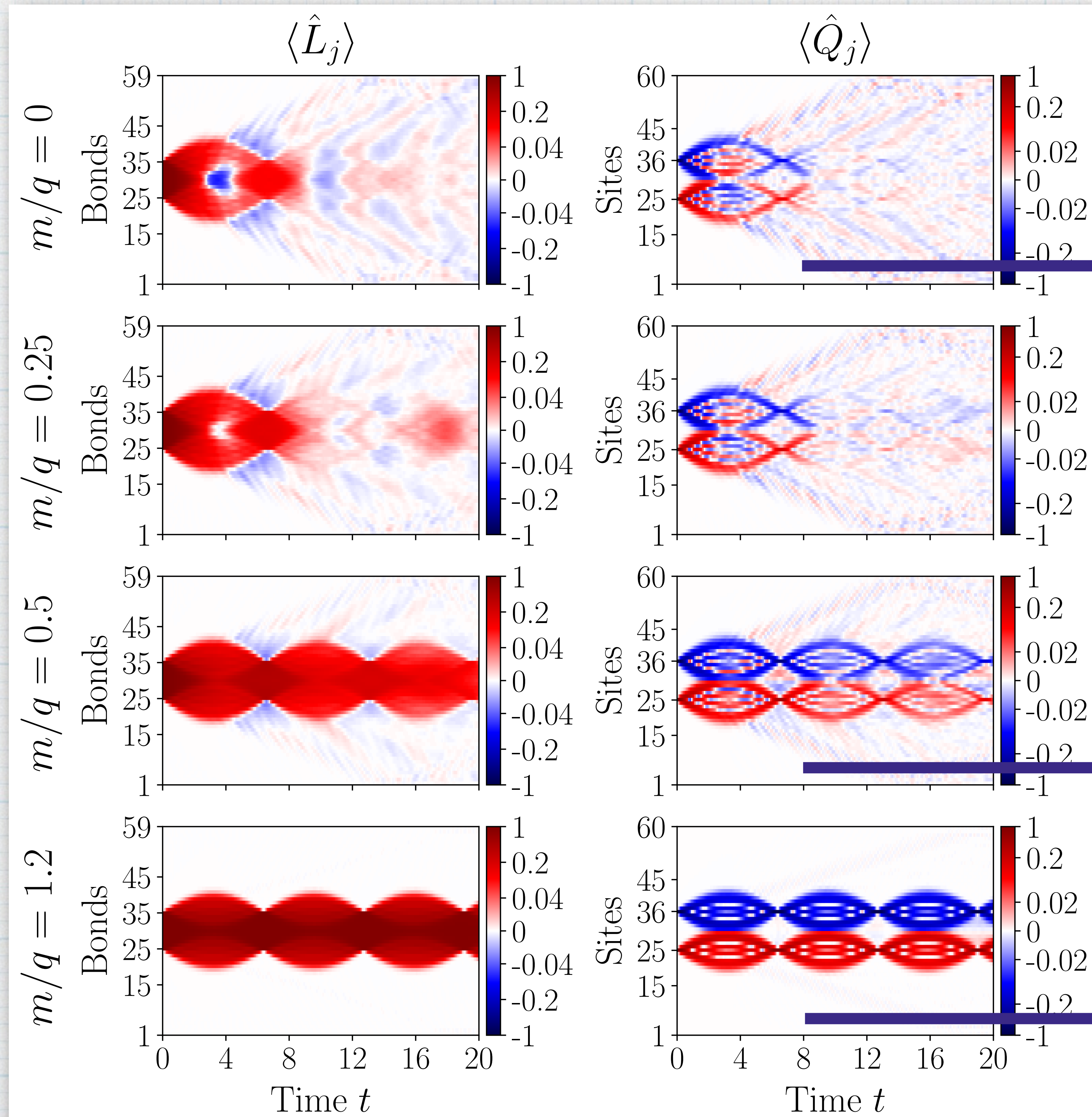


More confinement
less meson radiation

$N = 60$ *sites*, $N\text{-}1 = 59$ *bonds*, $R = 5$

Time evolution

Particle and gauge sector



Concentration of bosons in confined core vanishes at $t \approx 10$

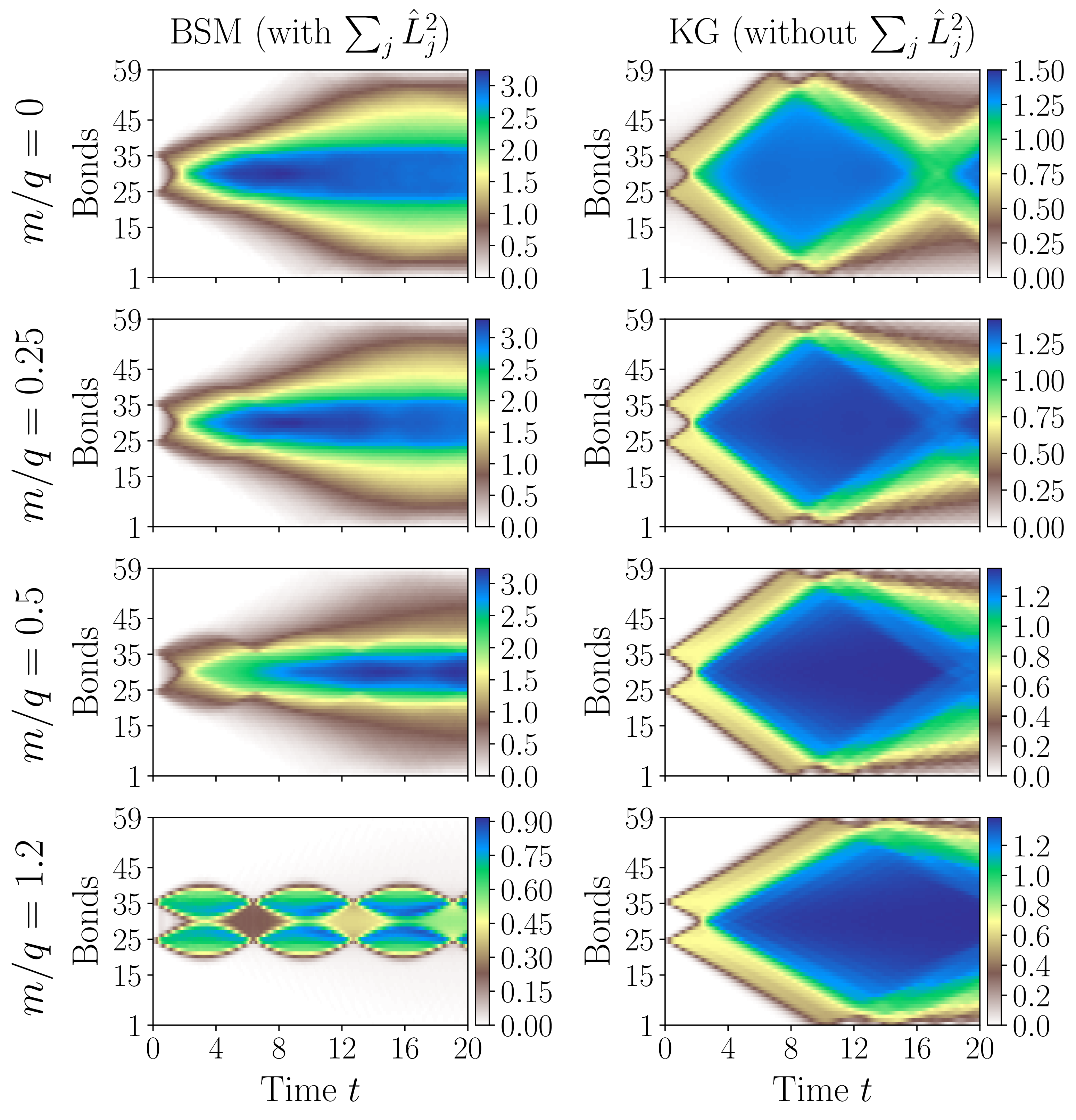
Two domains are clearly visible

Almost perfect periodic oscillation

