Gravitational Waves from fluid perturbations in First-Order Phase Transitions

Antonino Salvino Midiri

Ongoin works in collaboration with: Chiara Caprini, Daniel Figueroa, Kenneth Marschall, Simona Procacci, Alberto Roper Pol, Madeline Salomé

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Université de Genève

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«Exploring the early Universe with Gravitational Waves and Primordial Magnetic Fields»

Introduction: phase transitions in the early Universe

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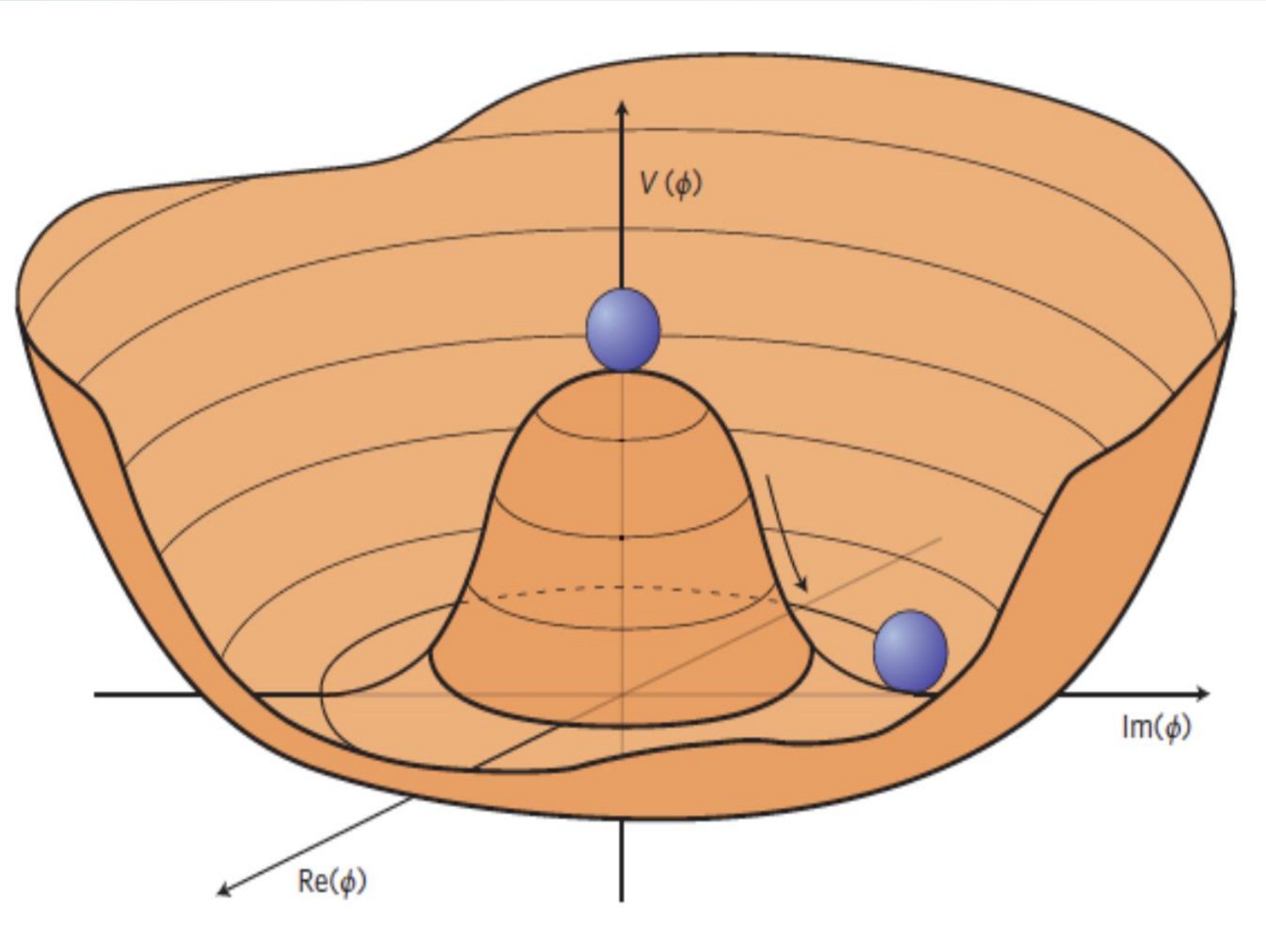
$$V_0(|\phi|) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda^2}{4}|\phi|^4$$



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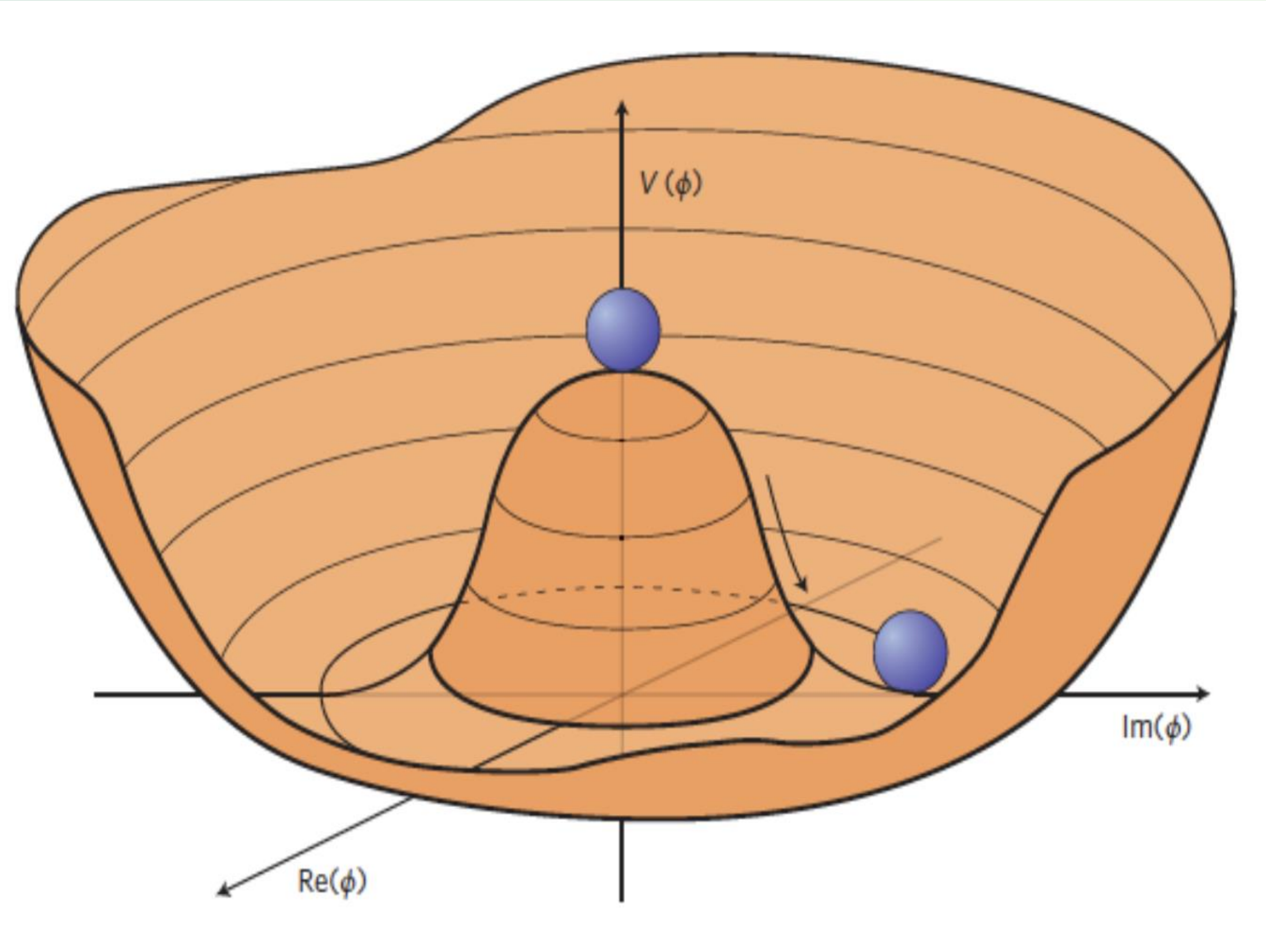
Minimum of the potential at $T = 0$

$$\frac{\partial V_0(|\phi|)}{\partial|\phi|} = 0$$

$$\xrightarrow{\text{vev}} |\phi| = \sqrt{\mu^2/\lambda^2} \equiv v \neq 0$$



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In the primordial plasma at finite temperature?

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As temperature decreases the Higgs vev goes from zero to $v \neq 0$ $\longrightarrow$ Electroweak Spontaneous Symmetry Breaking (EWSSB)

$SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \to SU(3)_C \bigotimes U(1)_{em}$

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Let us consider the Standard Model case [9203203]

$$V_{eff}(|\phi|, \mathrm{T}) = D^2(\mathrm{T}^2 - \mathrm{T}_0^2)|\phi|^2 - E^2\, \mathrm{T}\, |\phi|^3 + \frac{\lambda^2}{4}\, |\phi|^4$$

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If $E = 0$ the minimum is given by

$T \geq T_0 \qquad |\phi| = 0$

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Across the transition (at $T = T_0$)

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$\longrightarrow$ Second-Order Phase Transition

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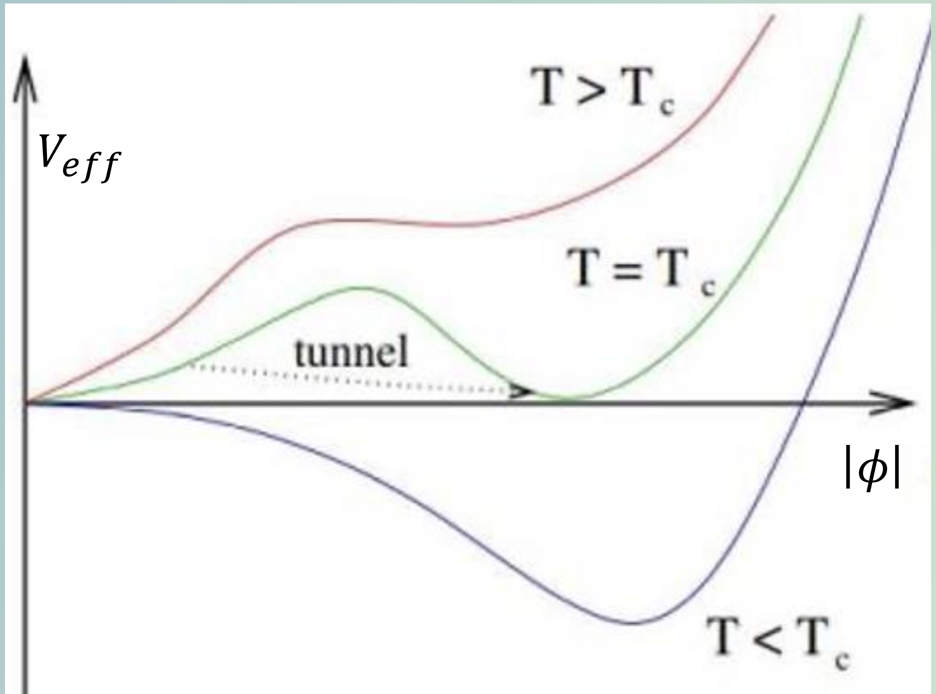

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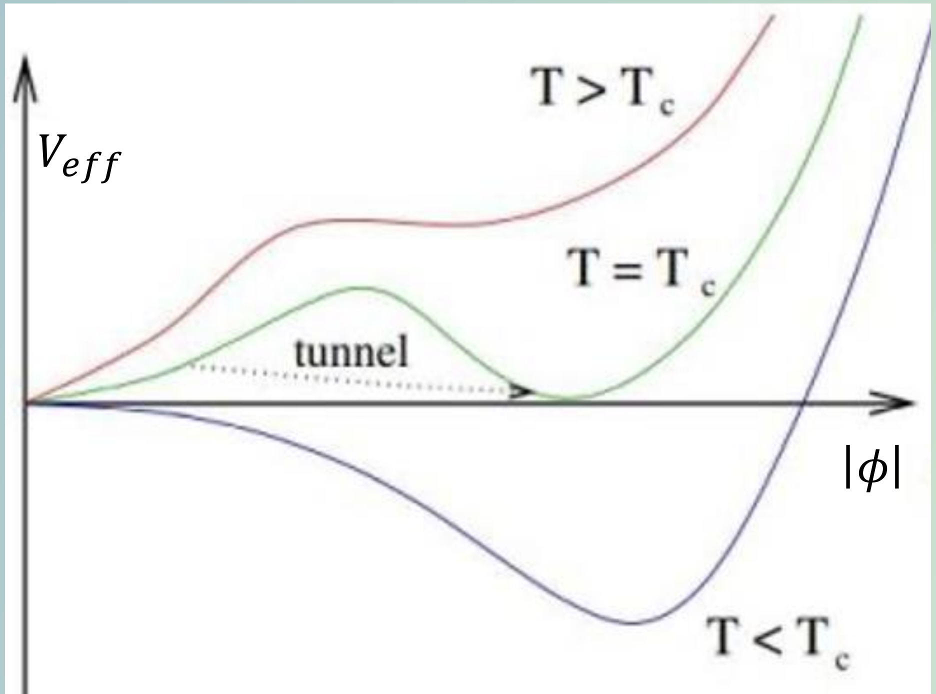

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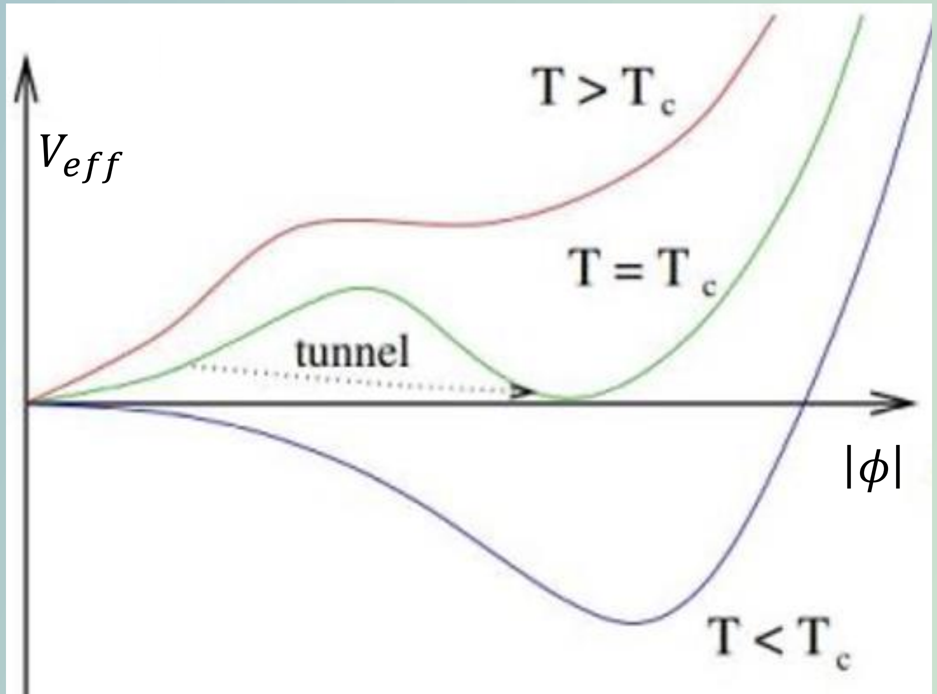

