On the interpretation of Lambda spin polarization in heavy-ion collisions at STAR

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On the interpretation of Λ spin polarization measurements

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On the interpretation of Lambda spin polarization in heavy-ion collisions at STAR

or

Elementary considerations about boosts and rotations

... the STAR experiment at Brookhaven National Laboratory (USA, Long Island) made the first positive measurements of spin polarization of Λ hyperons produced in relativistic heavy-ion collisions

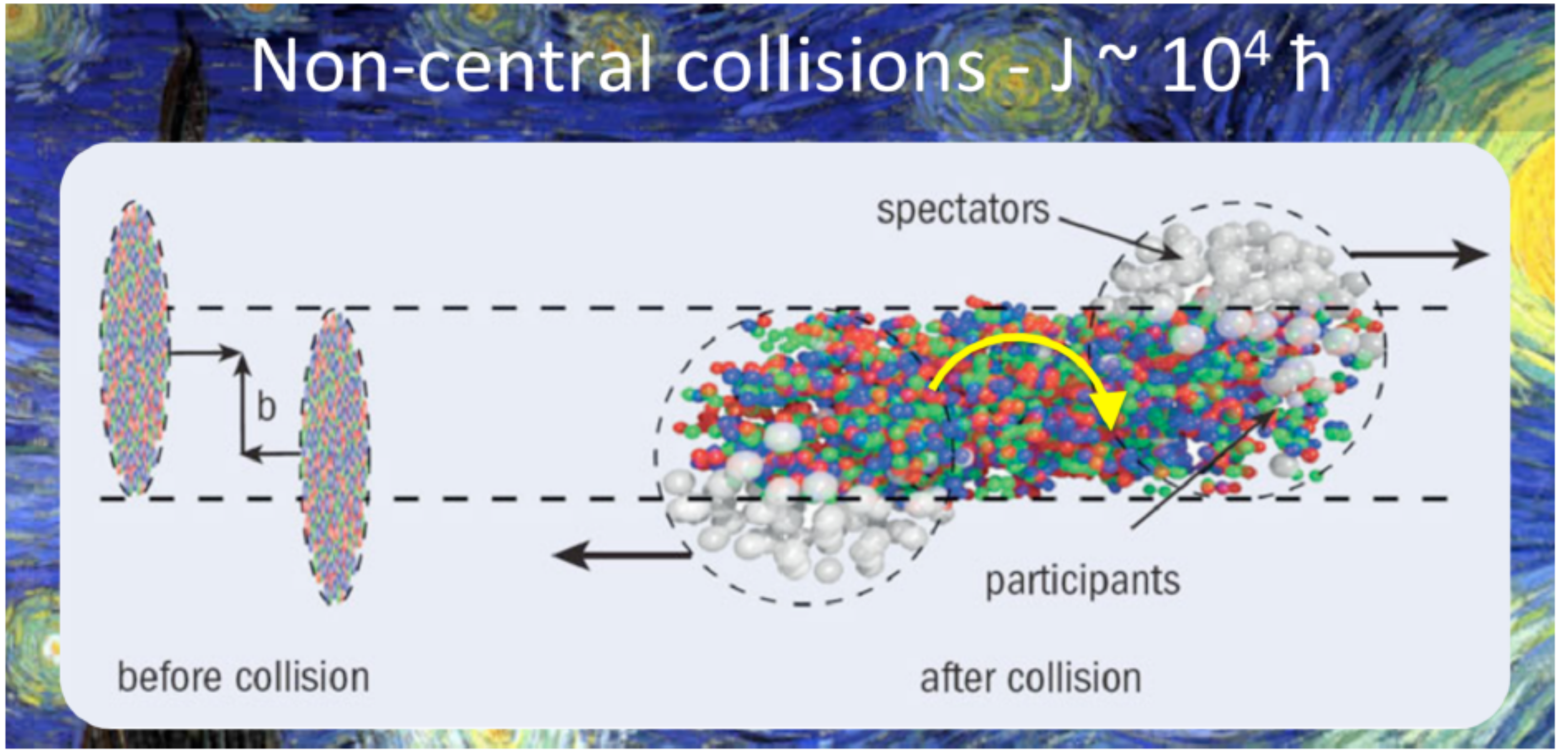
www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**

(from Sergiei Voloshin's talk at „Hirschegg 2019 Workshop")

Non-central heavy-ion collisions create fireballs with large global angular momenta, some part of the angular momentum can be transferred from the orbital to the spin part

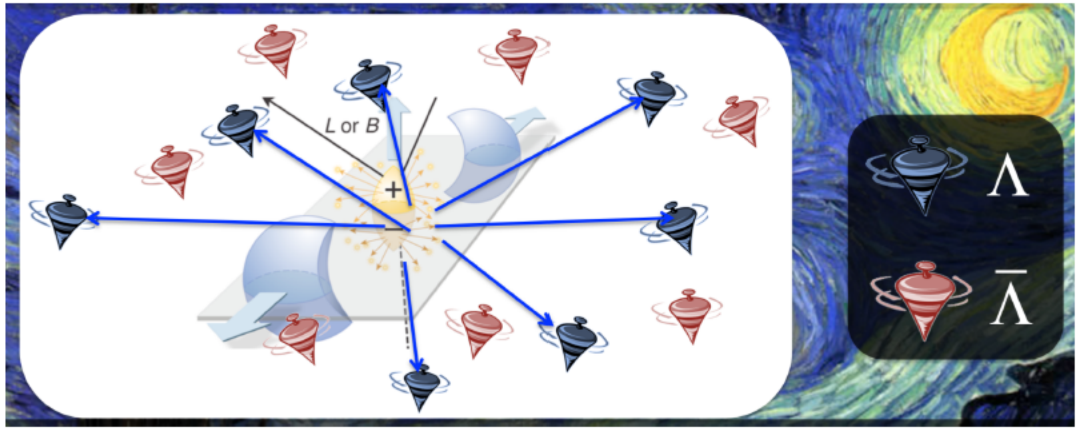
$$\boldsymbol{J}_{\text{init}} = \boldsymbol{L}_{\text{init}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$



e. g. $\pi^+ + \pi^- \to \rho^0$

(Michael Lisa, talk „Strangeness in Quark Matter 2016")