Time evolution

Entanglement dynamics

N = 60 sites, N-1 = 59 bonds, R = 5



In the Klein-Gordon scenario:

1. Entanglement spreads ballistically from the beginning.
2. Memory effect apparently disappears
3. Thermalization in the generalized sense.

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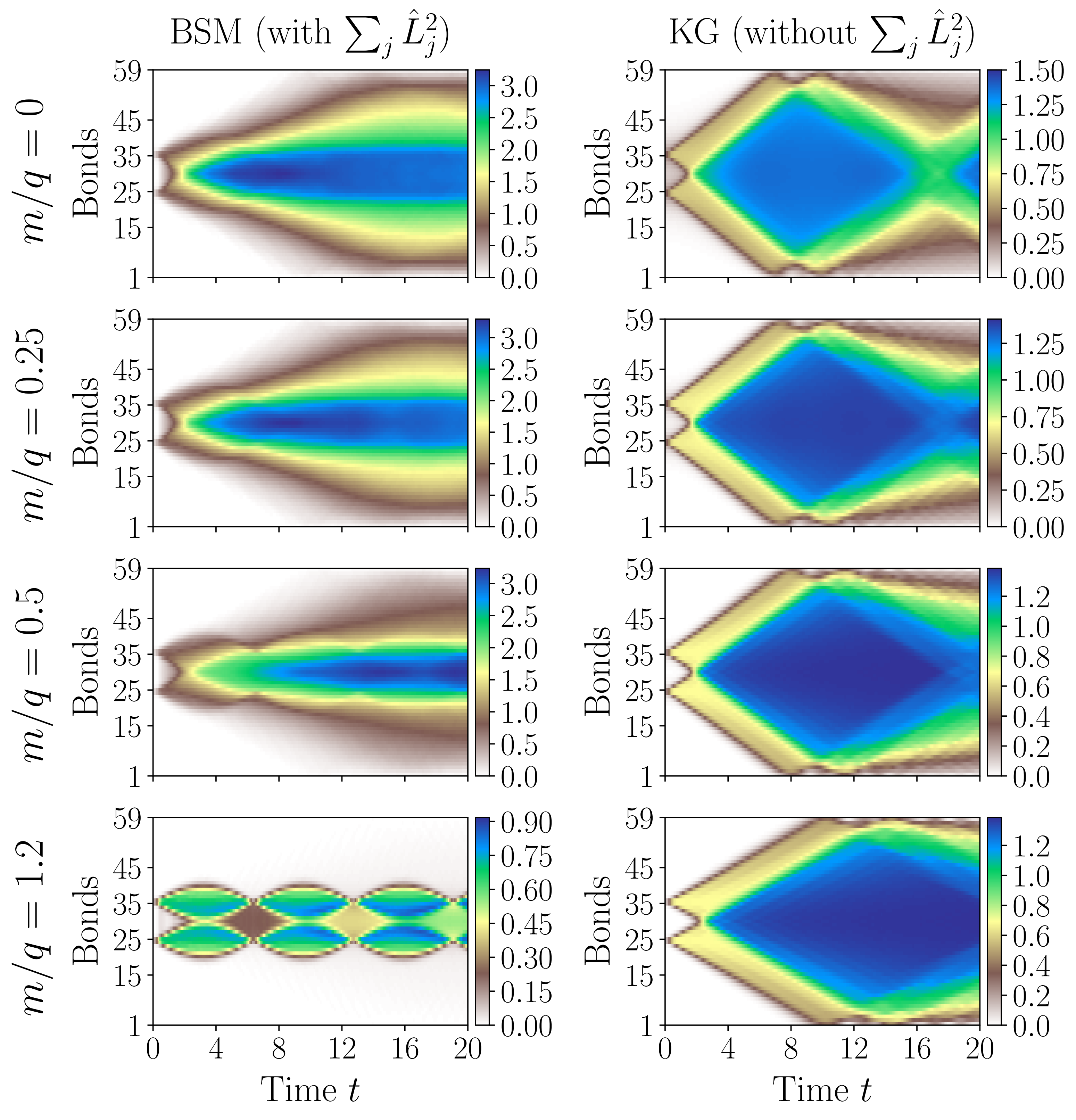
Time evolution