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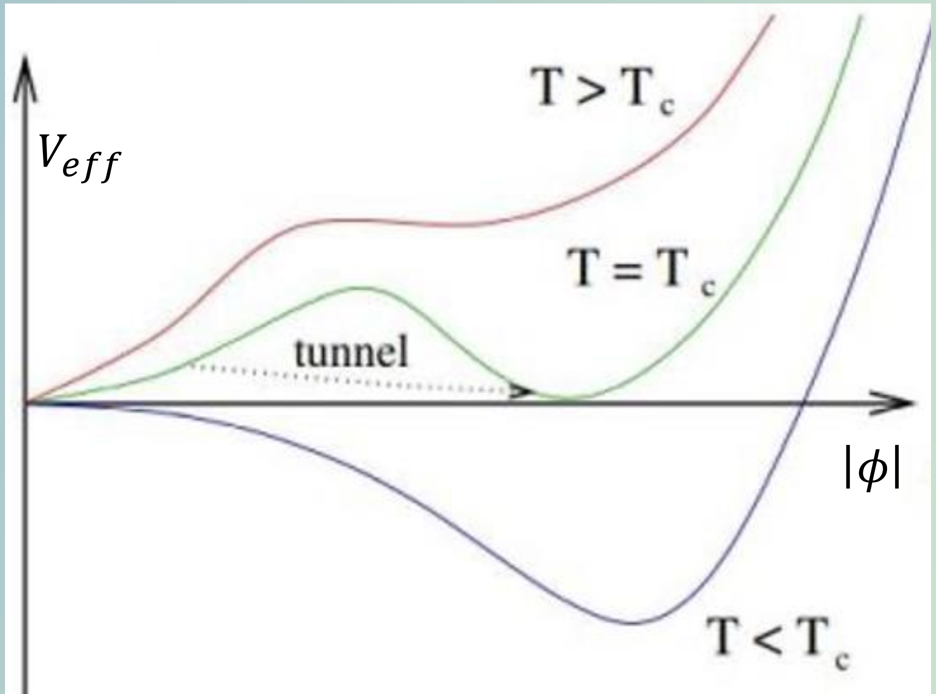

In the Standard Model the EW phase transition is a crossover ($E \neq 0$ but small)

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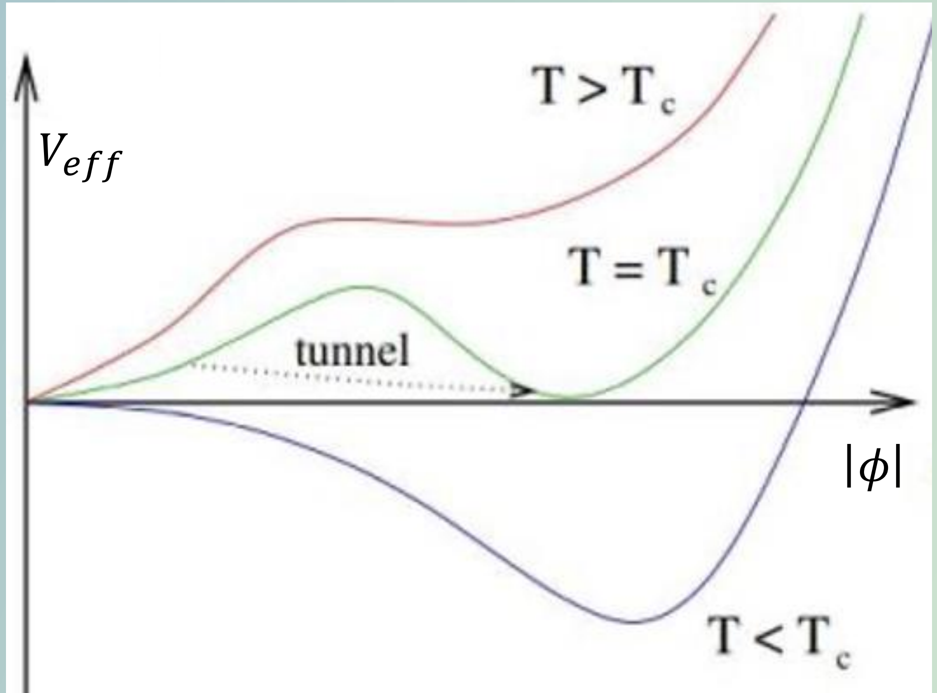
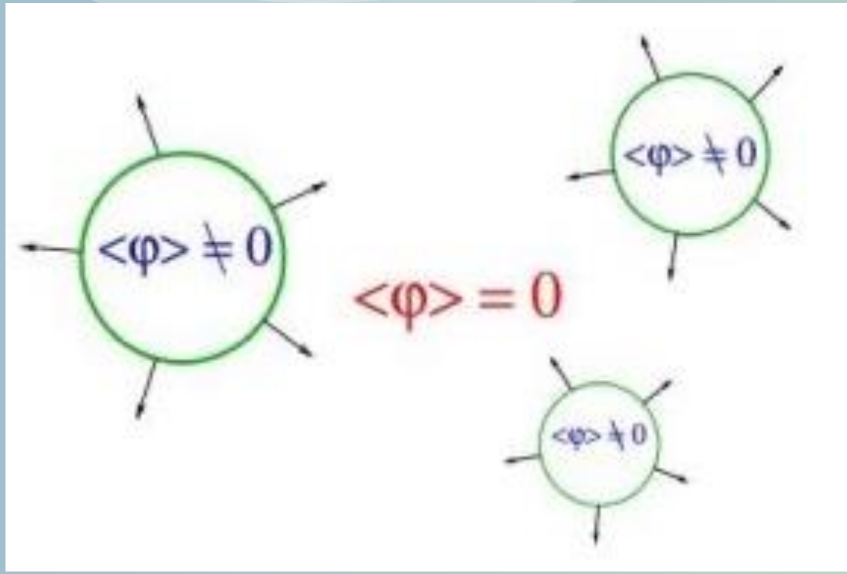


In the Standard Model the EW phase transition is a crossover ($E \neq 0$ but small)

However in BSM theories we can easily have first-order phase transitions (e. g. in SUSY already at tree level)

Introduction: first-order phase transitions and gravitational waves

First-Order Phase Transitions occur through the nucleation of broken phase bubbles



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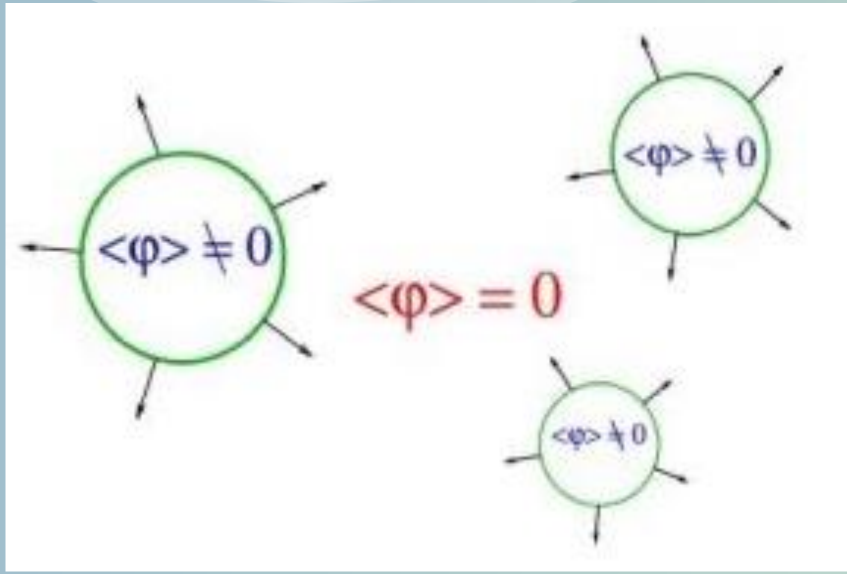
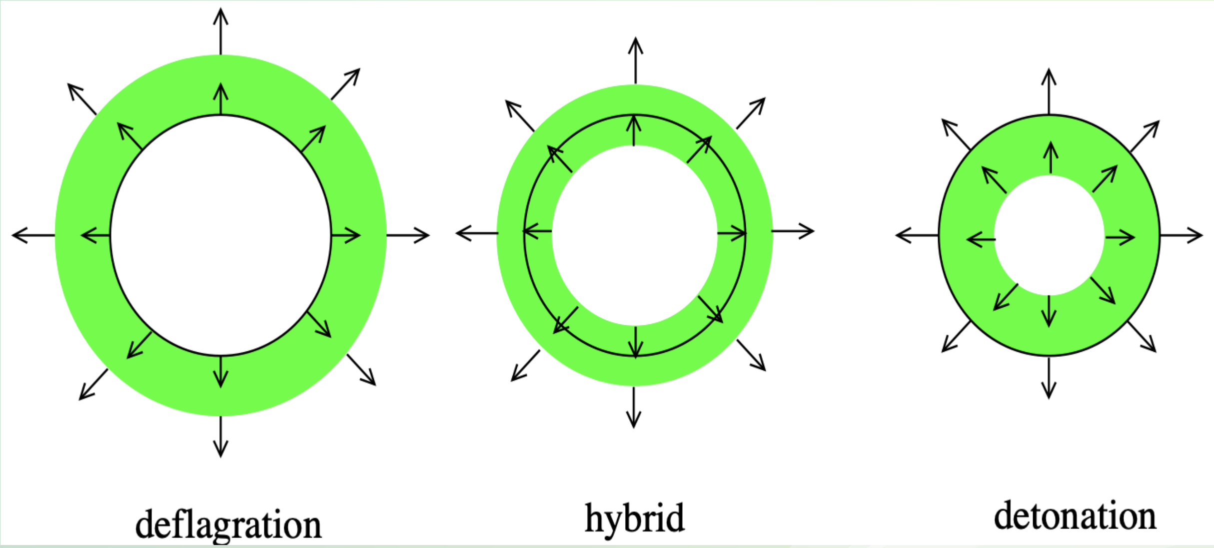


friction between scalar and plasma



Espinosa et al. [1004.4187]

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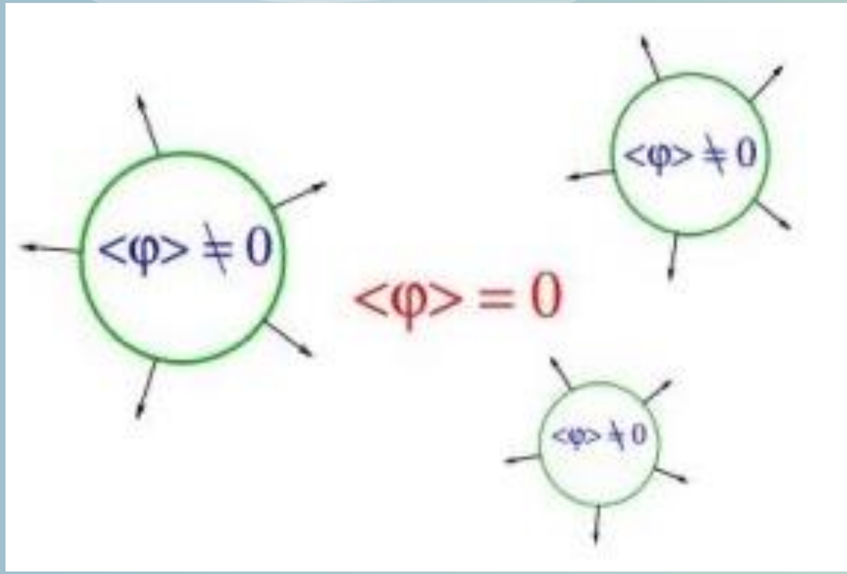

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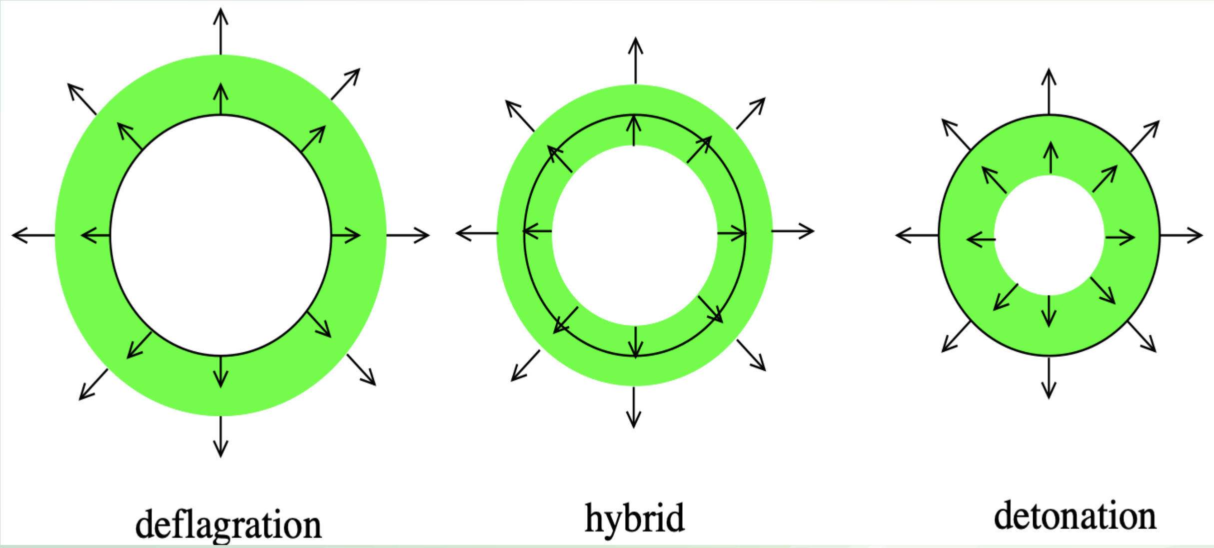

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Bubble expansion phase → scalar and fluid profiles are spherically symmetric

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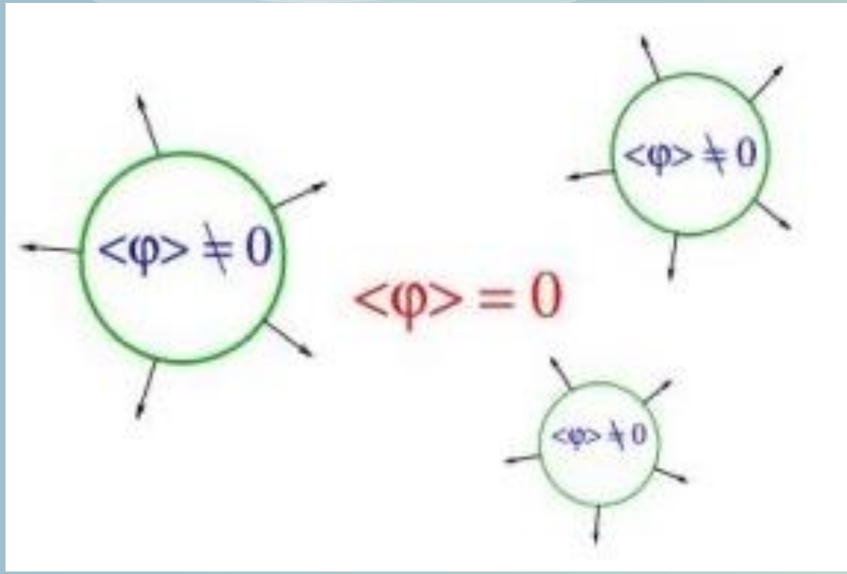