Production of a polarized Lambda

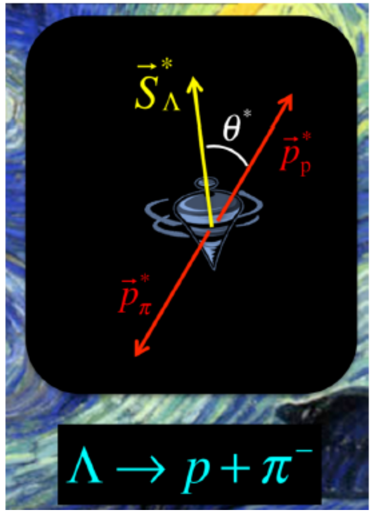


Polarization is measured through the analysis of the weak decay

$$\Lambda \rightarrow p + \pi^-$$

Proton prefers the emission direction that agrees with the spin orientation of Λ (in the rest frame of Λ)

(figures from Michael Lisa, talk at "Strangeness in Quark Matter 2016")

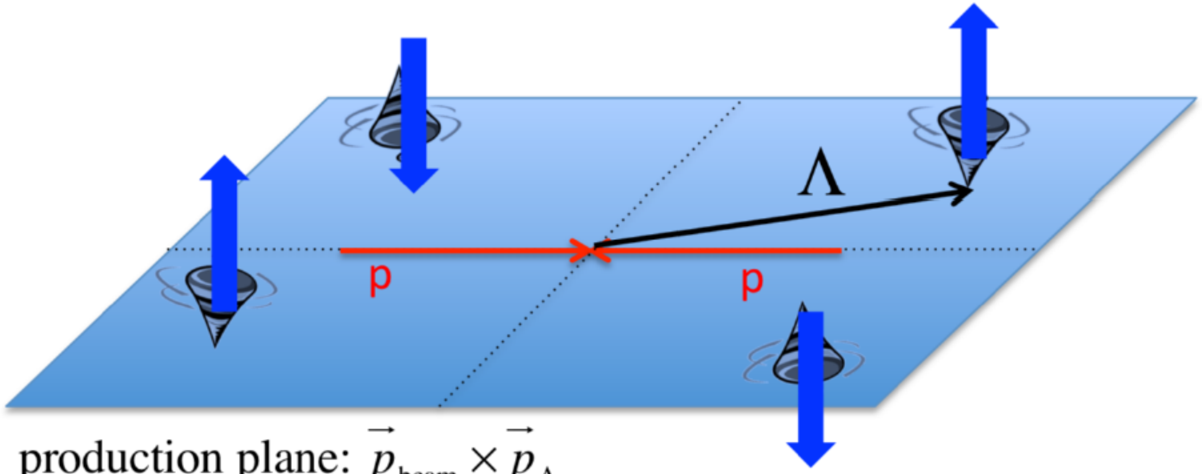


$$\frac{dN}{d\cos\theta^*} \sim 1 + \alpha_H \; P_H \; \cos\theta^*$$

$$P_H \; = \; \frac{3}{\alpha_H} \langle \cos\theta^* \rangle$$

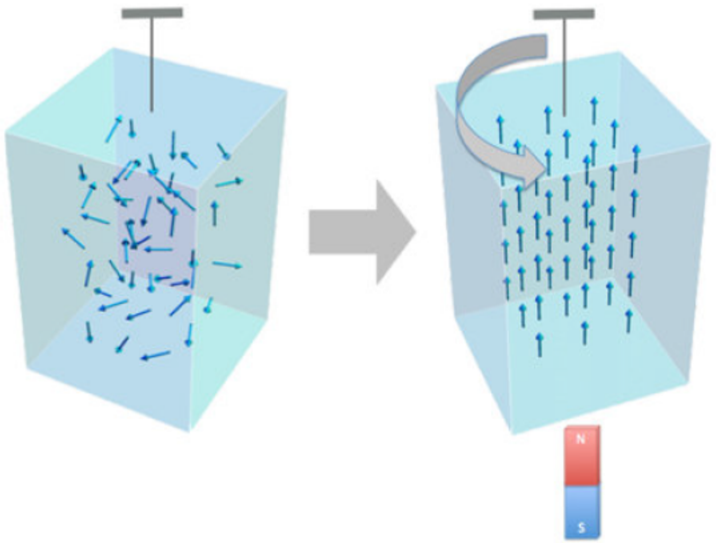
Known effect in p+p collisions [e.g. Bunce et al, PRL 36 1113 (1976)]