Entanglement dynamics



In the Klein-Gordon scenario:

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In the bosonic Schwinger model:

1. Initial spreading of entanglement slows down.
2. Starts to spread ballistically in correspondence with the radiation of lighter mesons (for lower masses).
3. Entanglement stays concentrated in the confined core, even long after the accumulation of bosons disappears.
4. Strong memory effect.

Time evolution

Entanglement dynamics: Classical vs. distillable

Due to global *U(1)* symmetry…

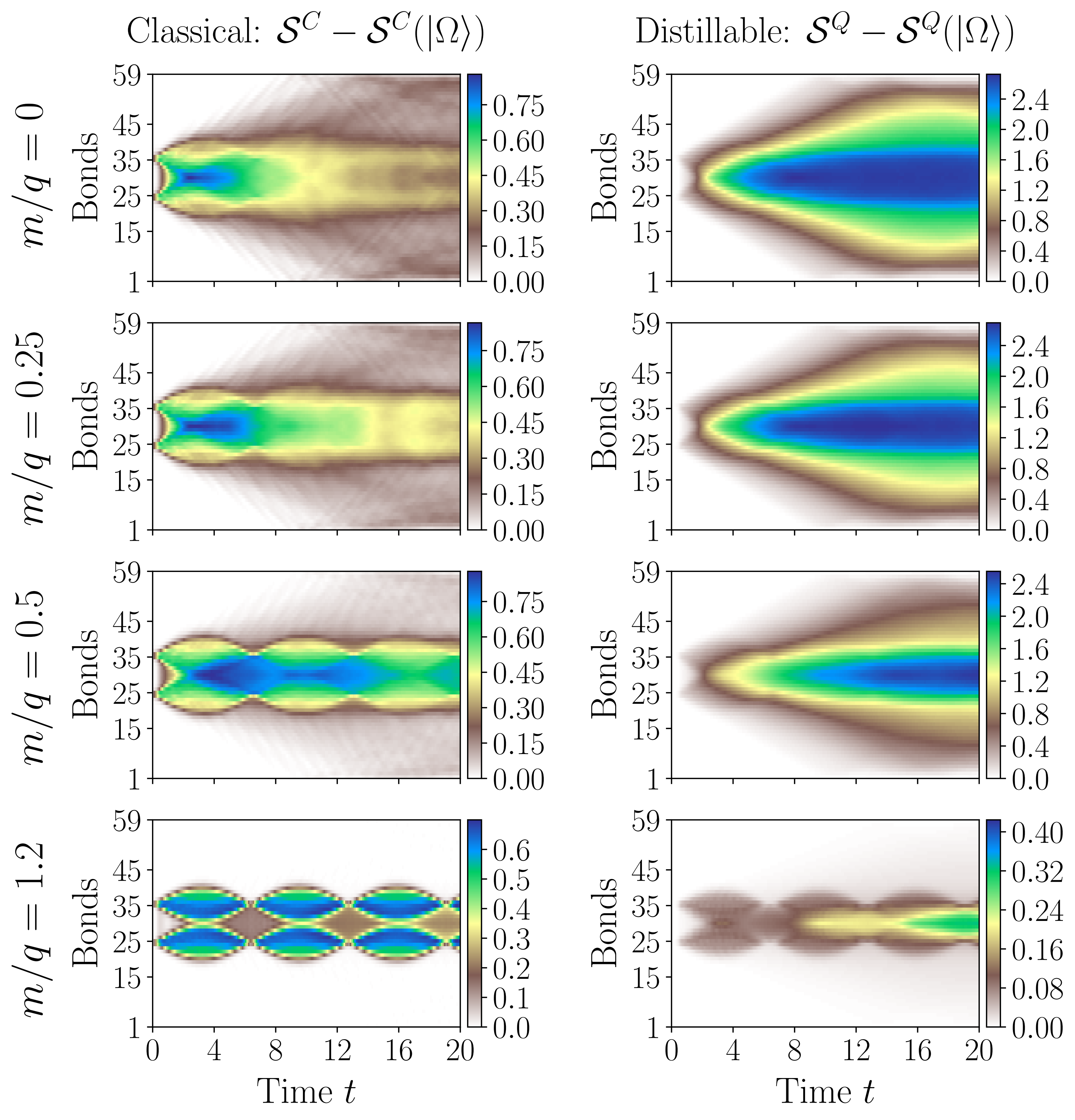
$$\rho = \bigoplus_Q \tilde{\rho}_Q = \bigoplus_Q p_Q \, \rho_Q$$

with $p_Q = \mathrm{Tr}\left[\tilde{\rho}_Q\right]$ and $\rho_Q = \tilde{\rho}_Q / p_Q$

$$\mathcal{S}(\rho) = \underbrace{-\sum_Q p_Q \ln p_Q}_{\mathcal{S}^C \text{ (classical)}} + \underbrace{\sum_Q p_Q \mathcal{S}(\rho_Q)}_{\mathcal{S}^Q \text{ (distillable)}}$$

Time evolution

Entanglement dynamics: Classical vs. distillable



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The classical part of the entropy remains *sharply* confined to the confined core, thereby demarcating confined domain from the deconfined one.