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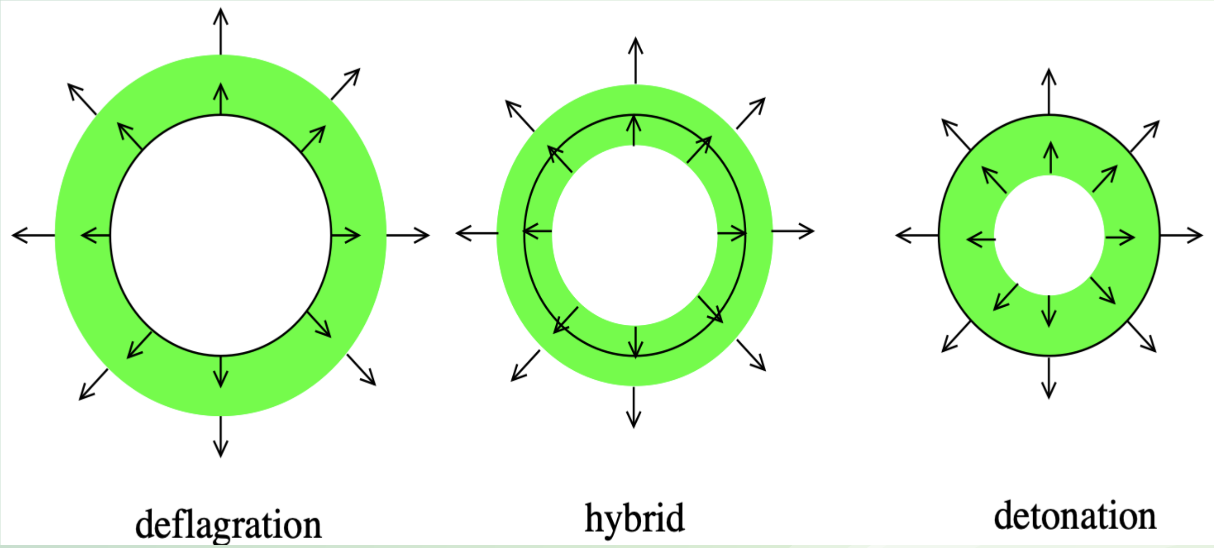

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Bubble expansion phase → scalar and fluid profiles are spherically symmetric

No anisotropic stresses → No gravitational wave production

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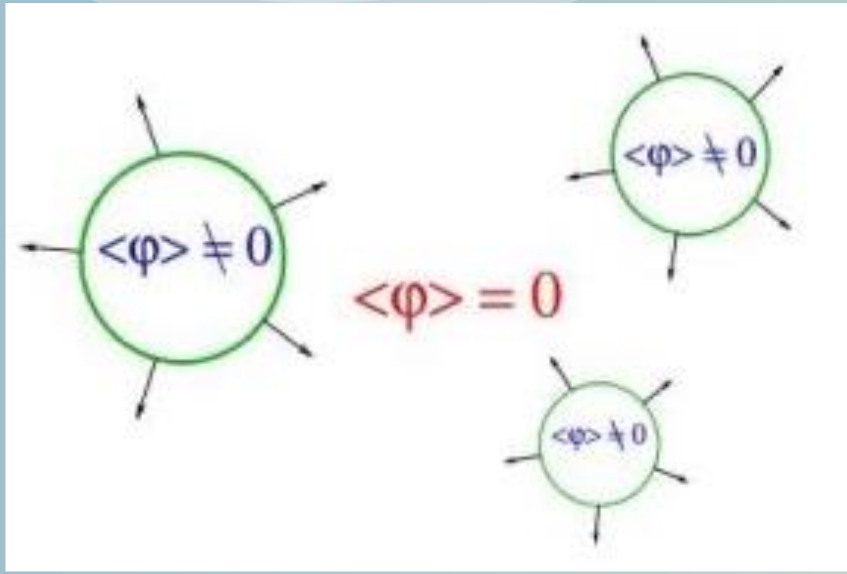

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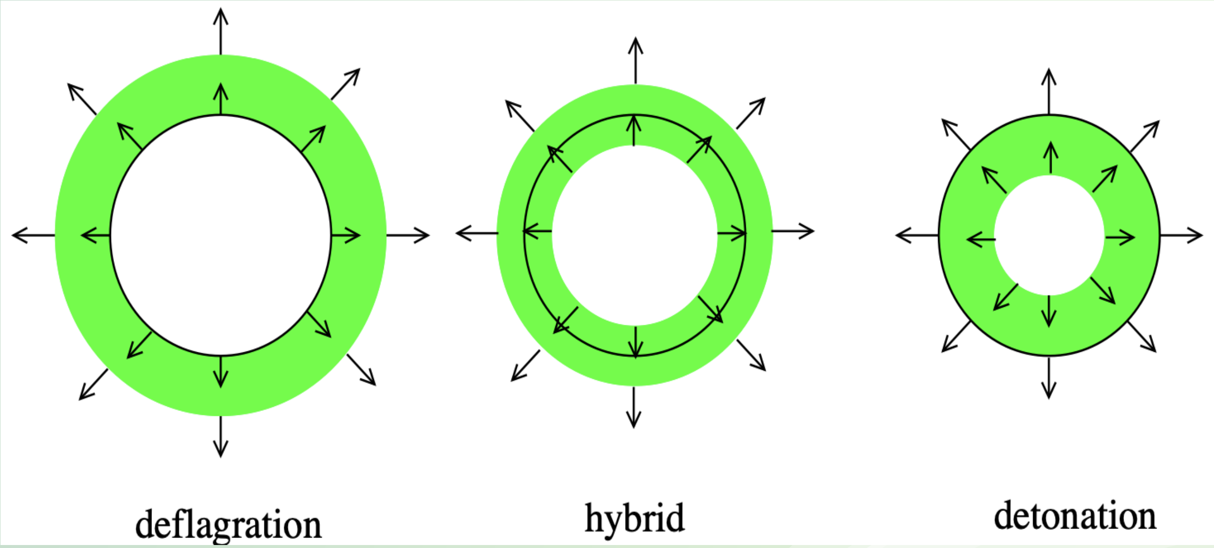

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Bubble collisions break spherical symmetry

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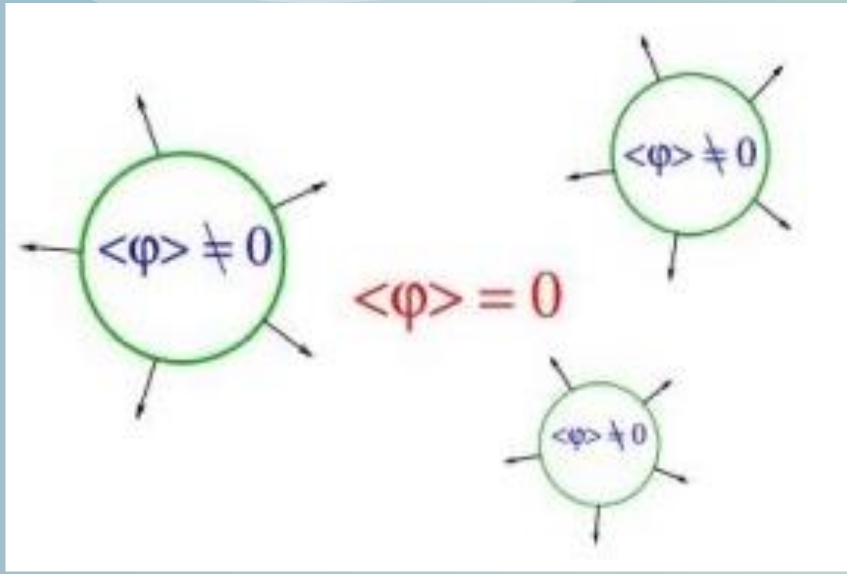

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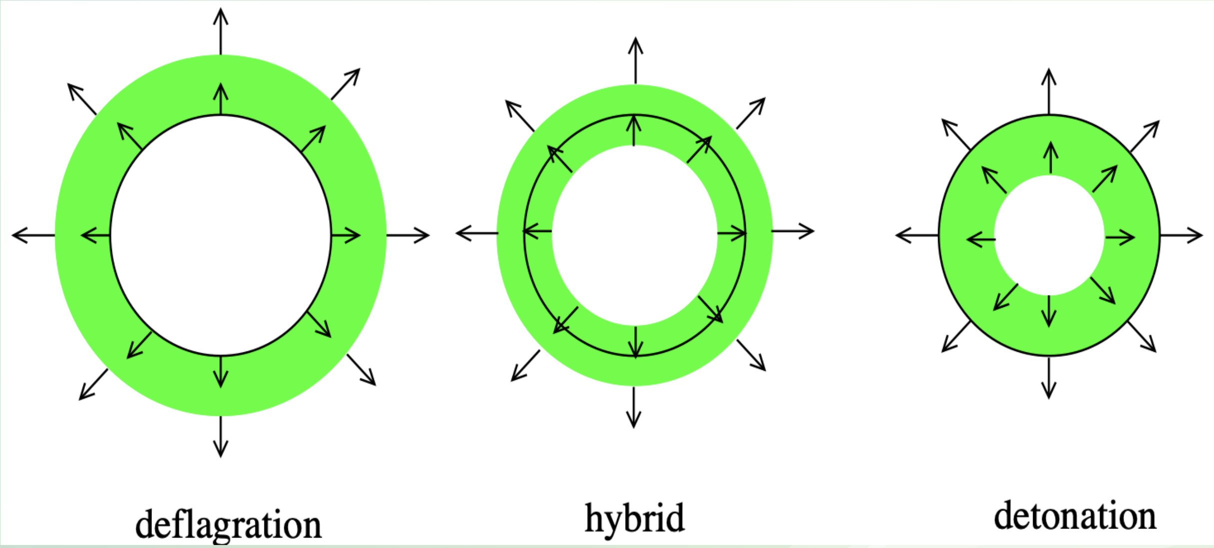

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Bubble collisions break spherical symmetry

Nonzero anisotropic stresses → scalar and fluid can produce gravitational waves

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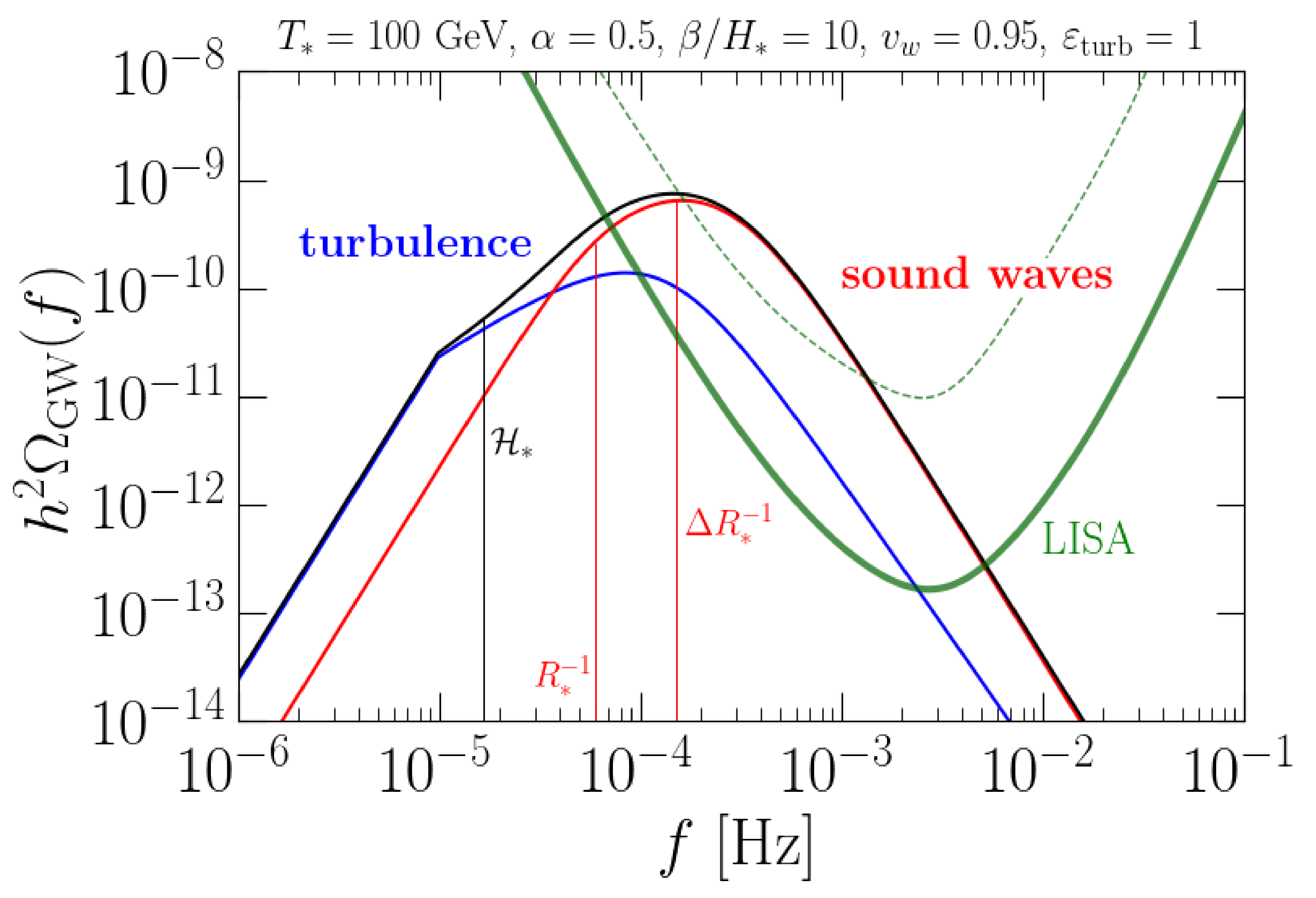
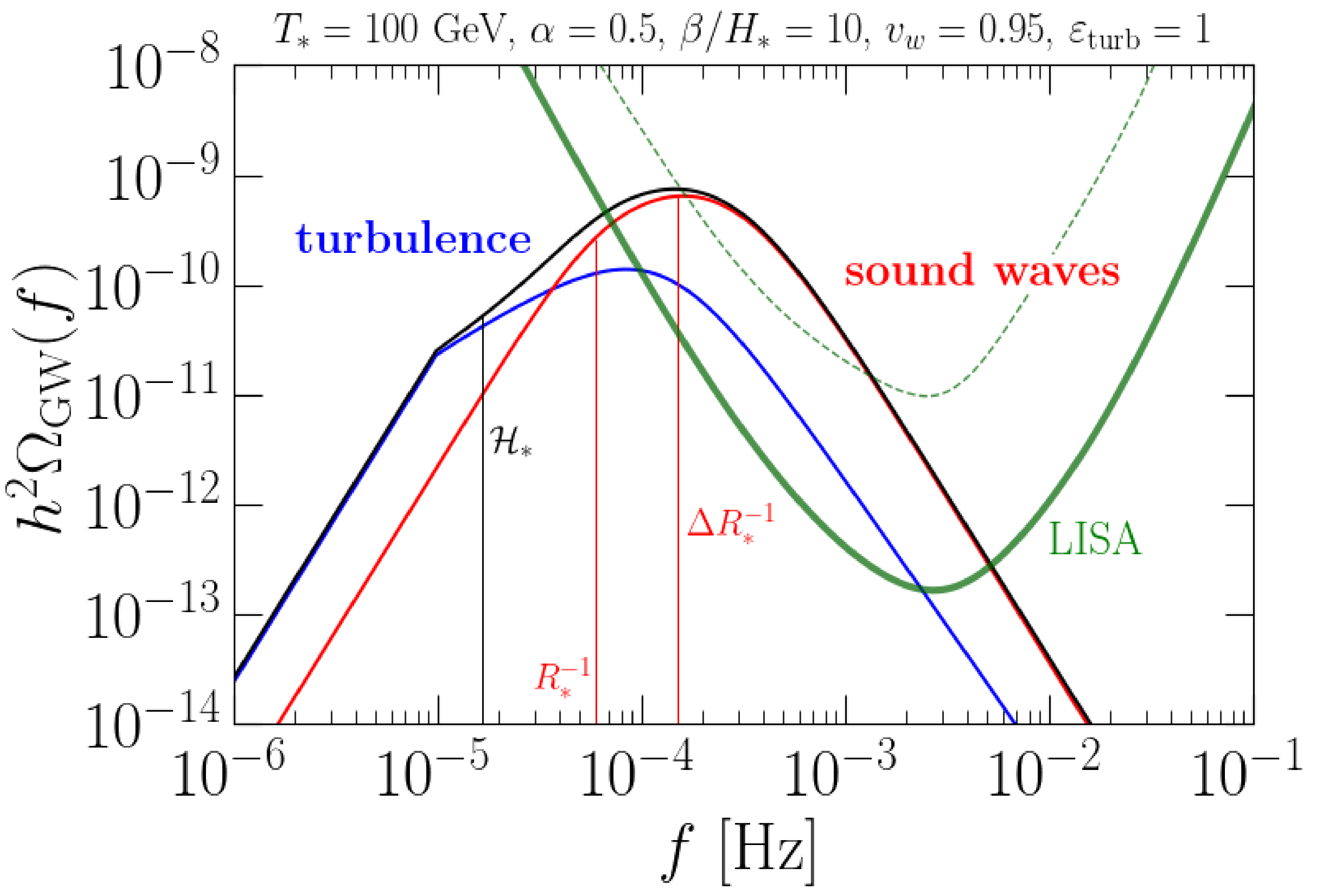


GW background from EW phase transition in the LISA sensitivity band!

Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Sound-shell model

Hindmarsh & Hijazi [1909.10040]

Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

GW background from EW phase transition in the LISA sensitivity band!

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Gravitational Waves from sound waves

[*Ongoing work* in collaboration with C. Caprini, S. Procacci, A. Roper Pol]

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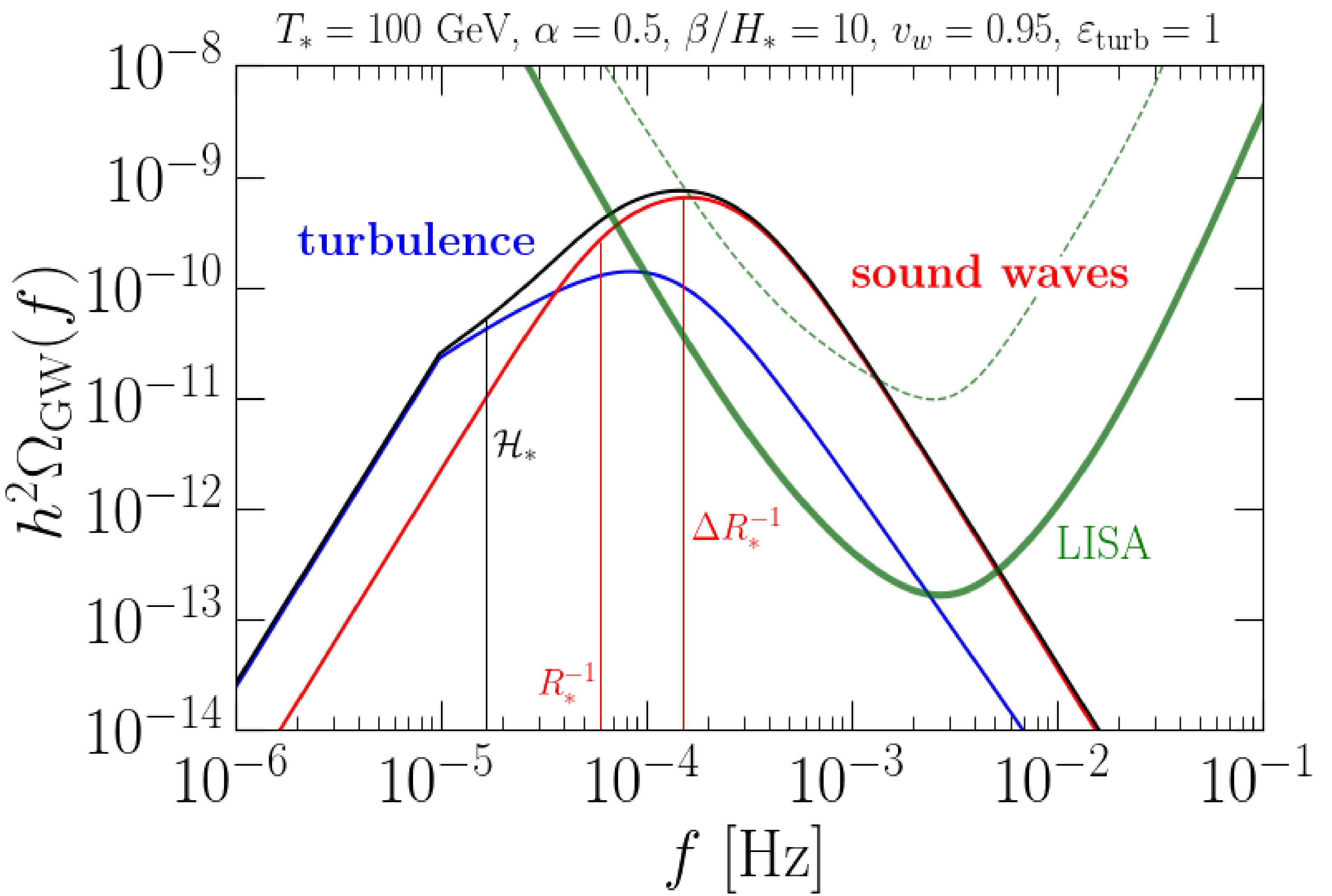

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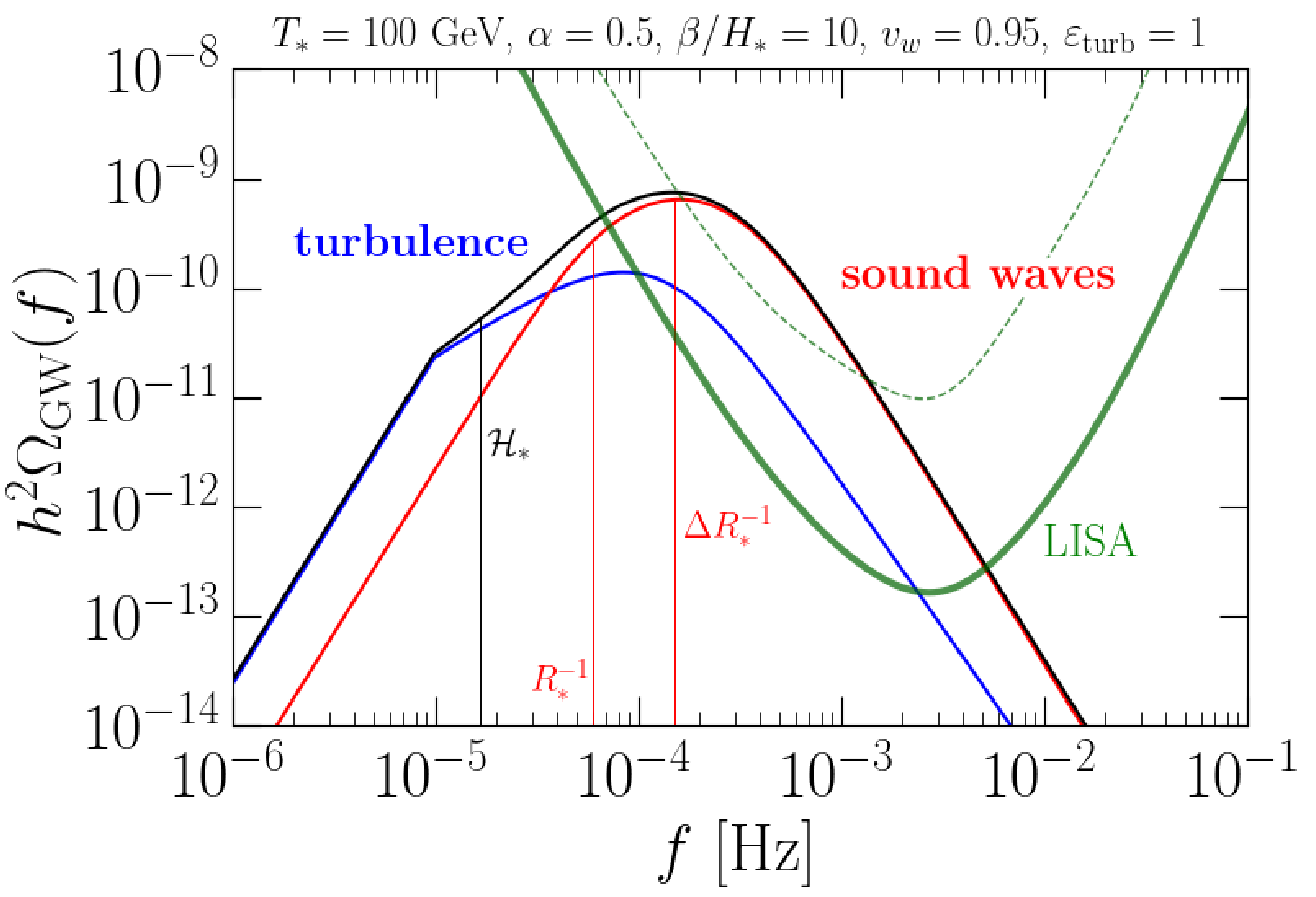

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Our work → What is the origin of the peak scales in the GW spectrum from sound waves?

← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Fluid perturbations from expanding scalar bubbles

$$T_{\mu\nu}^{tot} = w_{tot}\, u_\mu u_\nu + p_{tot}\, g_{\mu\nu} + \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi\, \partial^\sigma \phi \right)$$

$$w_{tot} = w - T \frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

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$$w_{tot} = w - T\frac{\partial V_{eff}(\phi, T)}{\partial T}$$

$$p_{tot} = p - V_{eff}(\phi, T)$$

$$\begin{cases} \nabla_\mu T^{\mu\nu}_{tot} = 0 \\ \nabla_\sigma(\partial^\sigma\phi) - \dfrac{\partial V}{\partial\phi} = \delta_{friction} \end{cases}$$

$\eta\, u^\mu \partial_\mu \phi$?

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$\eta\, u^\mu \partial_\mu \phi$?

Full picture requires lattice simulations

[1504.03291] [2409.03651][2505.17824]

What can we understand analytically?

Fluid perturbations from expanding scalar bubbles

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Simplifying assumptions:

Fluid perturbations from expanding scalar bubbles

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- Flat spacetime $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$

Fluid perturbations from expanding scalar bubbles

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- Flat spacetime $\quad g_{\mu\nu} \rightarrow \eta_{\mu\nu}$
- Bag equation of state $\longrightarrow$ (+) Symmetric phase, (−) Broken phase $\longrightarrow$

$$p_{\mathrm{tot}}^{\pm} = \frac{1}{3}a_\pm T_\pm^4 - \epsilon_\pm$$

$$e_{\mathrm{tot}}^{\pm} = a_\pm T_\pm^4 + \epsilon_\pm$$

$$w_{\mathrm{tot}}^{\pm} = e_{\mathrm{tot}}^{\pm} + p_{\mathrm{tot}}^{\pm}$$

Fluid perturbations from expanding scalar bubbles

$$T^{\mathrm{tot}}_{\mu\nu} = w_{\mathrm{tot}}\, u_\mu u_\nu + p_{\mathrm{tot}}\, g_{\mu\nu} + \cancel{\partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\, \partial^\sigma\phi\right)}$$

Simplifying assumptions:

- Flat spacetime $g_{\mu\nu} \to \eta_{\mu\nu}$
- Bag equation of state $\longrightarrow$ (+) Symmetric phase (−) Broken phase $\longrightarrow$
- Neglect scalar field profiles

$$p^{\pm}_{\mathrm{tot}} = \frac{1}{3} a_\pm T^4_\pm - \epsilon_\pm$$

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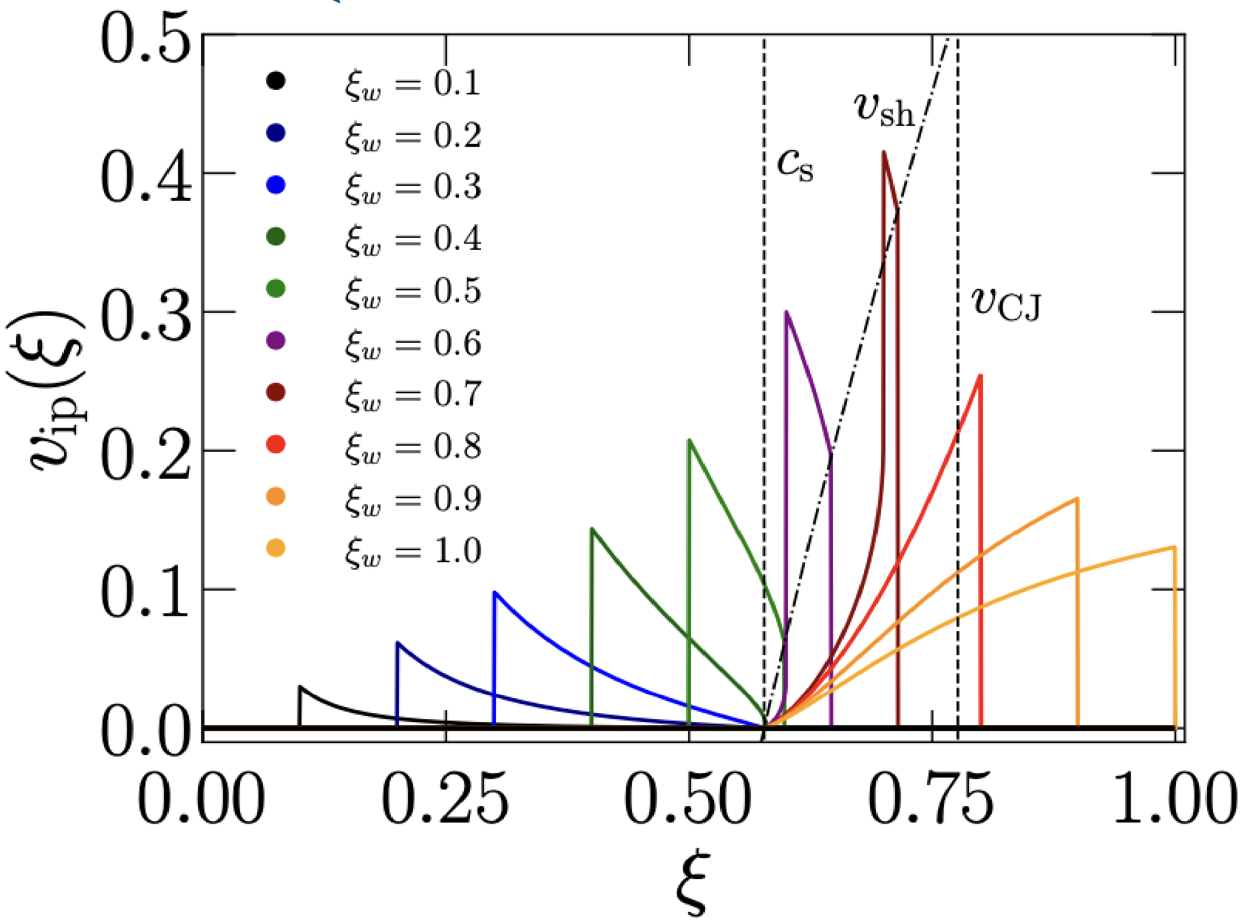
Fluid perturbations from expanding scalar bubbles