- Lambda polarization at *forward* rapidity relative to *production plane*

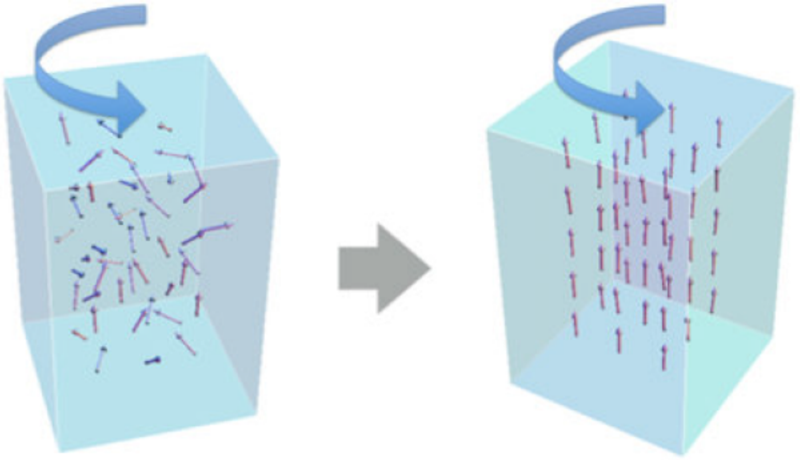


production plane: $\vec{p}_{\text{beam}} \times \vec{p}_{\Lambda}$

no integrated effect at midrapidity



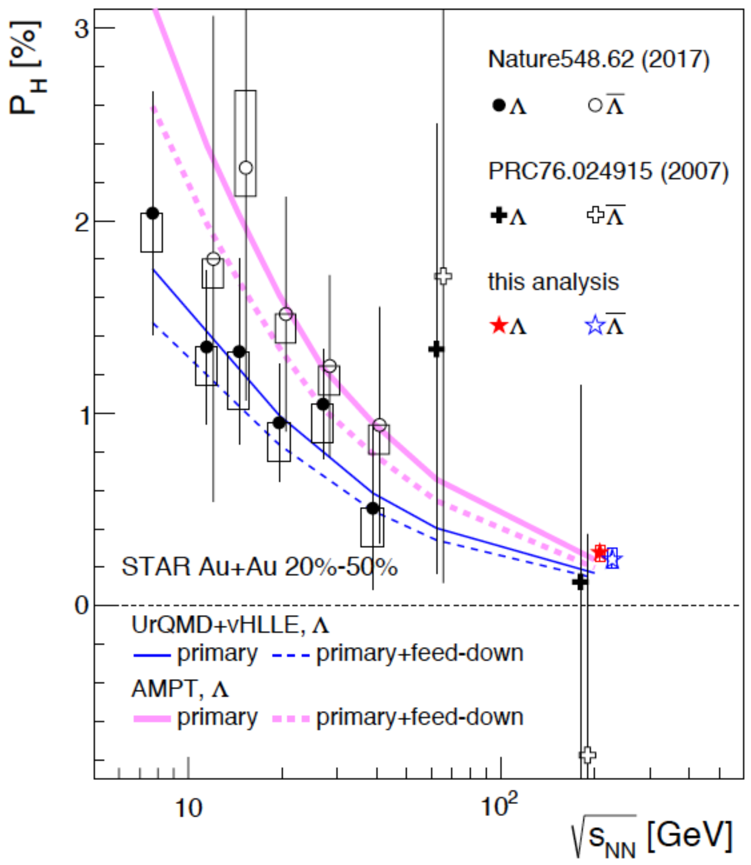
Einstein-de-Haas effect, 1915 (Richardson, 1908)
magnetization induces rotation



Barnett effect, 1915: rotation induces magnetization

Warning: the magnetic field aligns magnetic moments, those are opposite with respect to spin projection for particles and antiparticles, for systems with zero baryon number the magnetic field cannot induce the spin polarization

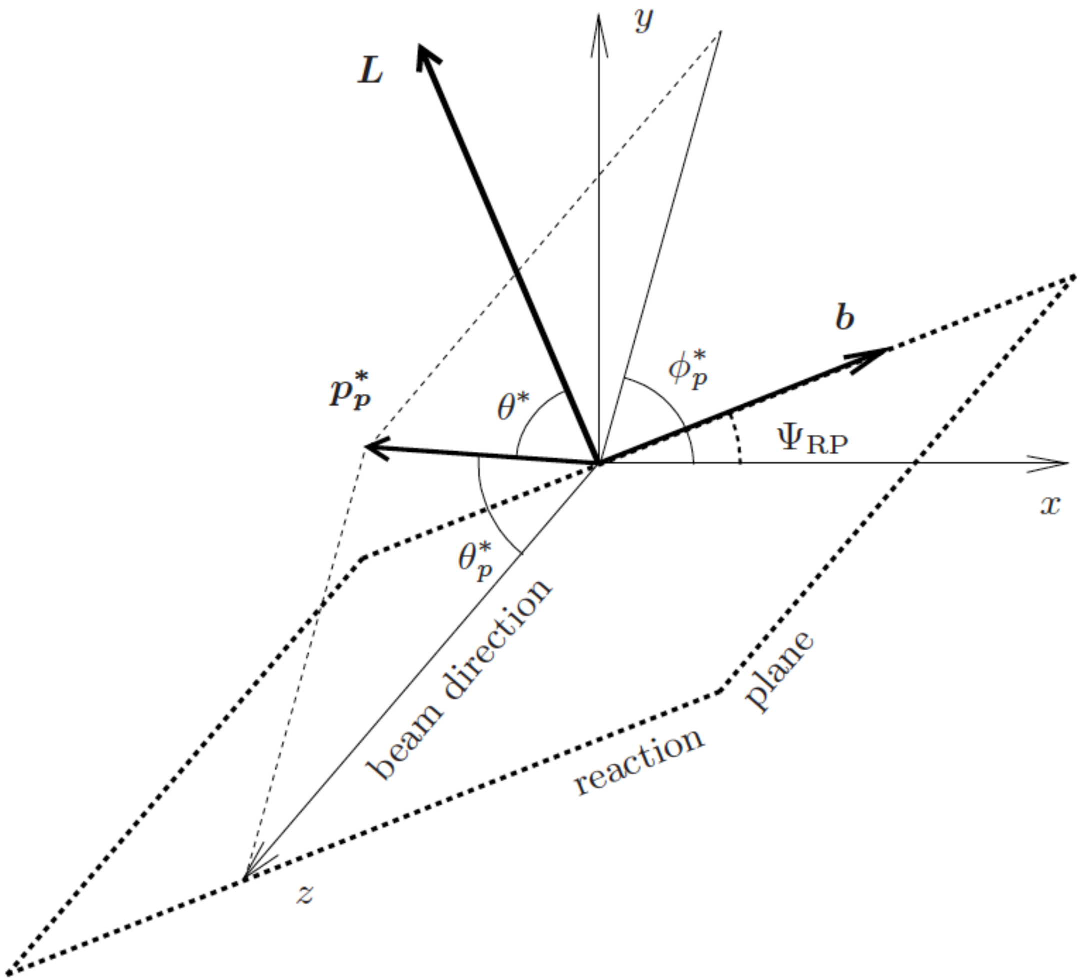
- polarization grows with decreasing beam energy, non-zero even for the highest RHIC energies
- within the exp. errors, the spin polarization is the same for particles and antiparticles — most likely, the observed effect has no connection to magnetic fields