Lack of thermalization

Thermalization

$\langle \hat{O}(\psi(t)) \rangle \to \bar{O}_{microcann.}$ as $t \to \infty$… Described by only one parameter (T)… no memory

$\mathcal{S}(t)$ should grow proportional to the bipartition size for sufficiently long t

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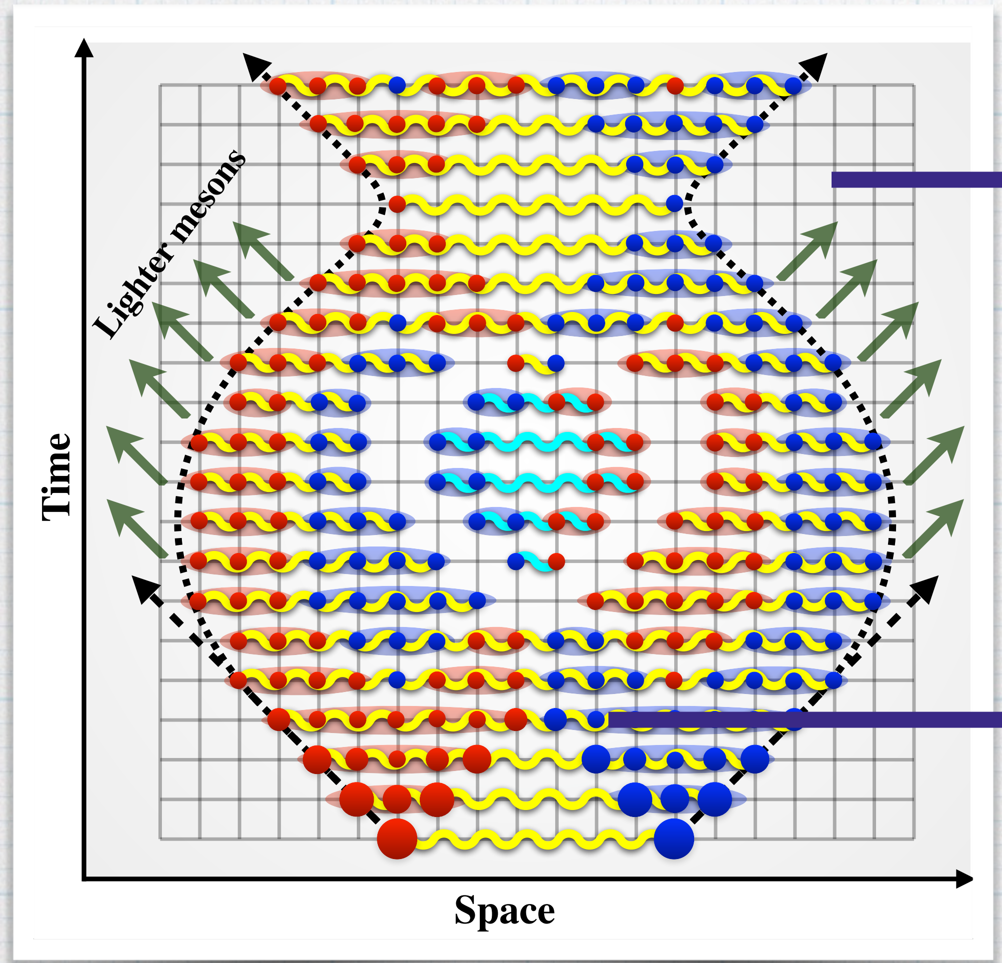
Expectation…