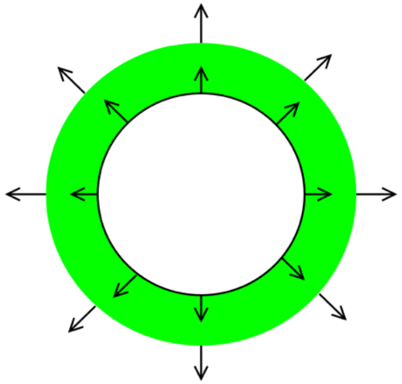
pip install cosmoGW



Fluid perturbations from expanding scalar bubbles

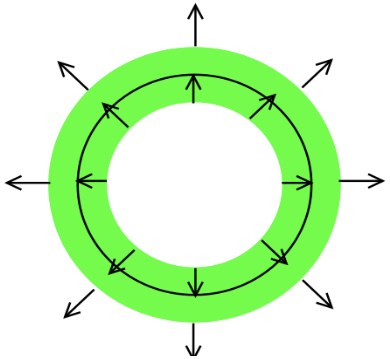
DEFLAGRATIONS

$\xi_w < c_s$



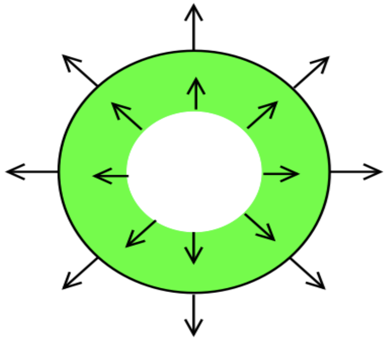
HYBRIDS

$c_s < \xi_w < v_{CJ}(\alpha)$



DETONATIONS

$\xi_w > v_{CJ}(\alpha)$



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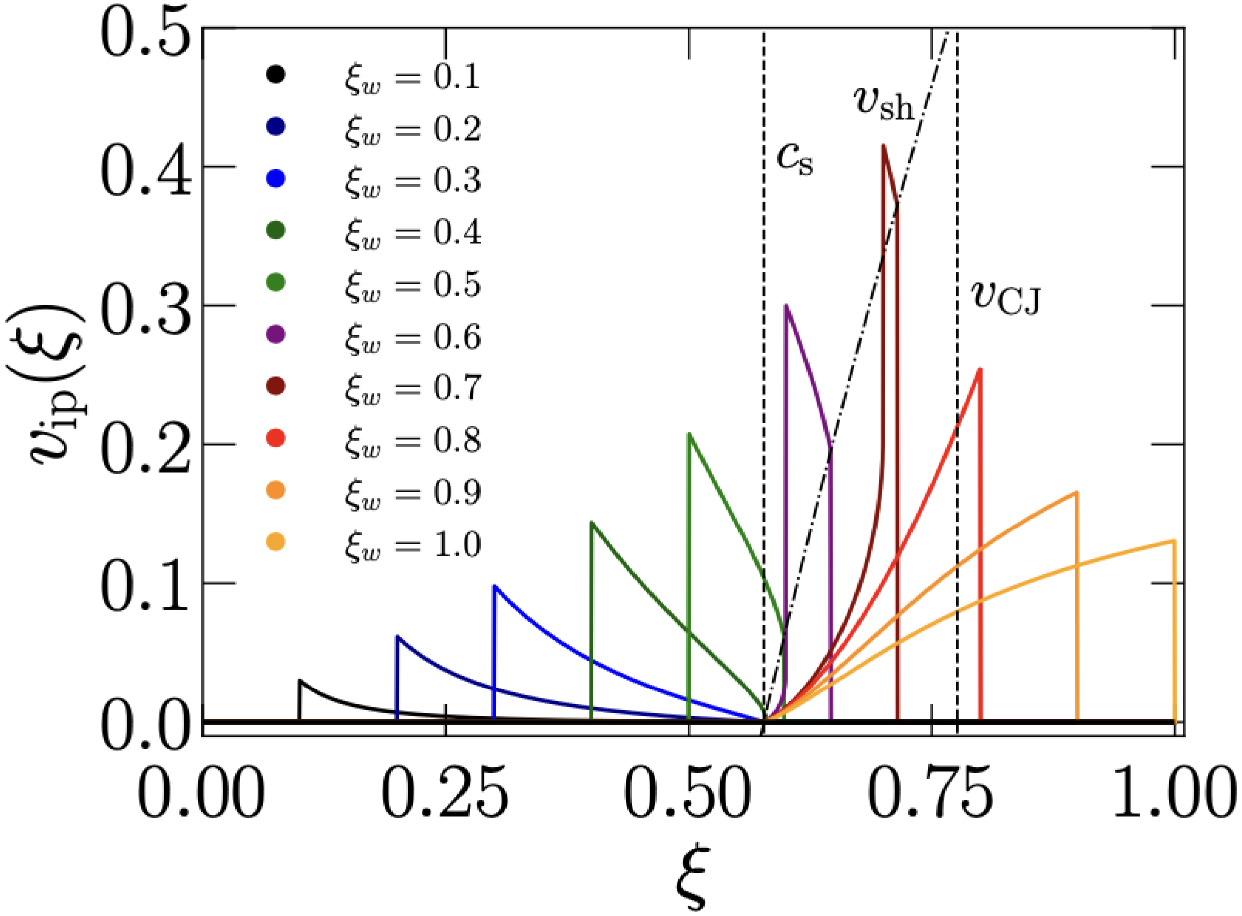
$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2+3\alpha)}}{\sqrt{3}(1+\alpha)}$$



Fluid perturbations from expanding scalar bubbles

Properties of the profiles:

- Compact support
  $v_{ip}(\xi) \neq 0 \quad for \;\; \xi_b < \xi < \xi_f$

- Discontinuity at ξ_w

- Deflagrations and hybrids have an additional discontinuity at $\xi = v_{sh}$

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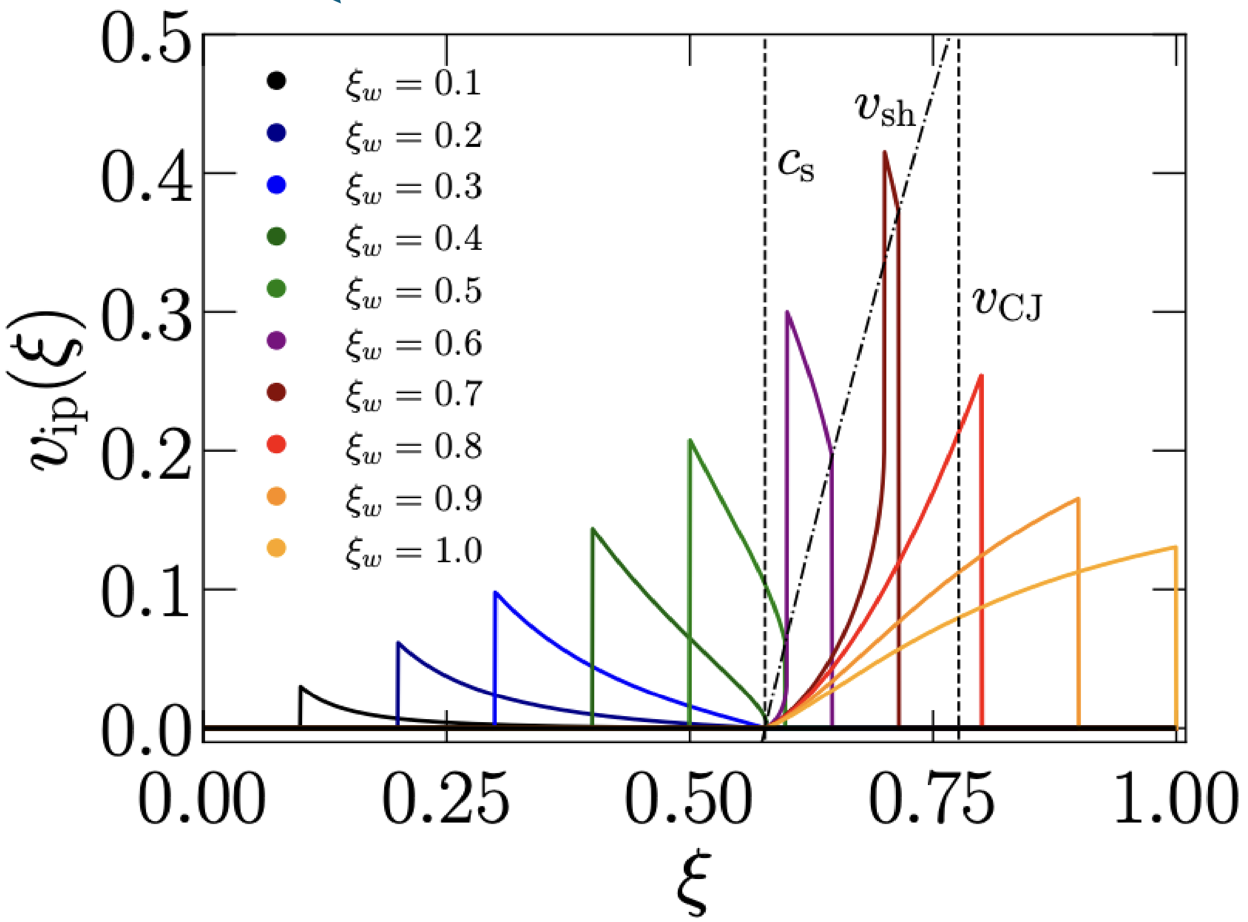
$$v_{CJ}(\alpha) = \frac{1 + \sqrt{\alpha(2 + 3\alpha)}}{\sqrt{3}(1 + \alpha)}$$



Evolution of the fluid perturbations: *before* collisions

The kinetic spectrum in the bubble expansion phase is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

$$\mathbf{v}^{(\mathrm{n})}(\mathrm{t},\mathbf{k}) = -\mathrm{i}\left[\mathrm{t}^{(\mathrm{n})}\right]^3 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}_0^{(\mathrm{n})}}\,\hat{\mathbf{k}}\, f'(z)$$

$$f'(z) = -4\pi \int_0^\infty \mathrm{j}_1(\mathrm{z}\xi)\,\xi^2\,\mathrm{v}_{\mathrm{ip}}(\xi)\,\mathrm{d}\xi$$

Evolution of the fluid perturbations: *before* collisions

The kinetic spectrum in the bubble expansion phase is an average over stochastic realizations

$$\mathbf{v}^{(n)}(t,\mathbf{k}) = -i\left[t^{(n)}\right]^3 e^{i\mathbf{k}\cdot\mathbf{x}_0^{(n)}}\,\hat{\mathbf{k}}\, f'(z)$$

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$$\left\langle v_i(t,\boldsymbol{k})\, v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

$$\left\langle v_i(t,\boldsymbol{k})\, v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)}} = \widehat{\boldsymbol{k}}_i\, \widehat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, n_b(t)(t-t_0)^6 |f'(z)|^2$$

Average over nucleation locations (homogeneously distributed)

Properties of $|f'(z)|^2$

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)}} = \widehat{\boldsymbol{k}}_i\,\widehat{\boldsymbol{k}}_j\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,n_b(t)(t-t_0)^6|f'(z)|^2$$

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From compact support of $v_{ip}(\xi)$

Large scales $k = z/t^{(n)} \to 0$

$$|f'(z)|^2 \to |f_0'|^2\, z^2$$

Properties of $|f'(z)|^2$

$$\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\rangle_{\boldsymbol{x}_0^{(n)}} = \hat{\boldsymbol{k}}_i\, \hat{\boldsymbol{k}}_j\, \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\, n_b(t)(t-t_0)^6 |f'(z)|^2$$

From compact support of $v_{ip}(\xi)$

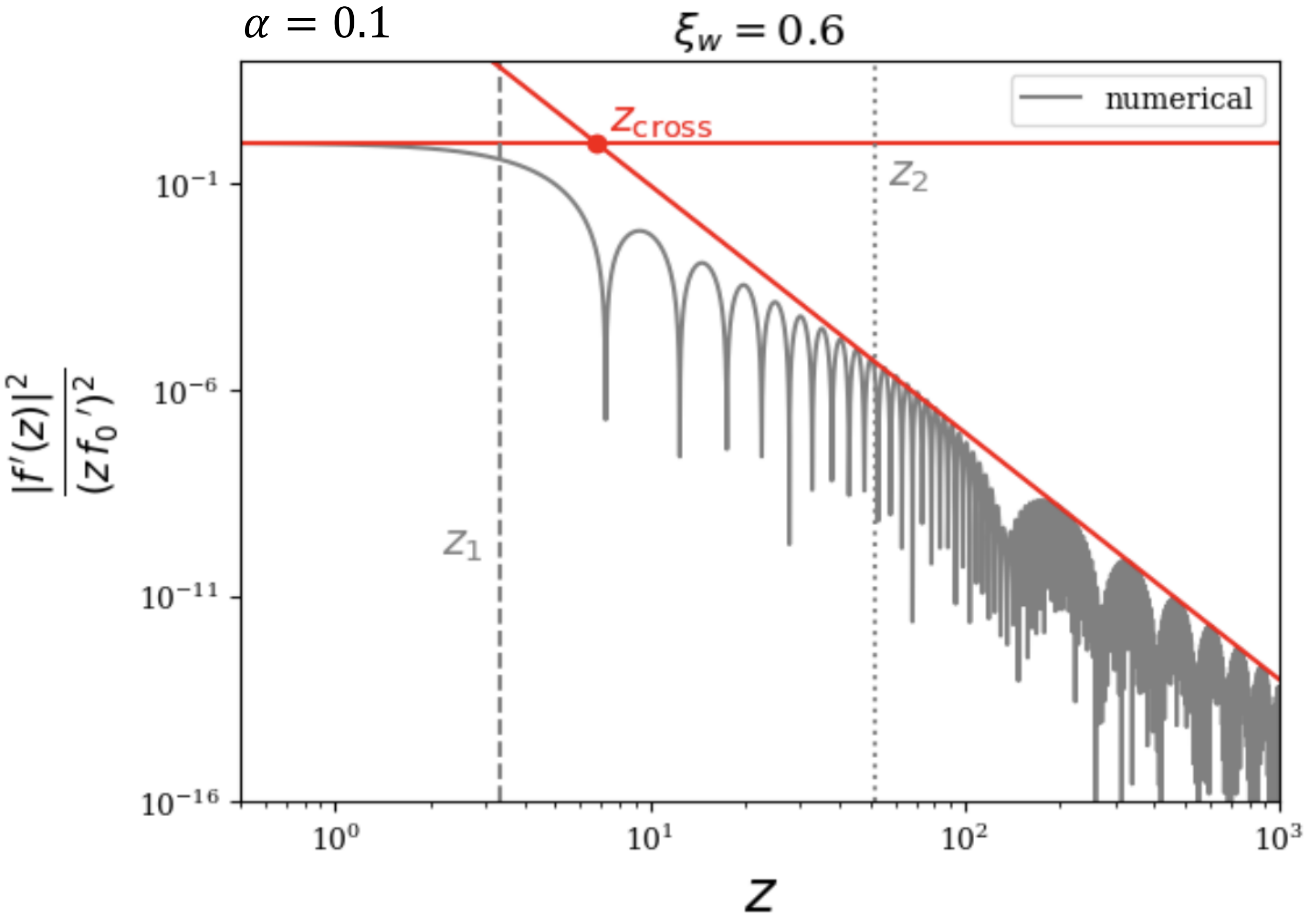
Large scales $k = z/t^{(n)} \to 0$ $\quad |f'(z)|^2 \to |f_0'|^2\, z^2$