(Takafumi Niida, arXiv:1808.10482, talk at "Quark Matter 2016")

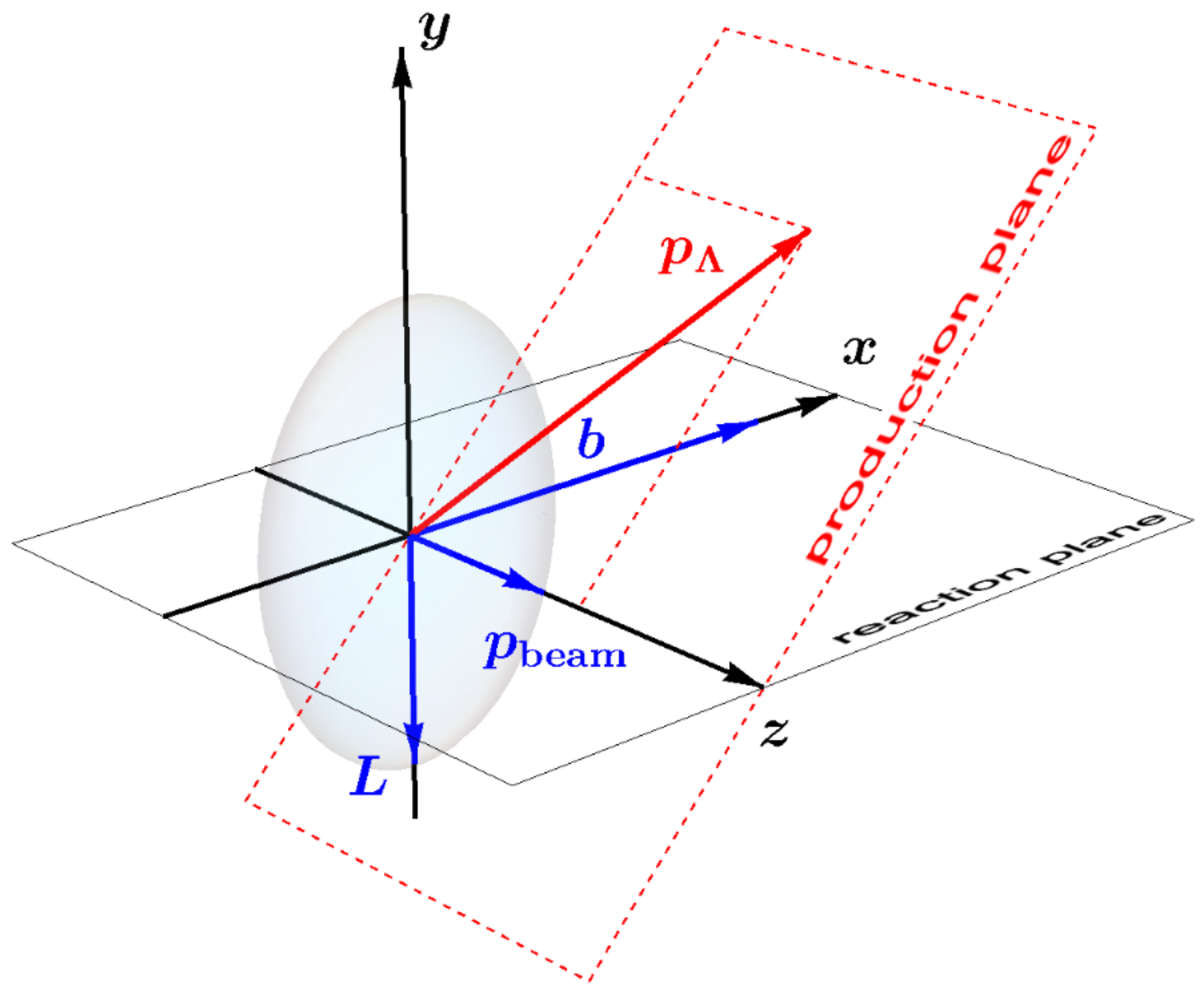
Global polarization measurement in Au+Au collisions

B. I. Abelev *et al.* (STAR Collaboration)
Phys. Rev. C **76**, 024915 – Published 29 August 2007; Erratum Phys. Rev. C **95**, 039906 (2017)



$$P_H = \frac{3}{\alpha_H} \langle \cos \theta^* \rangle$$

$$P_H = \frac{8}{\pi \alpha_H} \langle \sin\left(\phi_p^* - \Psi_{\mathrm{RP}}\right) \rangle$$



y
p_Λ
x
b
$p_{\rm beam}$
z
L
production plane
reaction plane

The Poincare algebra

$$[J_i, J_j] = i\,\epsilon_{ijk} J_k \;,$$

$$[J_i, K_j] = i\,\epsilon_{ijk} K_k \;,$$

$$[K_i, K_j] = -i\,\epsilon_{ijk} J_k \;,$$

$$\boxed{[J_i, P_j] = i\,\epsilon_{ijk} P_k \;,}$$

$$[K_i, P_j] = i H \delta_{ij} \;,$$

$$[J_i, H] = [P_i, H] = [H, H] = 0$$

$$[K_i, H] = i\,P_i \;,$$

The spin can be defined (together with three-momentum) in the particle rest frame
Alernatively we can use helicity