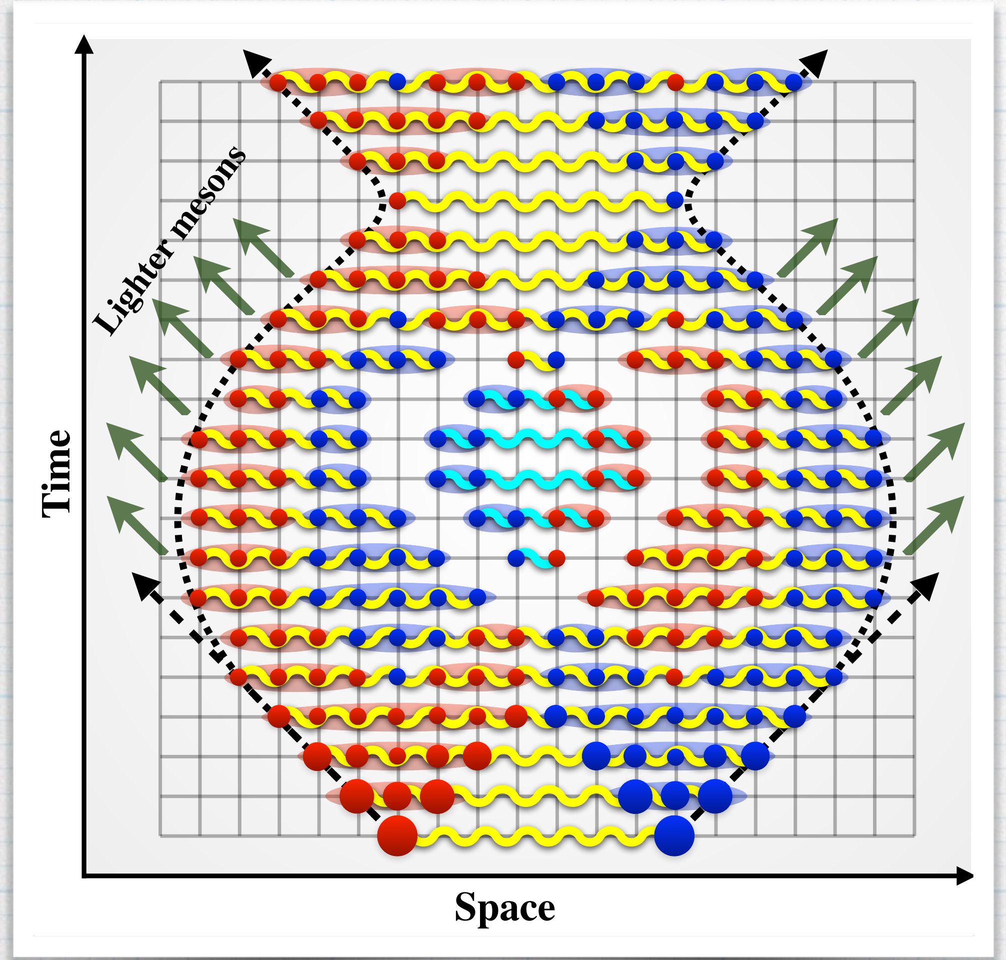
Deconfined domain.
Populated by free lighter mesons.
Should thermalize.
Should show volume-law of entropy.

Confined domain.
Coherent oscillations.
Memory effect.
Should remain **non**-thermal.
Entropy should **not** grow proportional to the bipartition size, but slower.