Small scales $k = z/t^{(n)} \to \infty$ $\quad |f'(z)|^2 \to |f_\infty'|^2\, z^{-4}$

From the discontinuities of $v_{ip}(\xi)$

Properties of $|f'(z)|^2$

The $\sim z^2$ ends around $z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$



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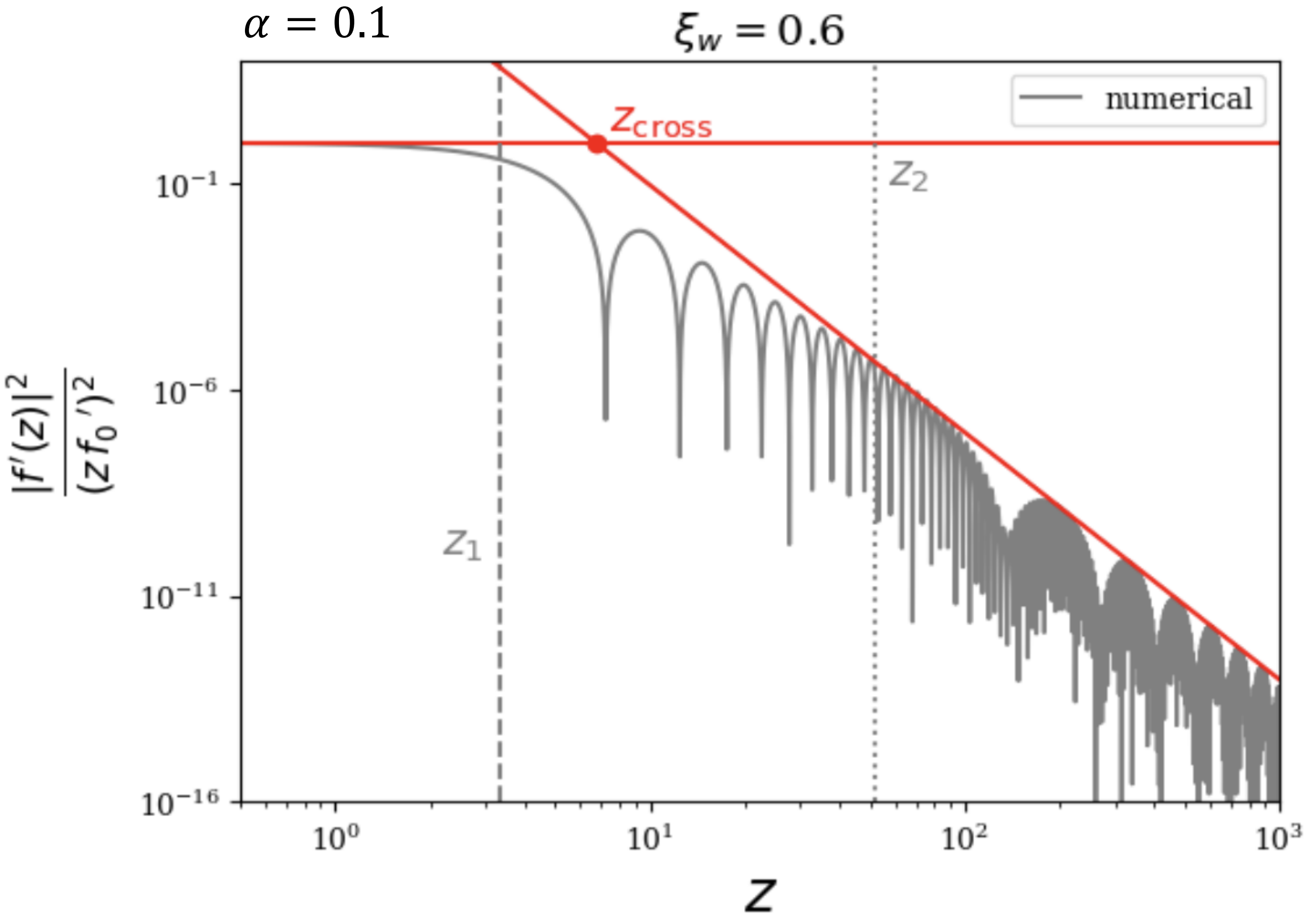

The $\sim z^{-4}$ begins around

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$

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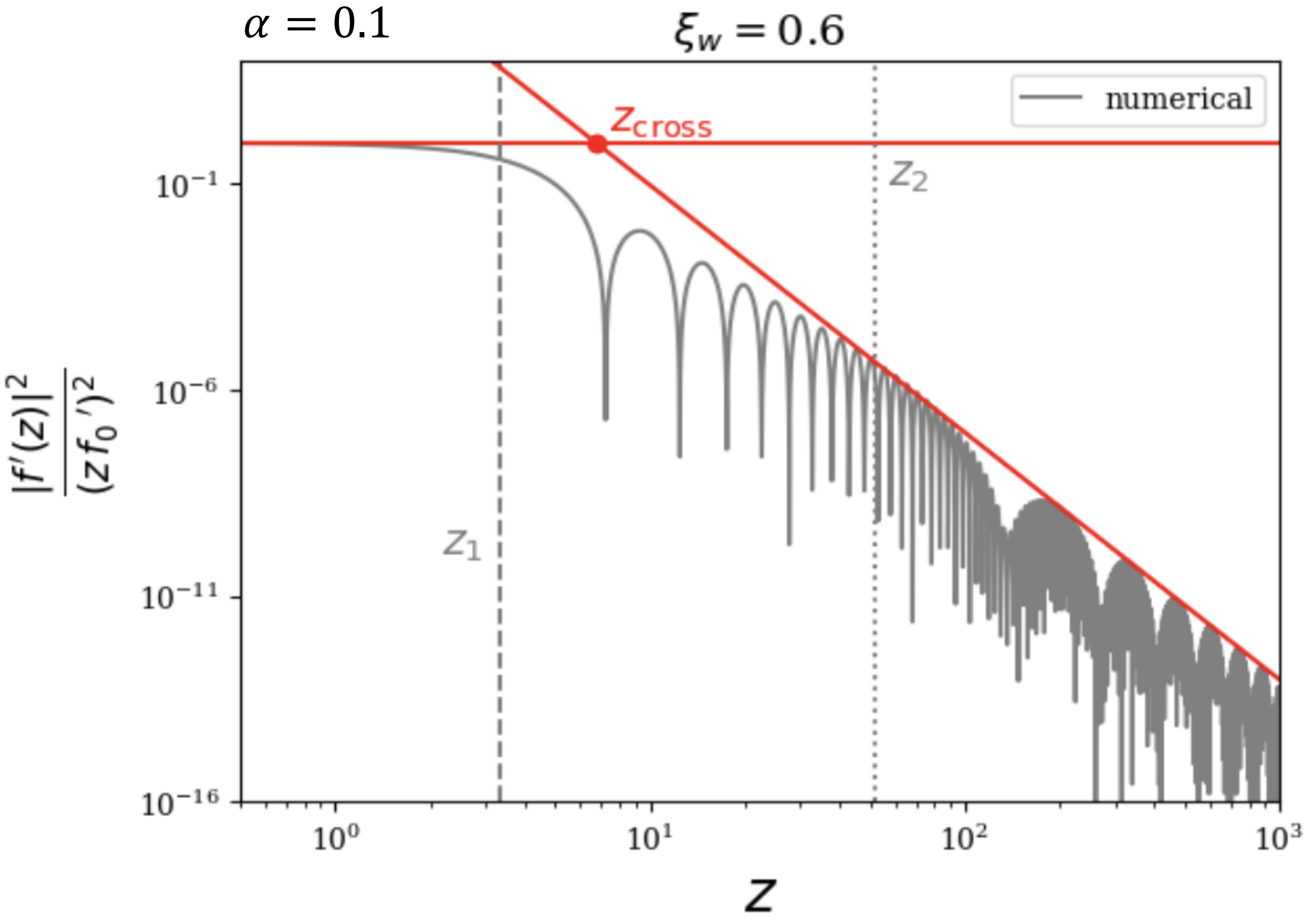

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$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

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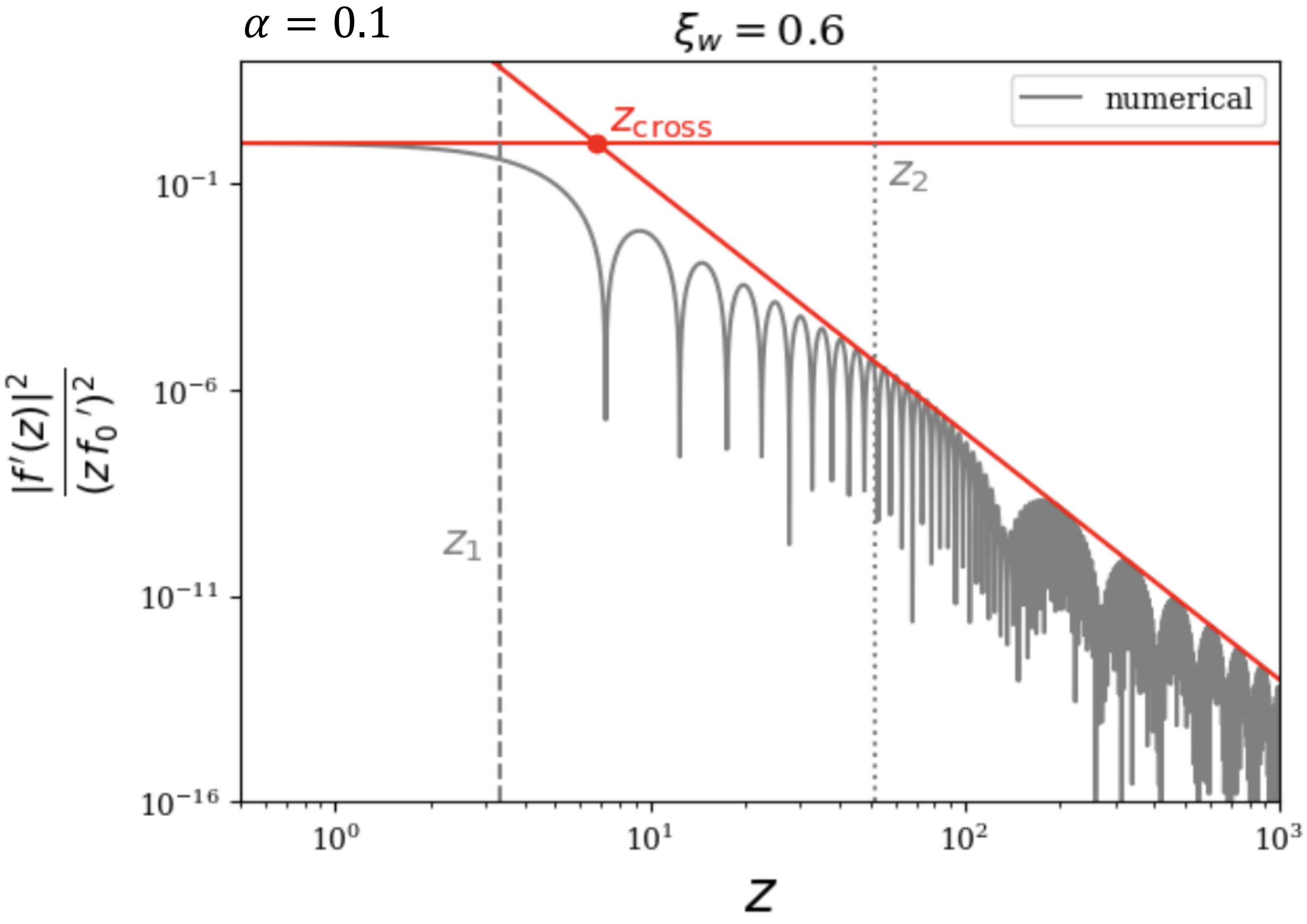

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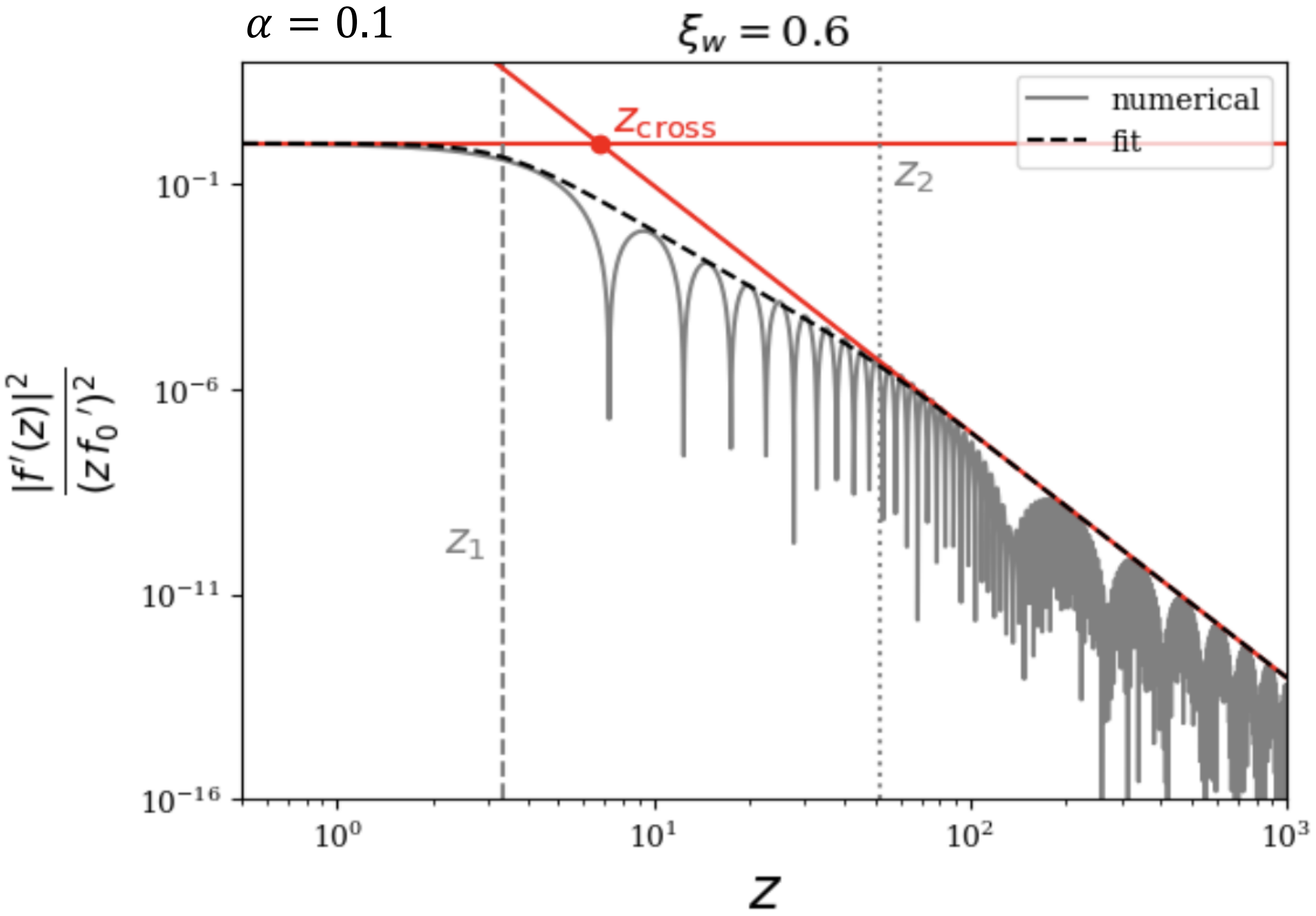
$\xi_f - \xi_b \propto \Delta R_*$ (sound shell thickness)