The canonical boost leads to the Lambda rest frame

$$p'^{\mu} = \mathcal{L}^{\mu}{}_{\nu}\left(-\boldsymbol{v}_{\Lambda}\right)p^{\nu}.$$

$$p'^{\mu}_{\Lambda} = (m_{\Lambda}, 0, 0, 0).$$

$$\mathcal{L}^{\mu}{}_{\nu}\left(-\boldsymbol{v}_{\Lambda}\right) = \begin{bmatrix} \frac{E_{\Lambda}}{m_{\Lambda}} & -\frac{p^{1}_{\Lambda}}{m_{\Lambda}} & -\frac{p^{2}_{\Lambda}}{m_{\Lambda}} & -\frac{p^{3}_{\Lambda}}{m_{\Lambda}} \\ -\frac{p^{1}_{\Lambda}}{m_{\Lambda}} & 1+\alpha p^{1}_{\Lambda}p^{1}_{\Lambda} & \alpha p^{1}_{\Lambda}p^{2}_{\Lambda} & \alpha p^{1}_{\Lambda}p^{3}_{\Lambda} \\ -\frac{p^{2}_{\Lambda}}{m_{\Lambda}} & \alpha p^{2}_{\Lambda}p^{1}_{\Lambda} & 1+\alpha p^{2}_{\Lambda}p^{2}_{\Lambda} & \alpha p^{2}_{\Lambda}p^{3}_{\Lambda} \\ -\frac{p^{3}_{\Lambda}}{m_{\Lambda}} & \alpha p^{3}_{\Lambda}p^{1}_{\Lambda} & \alpha p^{3}_{\Lambda}p^{2}_{\Lambda} & 1+\alpha p^{3}_{\Lambda}p^{3}_{\Lambda} \end{bmatrix}.$$

$$\mathcal{L} = \mathcal{R}^{-1}_{\Lambda}(\phi_{\Lambda}, \theta_{\Lambda})\mathcal{L}_{3}(-v_{\Lambda})\mathcal{R}_{\Lambda}(\phi_{\Lambda}, \theta_{\Lambda}).$$

$$\mathcal{R}_{\Lambda} = \mathcal{R}_{2}(\theta_{\Lambda})\mathcal{R}_{3}(\phi_{\Lambda})$$

$$\mathcal{R}_{3}\left(\phi_{\Lambda}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_{\Lambda} & \sin\phi_{\Lambda} & 0 \\ 0 & -\sin\phi_{\Lambda} & \cos\phi_{\Lambda} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{R}_{2}\left(\theta_{\Lambda}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_{\Lambda} & 0 & -\sin\theta_{\Lambda} \\ 0 & 0 & 1 & 0 \\ 0 & \sin\theta_{\Lambda} & 0 & \cos\theta_{\Lambda} \end{bmatrix}$$

$$\mathcal{L}_{3}(-v_{\Lambda}) = \begin{bmatrix} \gamma_{\Lambda} & 0 & 0 & -\gamma_{\Lambda}v_{\Lambda} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_{\Lambda}v_{\Lambda} & 0 & 0 & \gamma_{\Lambda} \end{bmatrix}$$

Transformation of the total orbital angular momentum (the same as of the magnetic field)

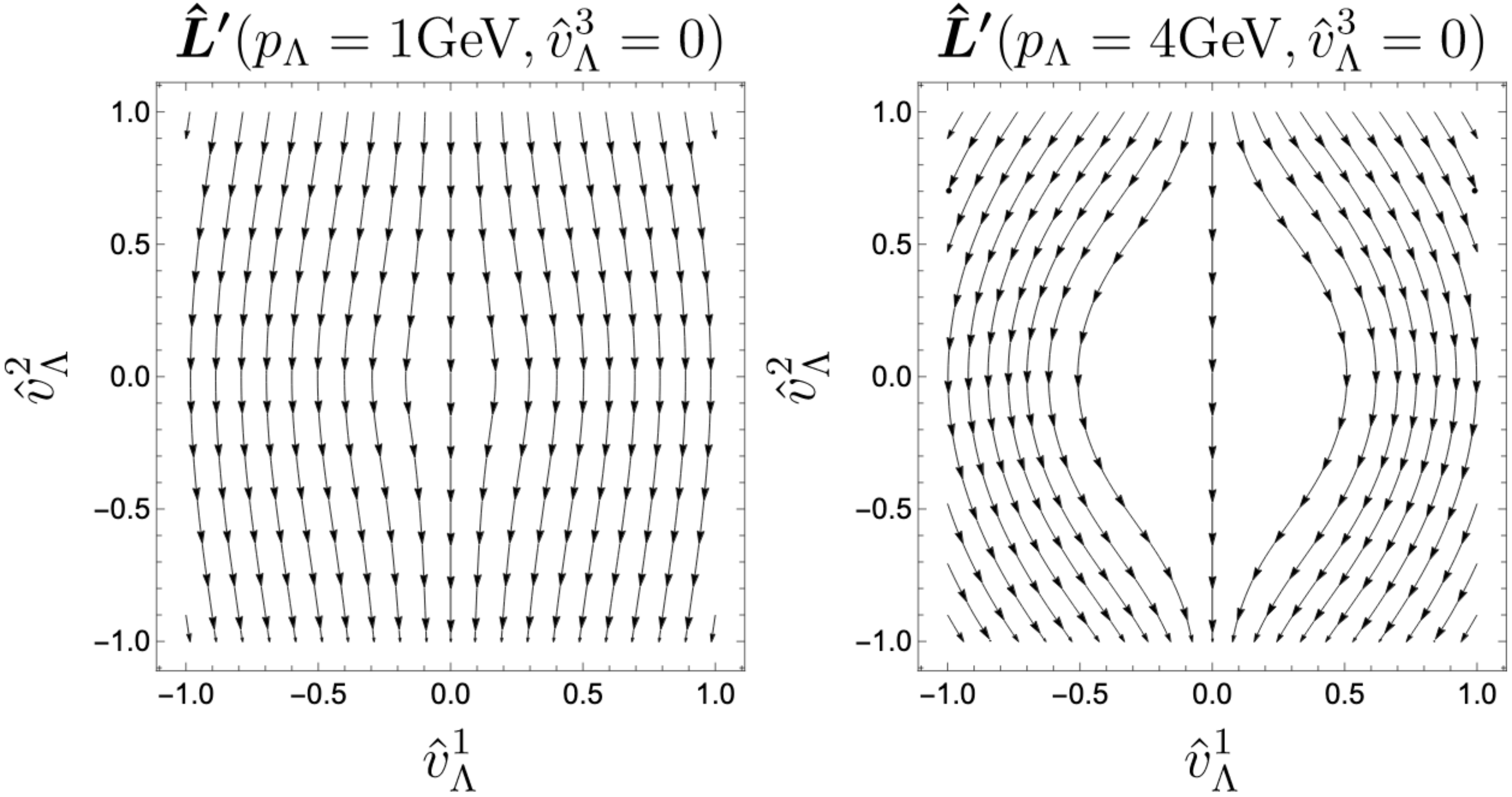
$$\boldsymbol{L}' = \gamma_\Lambda \boldsymbol{L} - \frac{\gamma_\Lambda^2}{\gamma_\Lambda + 1} \boldsymbol{v}_\Lambda (\boldsymbol{v}_\Lambda \cdot \boldsymbol{L}).$$