Lack of thermalization

N = 60, 90, 120 sites, with R = N/10

Extensive energy in the initial state: required for thermalization



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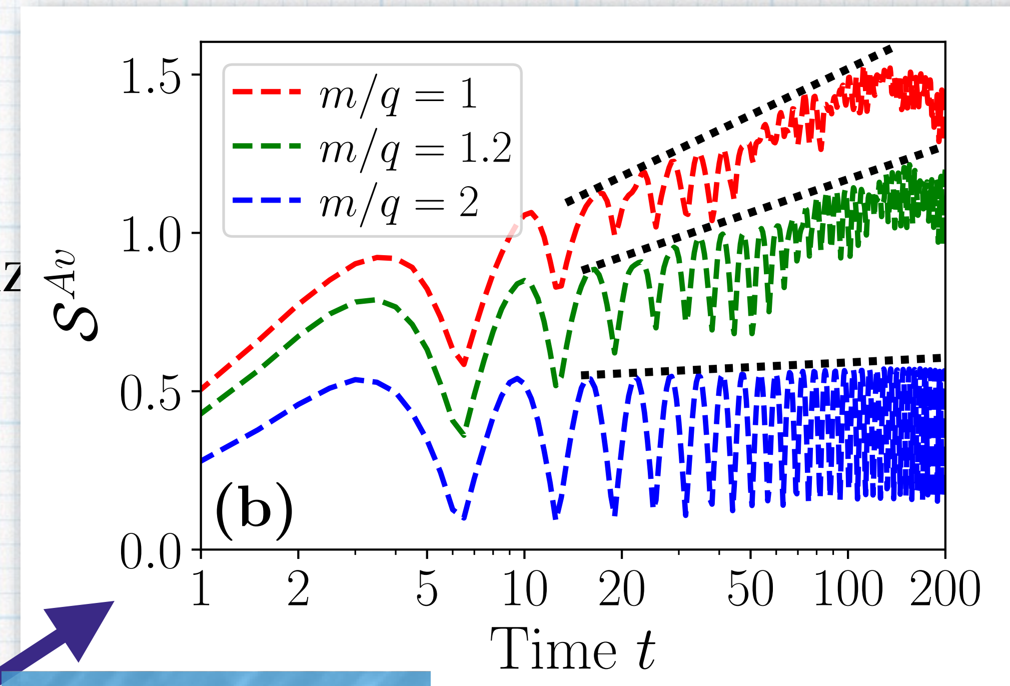
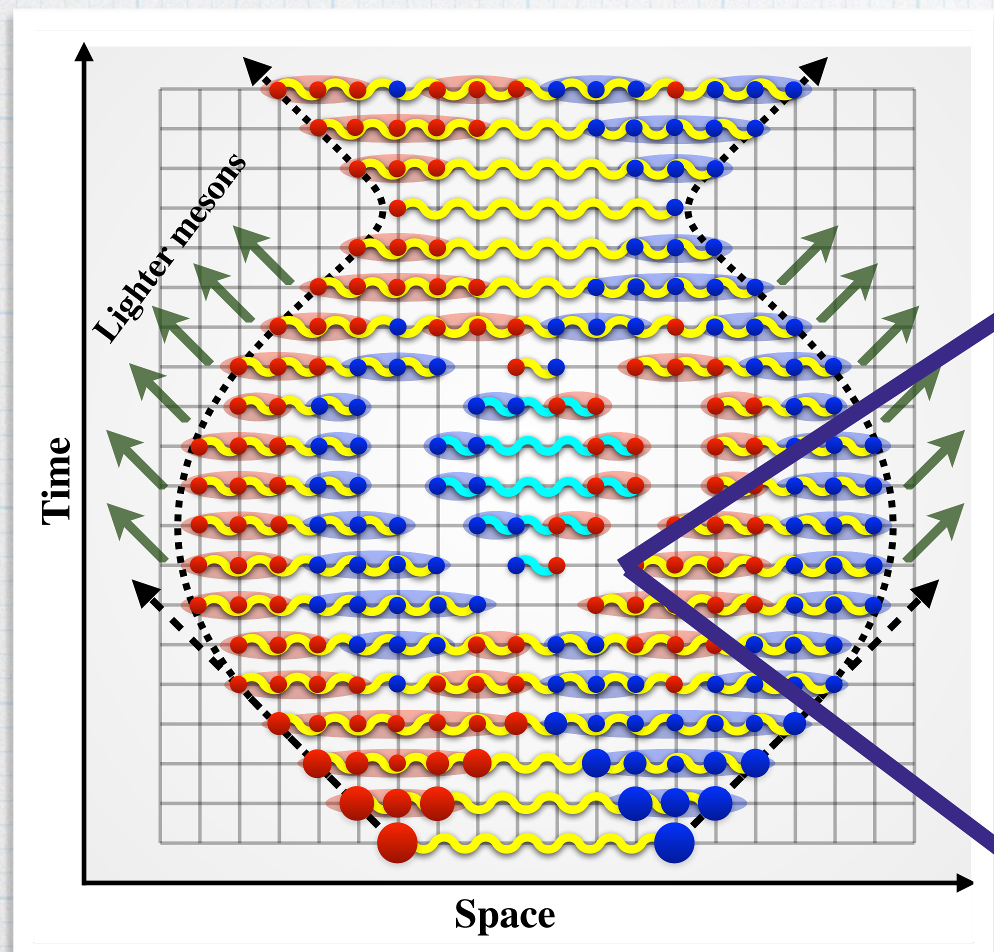
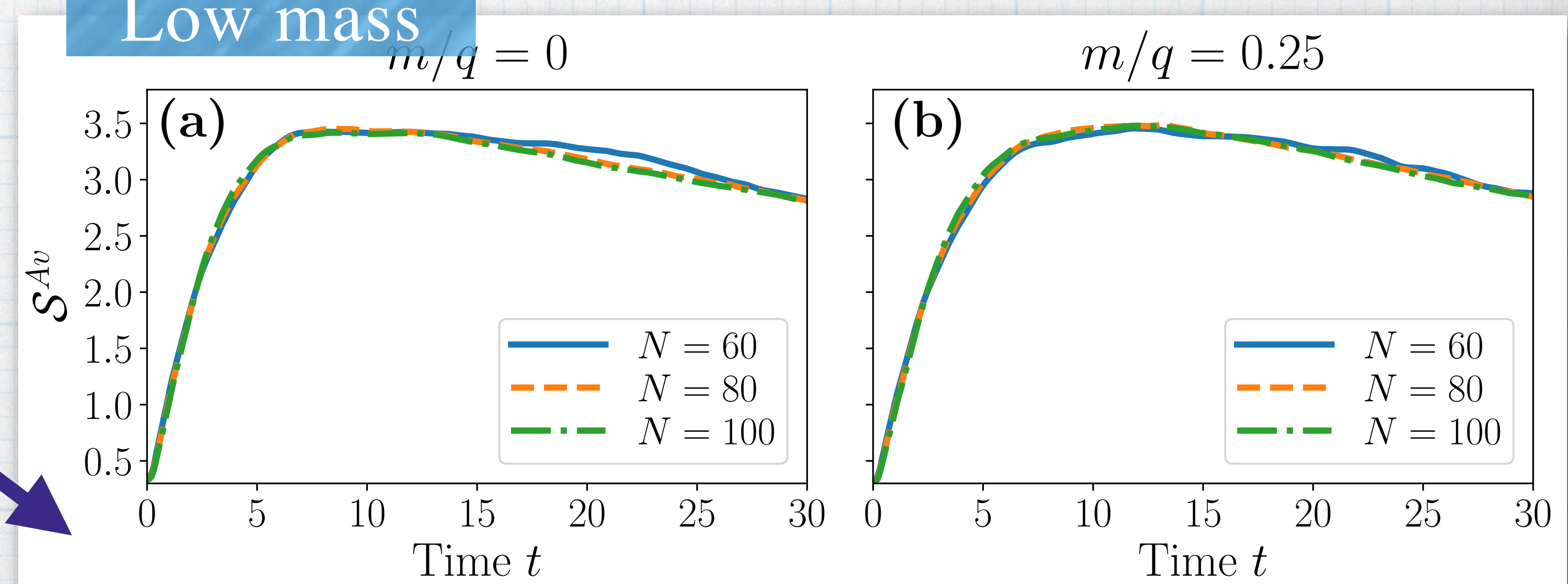


Higher mass



Low mass



$$\mathcal{S}^{Av} = \frac{1}{2R+1} \sum_{j=N/2-R}^{N/2+R} \mathcal{S}_j \longrightarrow \text{Shows perfect area-law}$$