$\xi_f - \xi_w = \xi_{sh} - \xi_w$ distance between discontinuities (for hybrids)

Properties of $|f'(z)|^2$

$$|f'(z)|^2_{env} = |f_0'|^2\, z^2 \left[1 + \left(\frac{z}{z_1}\right)^{a_1}\right]^{\frac{\gamma-2}{a_1}} \left[1 + \left(\frac{z}{z_2}\right)^{a_2}\right]^{\frac{-\gamma-4}{a_2}}$$



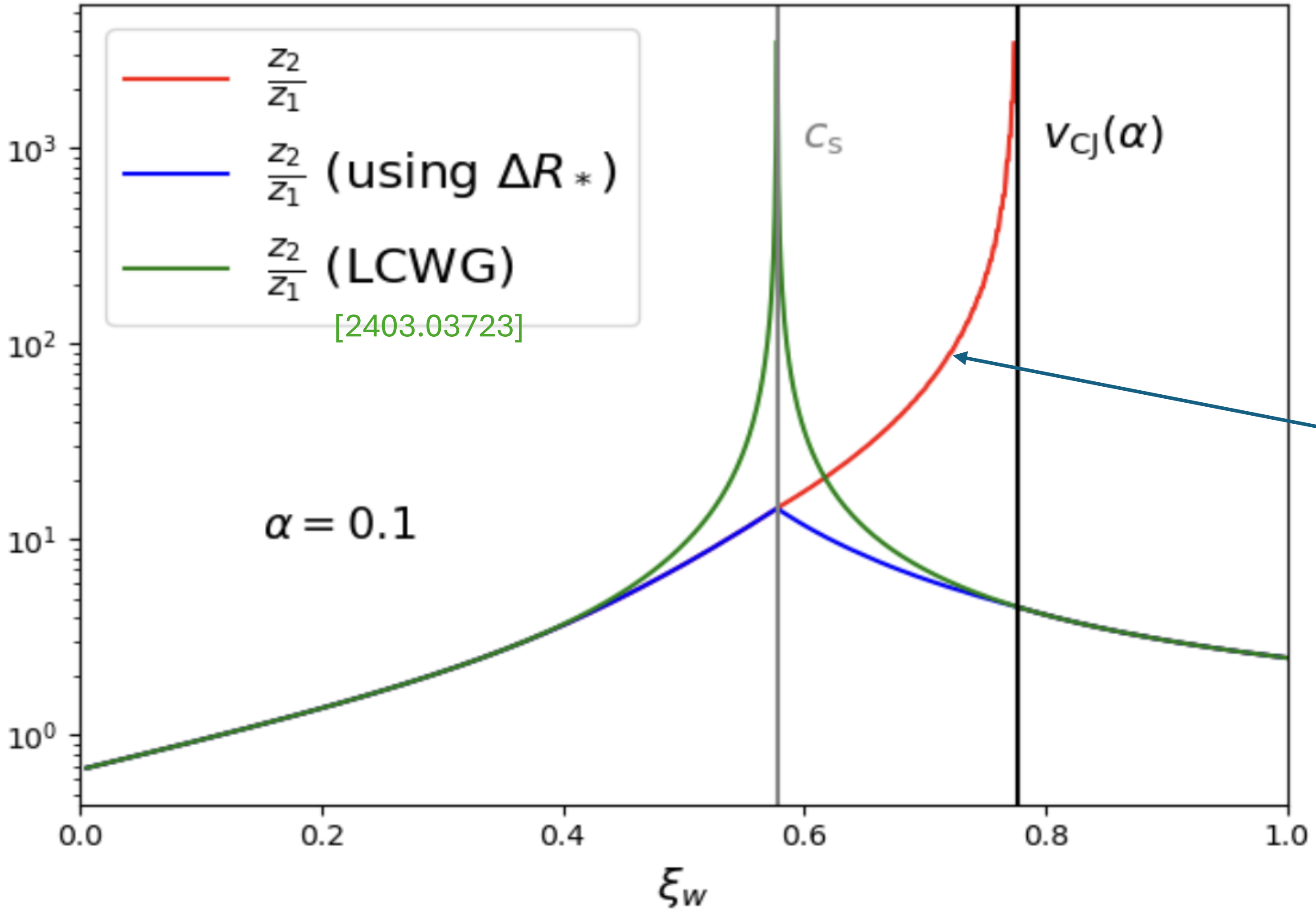
Double broken power law fit

$$\gamma = 2\left[1 - 3\frac{\log(z_2/z_{cross})}{\log(z_2/z_1)}\right]$$

Scales of $|f'(z)|^2$

$$z_1 \approx \frac{3\pi}{2}\left(\xi_f + \xi_b\right)^{-1}$$

$$z_2 \approx \pi \times \begin{cases} \left(\xi_f - \xi_b\right)^{-1} & (\xi_w < c_s) \\ \left(\xi_f - \xi_w\right)^{-1} & (c_s < \xi_w < v_{CJ}) \\ \left(\xi_f - \xi_b\right)^{-1} & (\xi_w > v_{CJ}) \end{cases}$$



Much broader spectrum for hybrids than using

$z_2 = \pi \times \left(\xi_f - \xi_b\right)^{-1} \propto \Delta R_*^{-1}$

$z_2 = \pi \times |c_s - \xi_w|^{-1}$ (Lisa Cosmology Working Group)

Evolution of the fluid perturbations: *before* collisions

The kinetic spectrum in the bubble expansion phase
is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k}) v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

Average over nucleation times

Evolution of the fluid perturbations: *across* collisions

The kinetic spectrum in the bubble expansion phase
is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k})\, v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\, t_0^{(n)}}$$

Average over nucleation times
and collision times

Evolution of the fluid perturbations: *across* collisions

The kinetic spectrum in the bubble expansion phase is an average over stochastic realizations

$$\left\langle v_i(t,\boldsymbol{k})v_j^*(t,\boldsymbol{k}')\right\rangle_{\boldsymbol{x}_0^{(n)},\,t_0^{(n)}}$$

Average over nucleation times and collision times

We can model the nucleation history with a normalized lifetime distribution $\nu(T)$

$$F_L(t_{coll},k) = n_b(t_{coll})\int_0^\infty dT\,\nu(T)T^6|f'(kT)|^2$$

Kinetic spectrum at collisions

Hindmarsh & Hijazi [1909.10040]

Evolution of the fluid perturbations: *across* collisions

$$F_L(t_{coll}, k) = n_b(t_{coll}) \int_0^\infty dT\, \nu(T) T^6 |f'(kT)|^2$$ ← Kinetic spectrum at collisions

Large scales $k \to 0$ $\quad F_L \to k^2 F_L^0$ $\quad$ k^2 ends around $\quad k_1 \simeq \beta \dfrac{z_1}{5.7}$

(exponential nucleation)

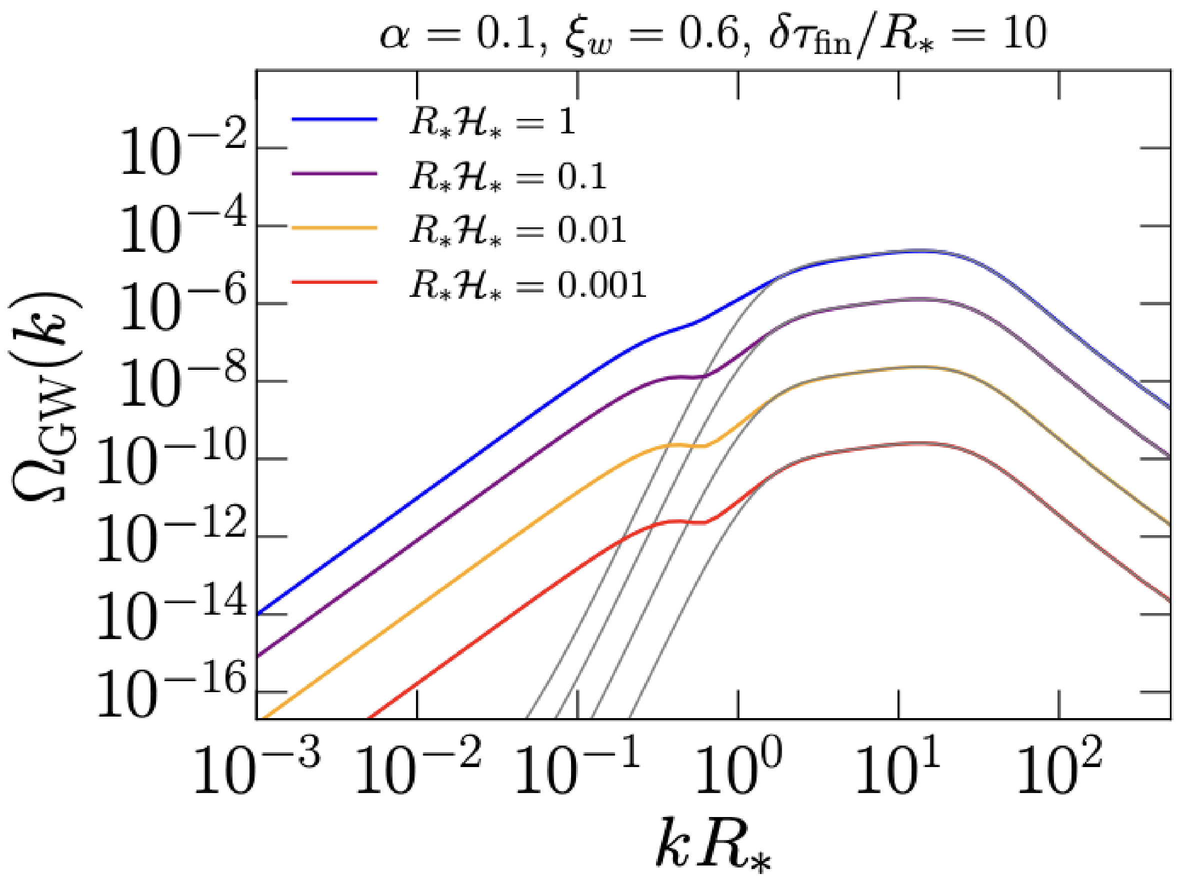
Small scales $k \to \infty$ $\quad F_L \to k^{-4} F_L^{env}$ $\quad$ k^{-4} starts around $\quad k_2 \simeq \beta \dfrac{z_2}{2.4}$

Consequences for the gravitational wave spectrum

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

UETC for sound-waves computed from the kinetic spectrum

Hindmarsh & Hijazi [1909.10040]



Roper Pol, Procacci, Caprini [2308.12943]

Double broken power law fit for the peak of Ω_{GW} with scales

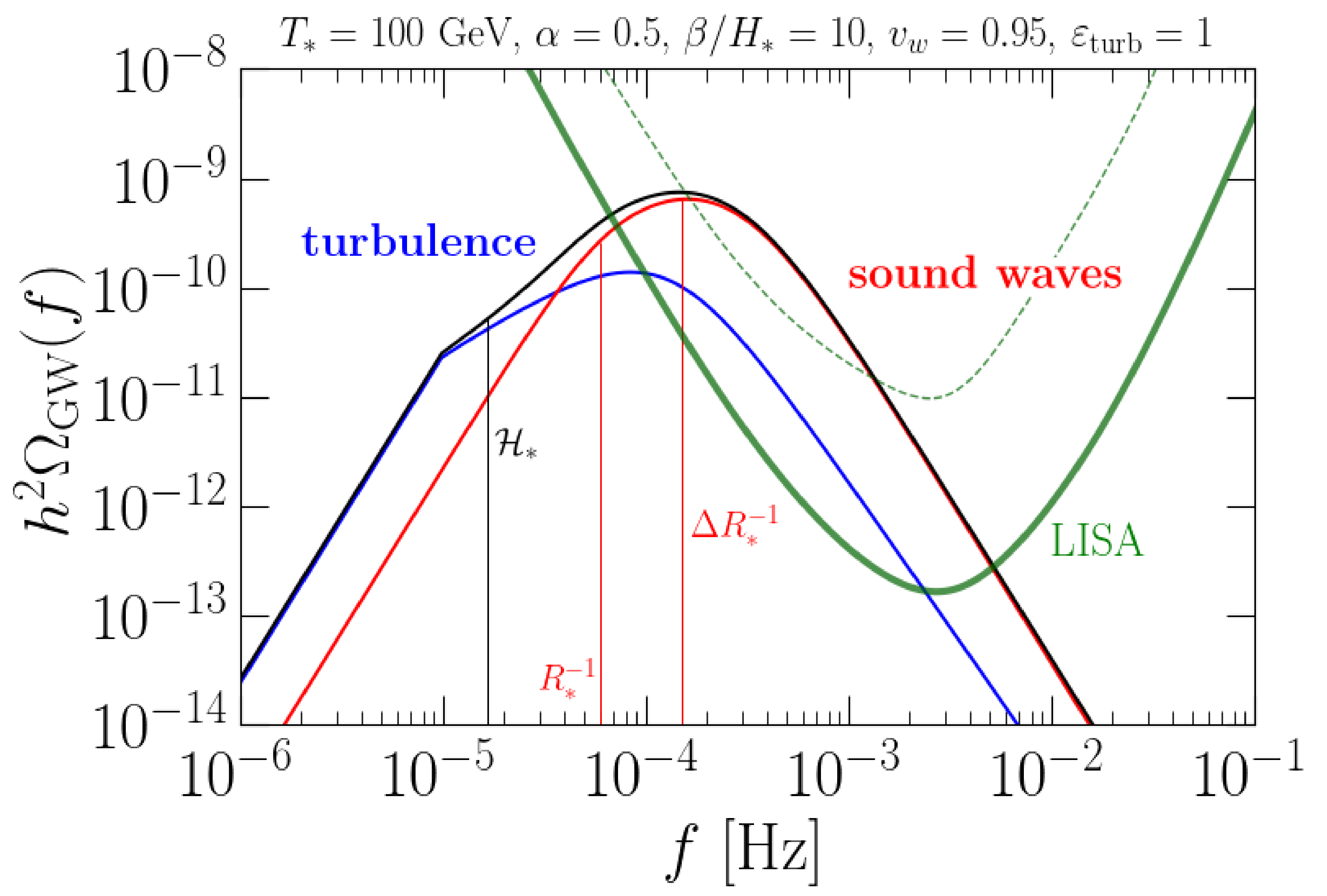
$$k_1^{GW} \approx 1.2 \times k_1$$

$$k_2^{GW} \approx 1.2 \times k_2$$

Gravitational Waves from turbulence

[*Ongoing works* in collaboration with C. Caprini, A. Roper Pol, M. Salomé (theory)
D. Figueroa, K. Marschall, A. Roper Pol (simulations)]