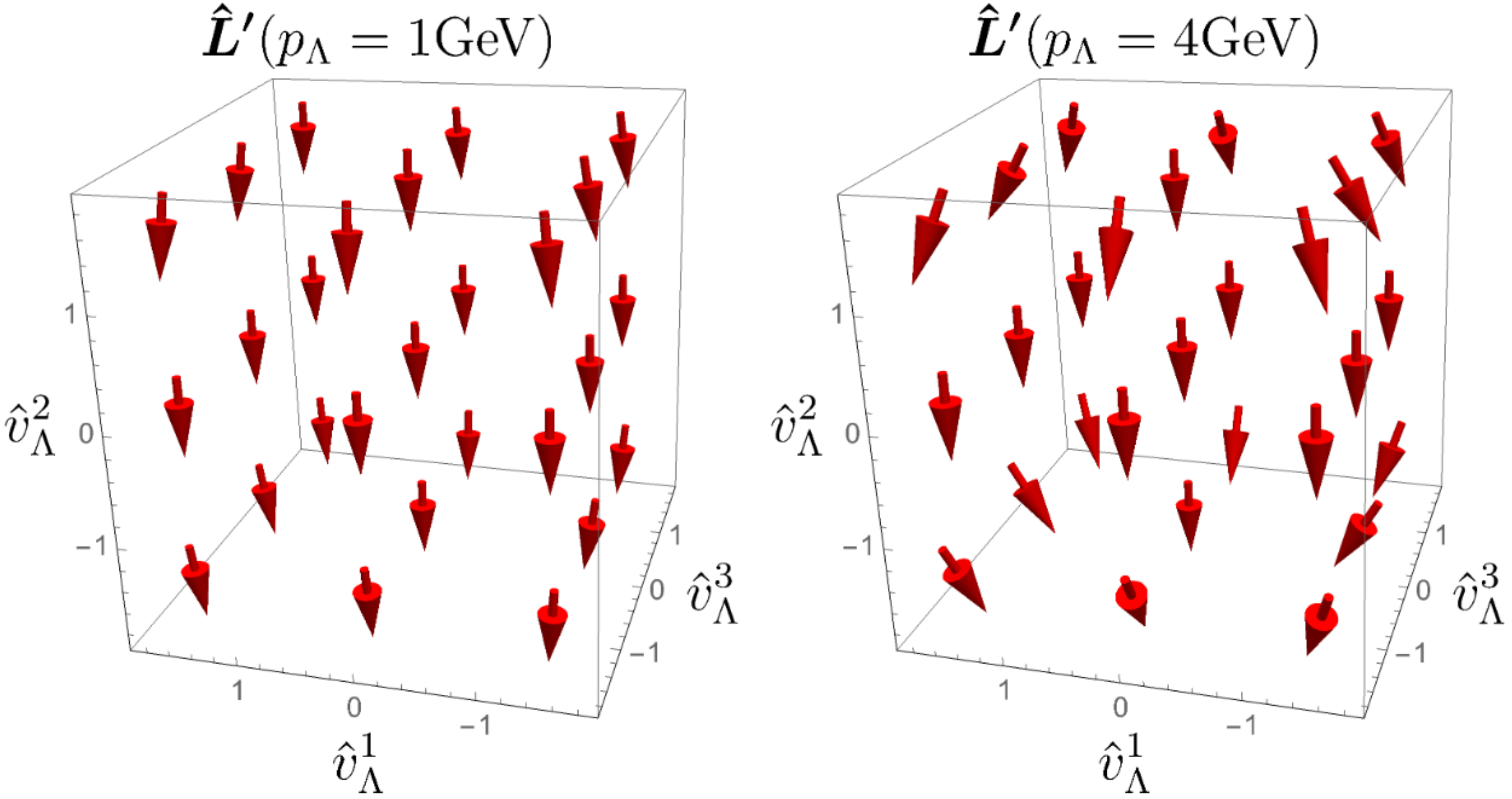
$$\frac{L'}{L} = \gamma_\Lambda \left(1 - (\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}})^2\right)^{1/2}.$$