Lack of thermalization

N = 60, 90, 120 sites, with R = N/10

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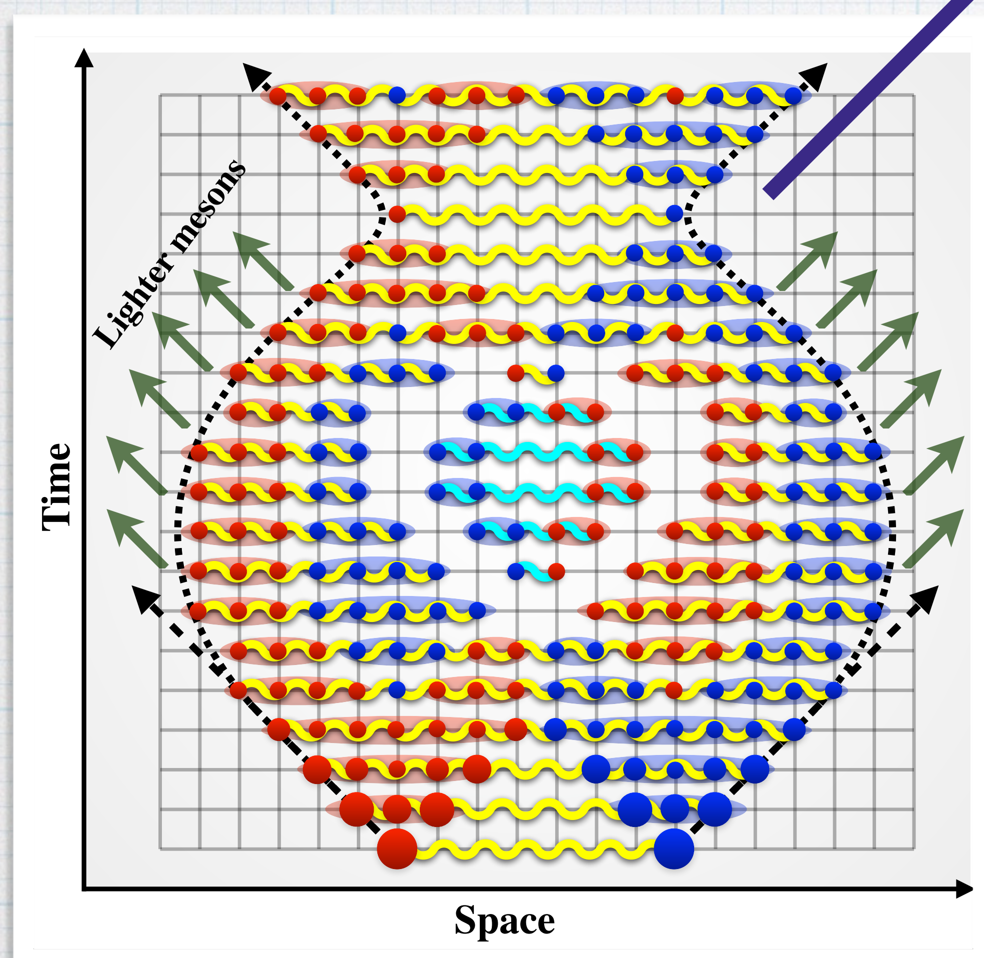
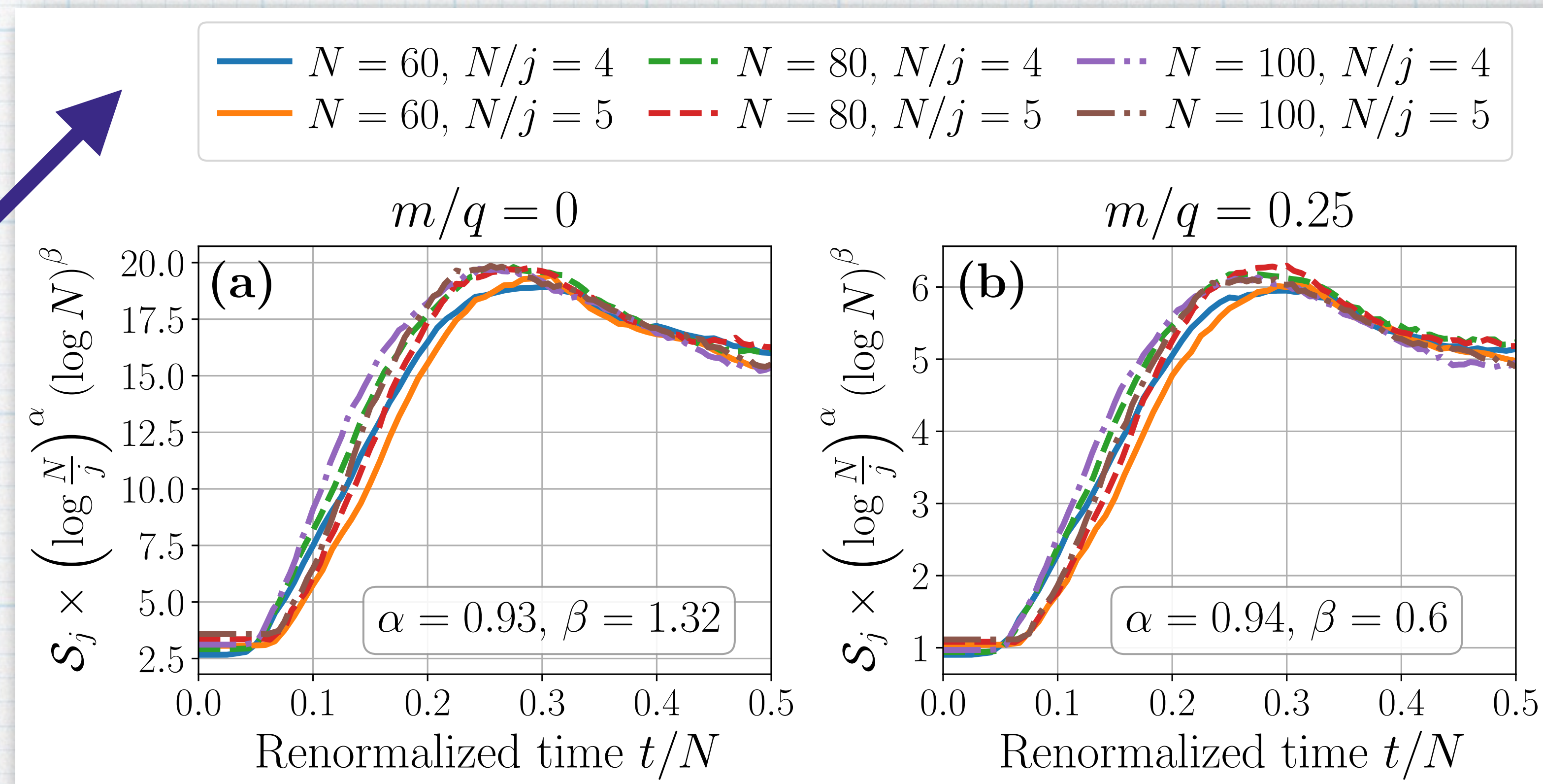