The conserved quantities J^{0i} corresponding to Lorentz boosts are of the form $J^{0i} = ER^i - tP^i$, where $R^i = (1/E)\int d^3x\, x^i\, T^{00}$ and $E = \int d^3x\, T^{00}$, with T^{00} being the energy density. In the center-of-momentum frame $P^i = 0$. Moreover, if the center-of-momentum frame is also the center-of-mass frame (strictly speaking, the center-of-energy for relativistic systems) then we also have $R^i = 0$.

$$\hat{\boldsymbol{L}}' = \left(1 - (\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}})^2\right)^{-1/2} \left(\hat{\boldsymbol{L}} - \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} \boldsymbol{v}_\Lambda (\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}})\right).$$

In the COM (center-of-mass frame) $\hat{\boldsymbol{L}} = (0, -1, 0)$


$\hat{\boldsymbol{L}}'(p_\Lambda = 1\mathrm{GeV}, \hat{v}^3_\Lambda = 0)$
$\hat{\boldsymbol{L}}'(p_\Lambda = 4\mathrm{GeV}, \hat{v}^3_\Lambda = 0)$
$\hat{v}^2_\Lambda$
$\hat{v}^1_\Lambda$
1.0
0.5
0.0
−0.5
−1.0



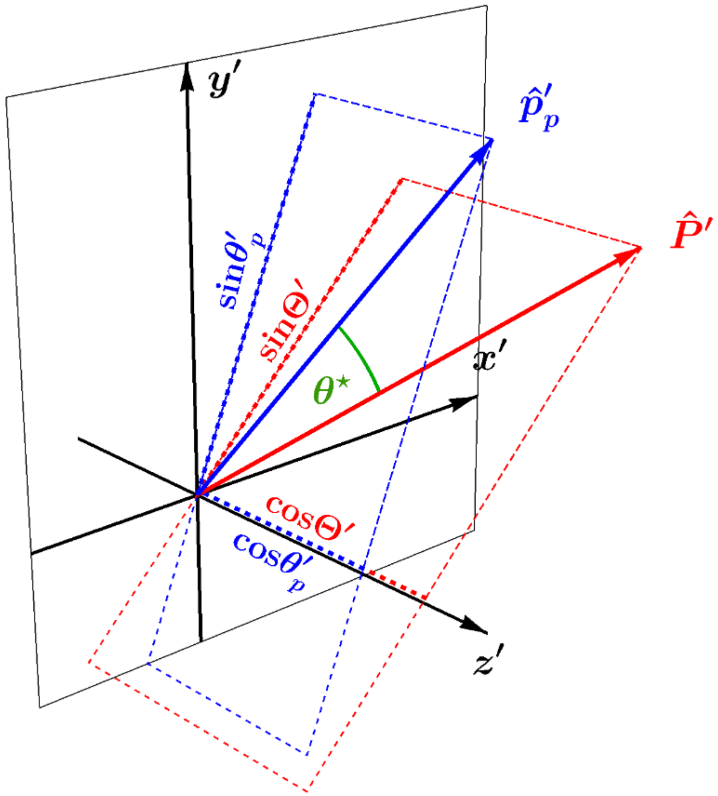
$\hat{\boldsymbol{L}}'(p_\Lambda = 1\text{GeV})$
$\hat{\boldsymbol{L}}'(p_\Lambda = 4\text{GeV})$
$\hat{v}^1_\Lambda$
$\hat{v}^2_\Lambda$
$\hat{v}^3_\Lambda$
1
0
−1

THE Λ REST FRAME $S'(\boldsymbol{p}_\Lambda)$

obtained by the canonical boost

the spin polarization three-vector of the Lambda hyperon

$$\hat{\boldsymbol{P}}' = (\sin\Theta' \cos\Phi', \sin\Theta' \sin\Phi', \cos\Theta')$$



the momentum direction of an emitted proton

$$\hat{\boldsymbol{p}}'_p = \left(\sin\theta'_p \cos\phi'_p, \sin\theta'_p \sin\phi'_p, \cos\theta'_p\right)$$

relativistic generalization: pure and mixed states

$$P'^{\mu} = (0, \boldsymbol{P}')\,.$$

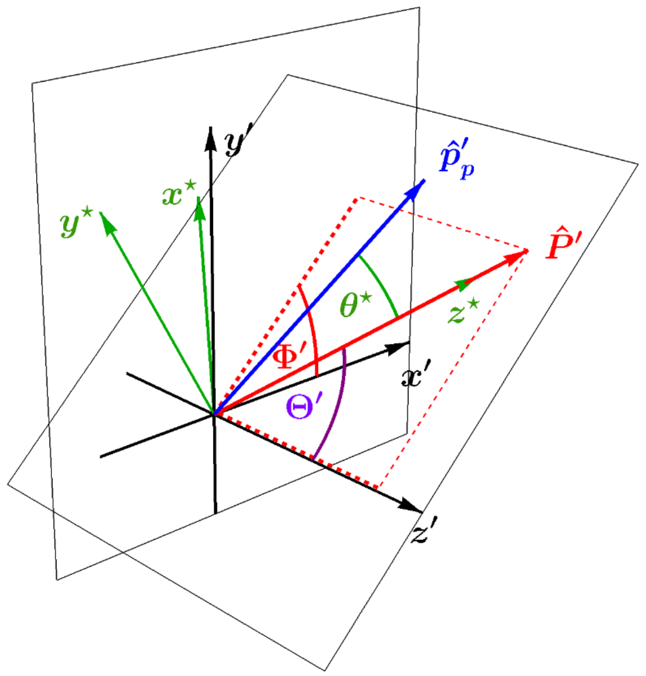
$$-1 \le P' \cdot P' \le 0.$$

THE Λ REST FRAME $S^*(\boldsymbol{p}_\Lambda)$

obtained by an additional rotation

$$\hat{\boldsymbol{P}}^* = \mathcal{R}_{y'}(\Theta')\,\mathcal{R}_{z'}(\Phi')\,\hat{\boldsymbol{P}}' = (0,0,1)$$