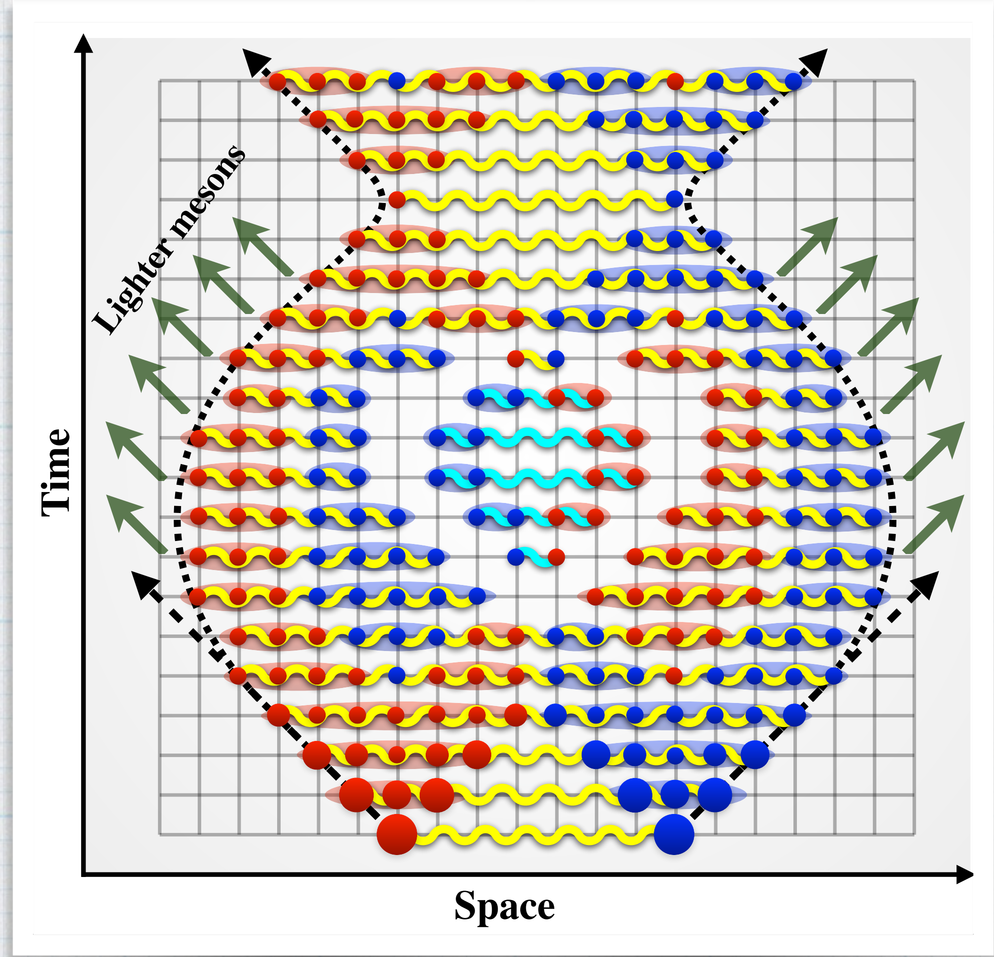


$$\mathcal{S}_j \propto \left(\log \frac{N}{j}\right)^{-\alpha} (\log N)^{-\beta} \ \text{with } \alpha \approx 1$$

For fixed N:

1. Sub-linear in j for small j.
2. Linear for intermediate j: volume-law.
3. Super-linear before saturating into the confined domain.

To summarize…



1. Bosonic Schwinger model shows strong confining dynamics.
2. Trajectories of the bosons bends inwards.
3. As a result, asymptotic states are exotic and highly non-thermal.
4. These states are made of —
   i. Strongly correlated confined core that obeys area-law of entropy.
   ii. Almost thermal outer region (for lower masses) or vacuum (higher masses).

Open questions…

1. Whether such exotic non-thermal states persists at very very long time. Can be answered by next generation tensor-network algorithms or quantum simulations.
2. Origin of the lack-of-thermalization/slow-dynamics in confining theories.

Thank you!!!

Collaborators…

Jakub Zakrzewski

Maciej Lewenstein

Luca Tagliacozzo