$$\mathcal{R}_{y'}(\Theta') = \begin{bmatrix} \cos\Theta' & 0 & -\sin\Theta' \\ 0 & 1 & 0 \\ \sin\Theta' & 0 & \cos\Theta' \end{bmatrix} \qquad \mathcal{R}_{z'}(\Phi') = \begin{bmatrix} \cos\Phi' & \sin\Phi' & 0 \\ -\sin\Phi' & \cos\Phi' & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



proton three-momentum components (directions)

$$\begin{aligned}
\hat{p}^*_{p,x} &= \cos(\Phi' - \phi'_p)\sin\theta'_p\cos\Theta' - \cos\theta'_p\sin\Theta' \equiv \sin\theta^*\cos\phi^*, \\
\hat{p}^*_{p,y} &= -\sin(\Phi' - \phi'_p)\sin\theta'_p \equiv \sin\theta^*\sin\phi^*, \\
\hat{p}^*_{p,z} &= \cos(\Phi' - \phi'_p)\sin\theta'_p\sin\Theta' + \cos\theta'_p\cos\Theta' \equiv \cos\theta^*.
\end{aligned}$$

THE WEAK DECAY LAW

$$\frac{dN_p^{\rm pol}}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_\Lambda \boldsymbol{P}^* \cdot \hat{\boldsymbol{p}}_p^*\right)$$

In the frame obtained by the boost

$$\frac{dN_p^{\rm pol}}{d\Omega'} = \frac{1}{4\pi}\left[1 + \alpha_\Lambda P\left(\cos(\Phi' - \phi_p')\sin\theta_p' \sin\Theta' + \cos\theta_p' \cos\Theta'\right)\right]$$

$$\langle \hat{p}'_{p,x}\rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right)(\sin\theta_p')^2 \cos\phi_p' \, d\theta_p' \, d\phi_p' = \frac{1}{3}P'\alpha_\Lambda \sin\Theta' \cos\Phi',$$

$$\langle \hat{p}'_{p,y}\rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right)(\sin\theta_p')^2 \sin\phi_p' \, d\theta_p' \, d\phi_p' = \frac{1}{3}P'\alpha_\Lambda \sin\Theta' \sin\Phi',$$

$$\langle \hat{p}'_{p,z}\rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right)\sin\theta_p' \cos\theta_p' \, d\theta_p' \, d\phi_p' = \frac{1}{3}P'\alpha_\Lambda \cos\Theta'.$$

thus the polarization components are directly obtained from the proton distributions

$$\boldsymbol{P}' = P'\left(\sin\Theta'\cos\Phi', \sin\Theta'\sin\Phi', \cos\Theta'\right) = \frac{3}{\alpha_\Lambda}\left(\langle \hat{p}'_{p,x}\rangle, \langle \hat{p}'_{p,y}\rangle, \langle \hat{p}'_{p,z}\rangle\right)$$

$$P_H = \frac{3}{\alpha_H}\langle \cos\theta^* \rangle$$

One can also find

$$\langle \cos\phi_p' \rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right) \sin\theta_p' \cos\phi_p' \, d\theta_p' \, d\phi_p' = \frac{\pi\alpha_\Lambda}{8} P' \sin\Theta' \cos\Phi',$$

$$\langle \sin\phi_p' \rangle = \int \left(\frac{dN_p^{\rm pol}}{d\Omega'}\right) \sin\theta_p' \sin\phi_p' \, d\theta_p' \, d\phi_p' = \frac{\pi\alpha_\Lambda}{8} P' \sin\Theta' \sin\Phi'.$$

The last expression rewritten in the form

$$P_H = \frac{8}{\pi\alpha_\Lambda} \langle \sin\phi_p' \rangle$$

The quantity P_H is the y-component of the polarization three-vector measured in the Lambda rest frame, namely, $P_H = P' \sin\Theta' \sin\Phi'$. Strictly speaking, it is not the component of the polarization along the total angular momentum vector as the y-directions in COM and the Lambda rest frame are different (although the differences for slowly moving Lambdas might be quite small).

The quanity that is measured (for Lambdas with a given three-momentum in COM) is

$$\frac{1}{3} P' \alpha_\Lambda \sin \Theta' \sin \Phi'$$

This involves all three polarization variables: the magnitude and direction. This result is Lambda-frame dependent.

Averaging of this quantity over Lambdas with different momenta is controversial – valid in non-relativistic physics, where angles do not change under Galilean transformations

We propose to measure/construct the variable

$$\hat{\boldsymbol{L}}' \cdot \boldsymbol{P}' = \left(1 - (\boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}})^2\right)^{-1/2} \left(\hat{\boldsymbol{L}} \cdot \boldsymbol{P}' - \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} \boldsymbol{v}_\Lambda \cdot \boldsymbol{P}' \;\; \boldsymbol{v}_\Lambda \cdot \hat{\boldsymbol{L}}\right)$$

Conclusions:

1) Interpretation in terms of the Barnett effect should be made with great care – there is no common frame for all Lambdas where they are at rest

2) The present measurements measure y-component of the polarization in the Lambda rest frame, y-directions are different for different Lambdas and they might be significantly different from the direction of the total angular momentum of the produced system

3) An expression is given that shows how the polarization along the physical L direction can be measured

4) If the y-components are measured, the measurement of x-components seems to be possible. Since the z-components are measured, then we suggest that the full polarization vector should be measured before physics interpretations are done

5) Similar problems may appear in the measurement of Lambda polarization in p+p collisions.

6) The effect of the TPC magnetic field should be recheked.

7) The effects discussed here are important for relativistic Lambdas – we do not expect radical quantitative changes