Introduction: first-order phase transitions and gravitational waves



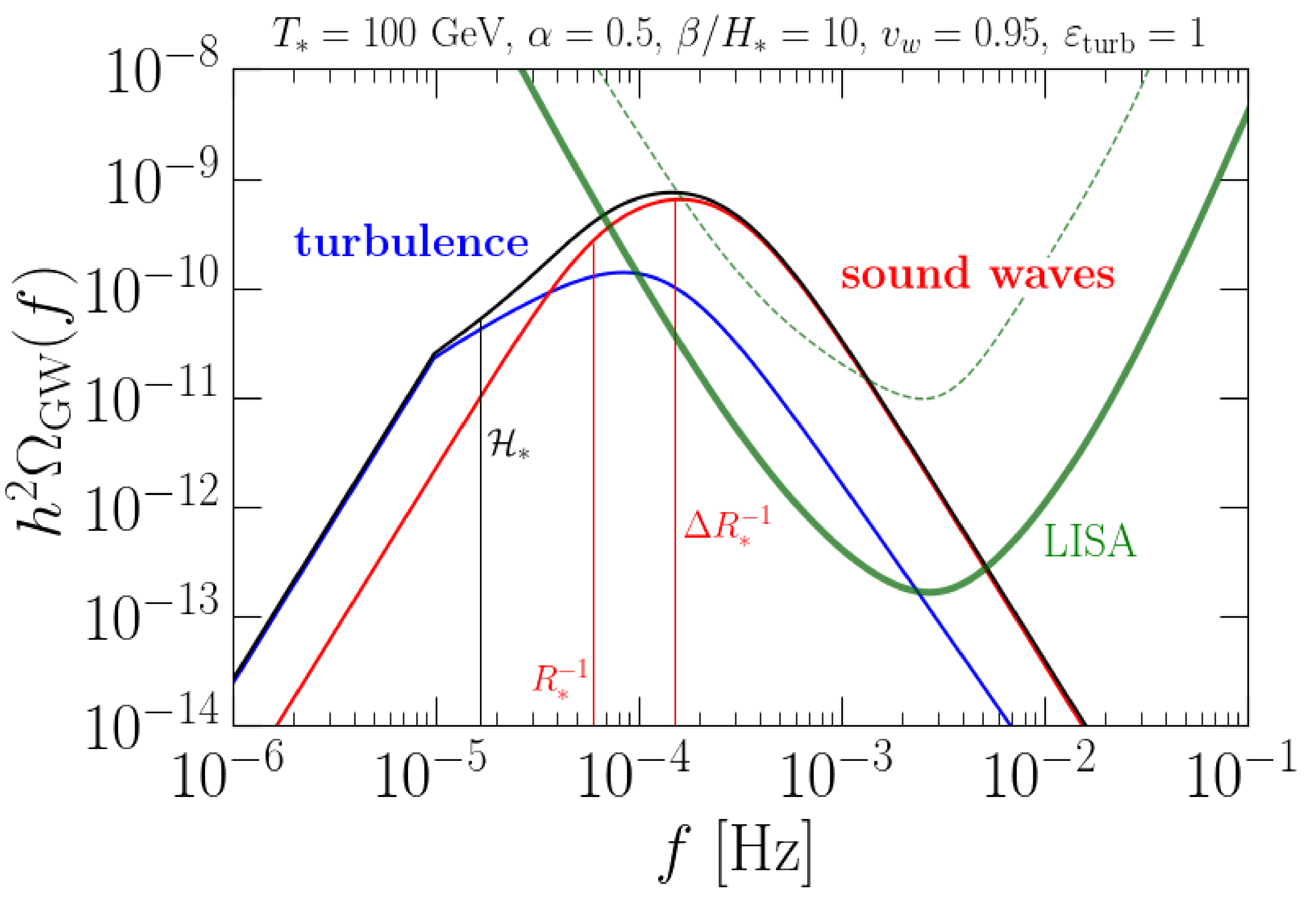
Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Introduction: first-order phase transitions and gravitational waves



Constant-in-time model

Roper Pol, Caprini et al. [2201.05630]

How long does it take for turbulence to develop?

Which fraction of energy goes into it?

How does the sourcing period affect the final GW spectrum?

How does turbulence evolve in the fully relativistic regime?

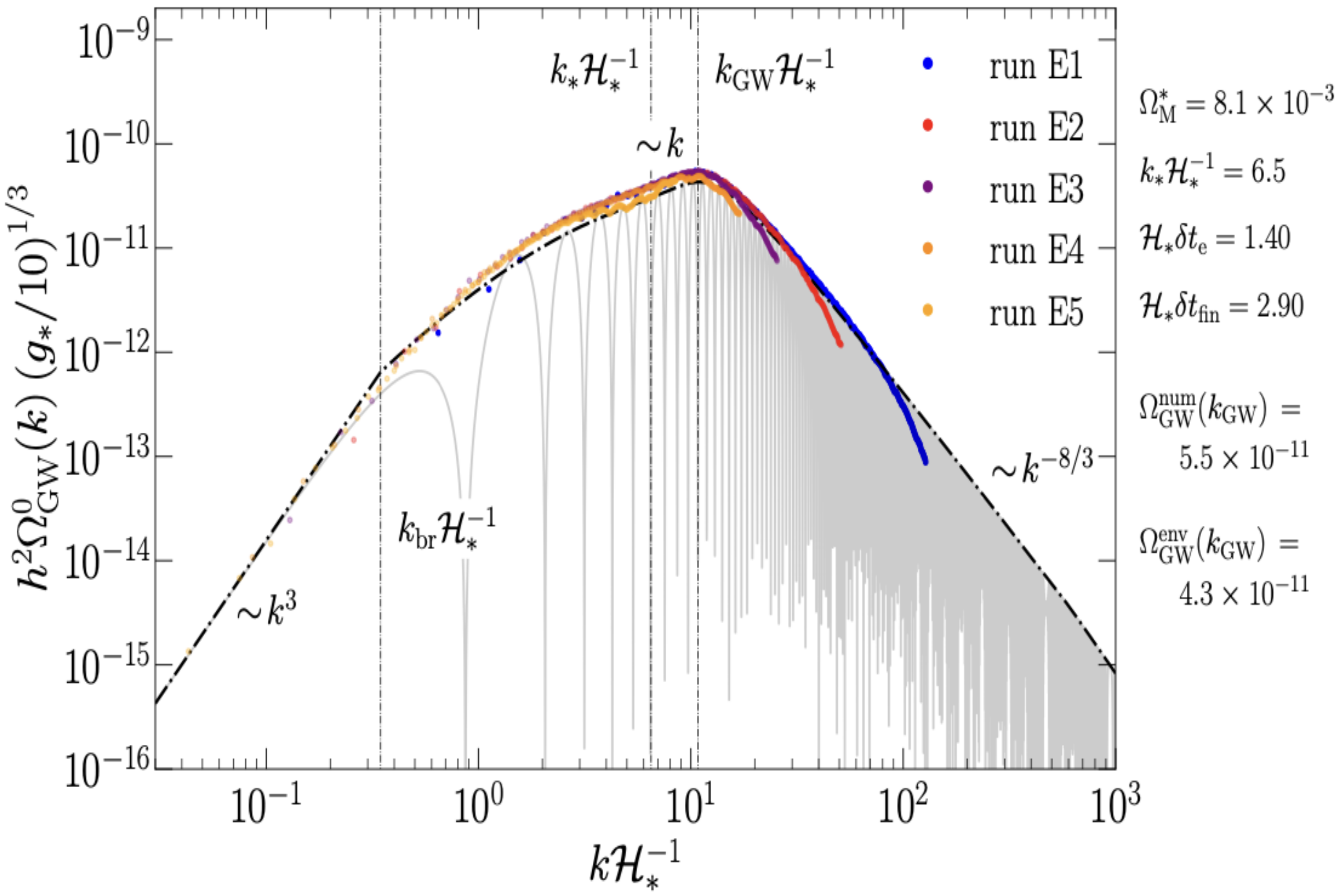
← Credits: Alberto Roper Pol

Lisa Cosmology Working Group [2403.03723]

Gravitational Waves from *decaying* MHD turbulence

- In a cosmological phase transition scalar field gradients can generate magnetic fields (Vachaspati et al. 2021) leading, due to the high conductivity of the primordial plasma (Arnold et al. 2003), to MHD turbulence

- The GW spectrum from numerical simulations of *decaying* MHD turbulence can be described with the constant-in-time model (Roper Pol et al. [2201.05630])



Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0, k) = 3\, \mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1} \frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

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Assuming that the source is slowly decaying* for $\tau_* < \tau < \tau_{fin}$ $\longrightarrow$ $E_\Pi(k, \tau_1, \tau_2) = E_\Pi^*(k)$

*with respect to the light crossing time at wavenumber k

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$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\, \Delta^2(k,\tau_0)$$

*with respect to the light crossing time at wavenumber k

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$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6\times 10^{-5}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

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Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

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$$E_\Pi^*(k) \propto \langle \Pi_{ij}(\tau_*,k)\Pi_{ij}^*(\tau_*,k)\rangle$$

*with respect to the light crossing time at wavenumber k

Constant-in-time model for the UETC of the source

$$\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\frac{d\tau_2}{\tau_2} \cos k(\tau_0-\tau_1)\cos k(\tau_0-\tau_2)\, E_\Pi(k,\tau_1,\tau_2)$$

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$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\,\Delta^2(k,\tau_0)$$

$$\Delta(k,\tau_0) \equiv \int_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{\mathrm{d}\tilde{\tau}}{\tilde{\tau}} \cos k(\tau_0-\tilde{\tau})$$

$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6\times 10^{-5}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

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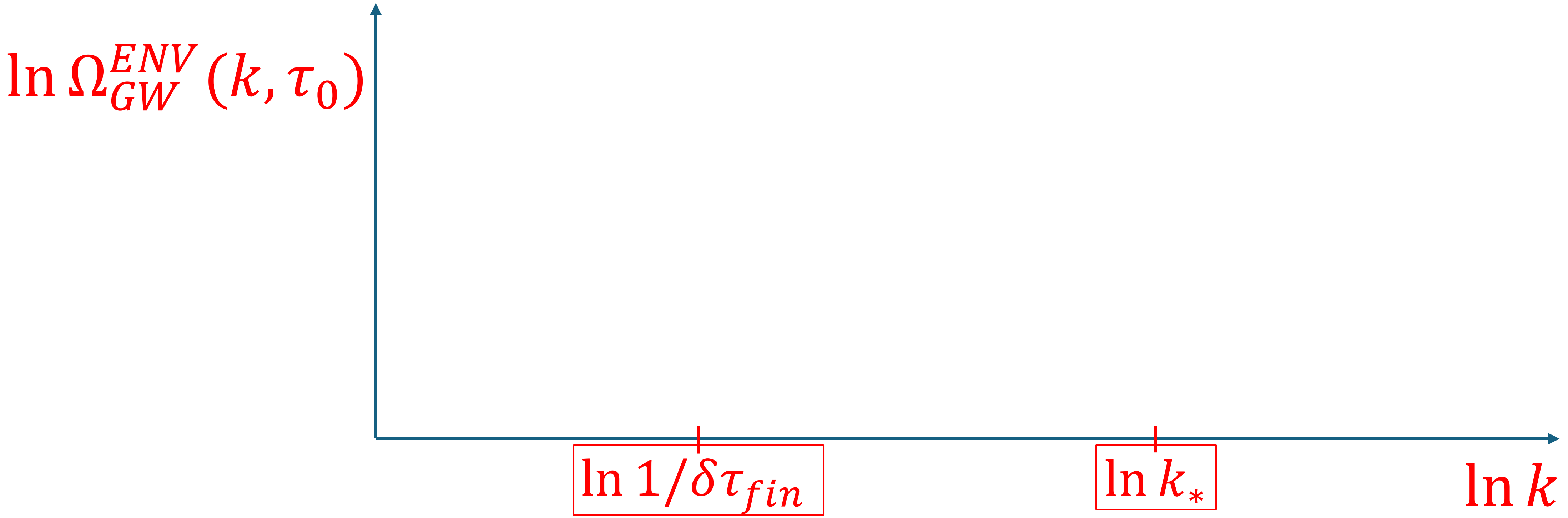
Constant-in-time model for the UETC of the source

$\delta\tau_{fin} = \tau_f - \tau_*$

$$\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW}\,E_\Pi^*(k)\,\Delta_0^2(k,\tau_{fin})$$

causality

Assuming for the UETC $E_\Pi^*(k) \sim \begin{cases} k^3 & k < k_* \\ k^{-b} & k > k_* \end{cases}$



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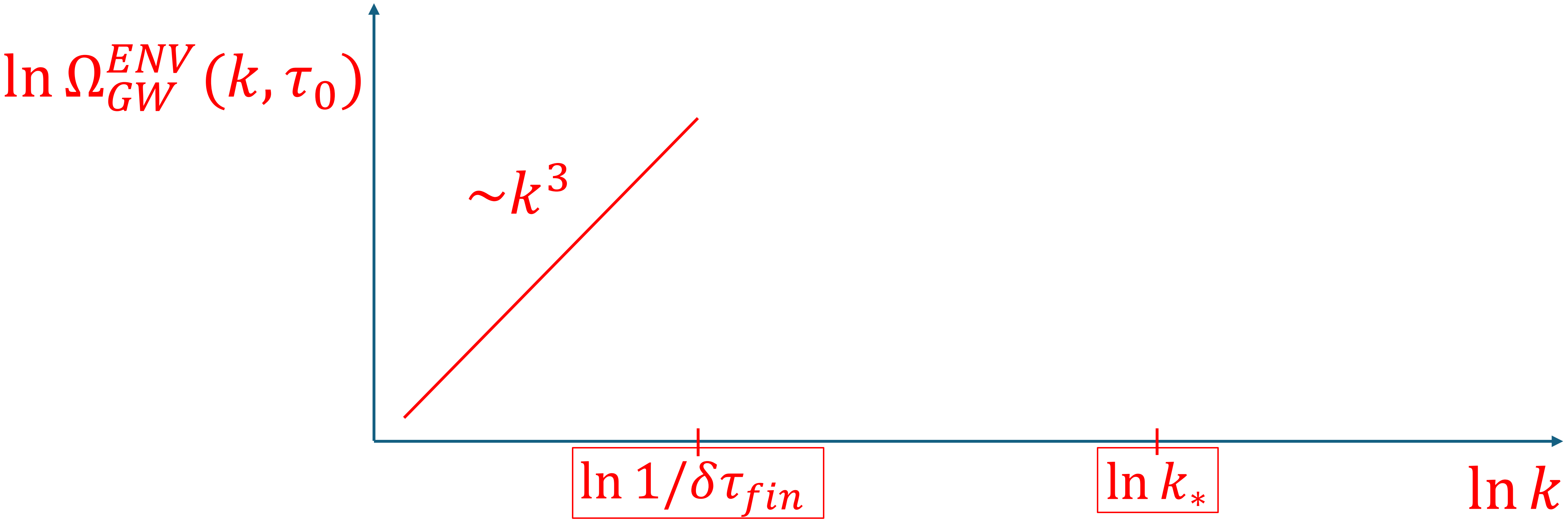

Constant-in-time model for the UETC of the source

$\delta\tau_{fin} = \tau_f - \tau_*$

$$\Omega_{GW}(k,\tau_0) \equiv 3\, \mathcal{T}_{GW} E^*_\Pi(k)\, \Delta_0^2(k,\tau_{fin})$$

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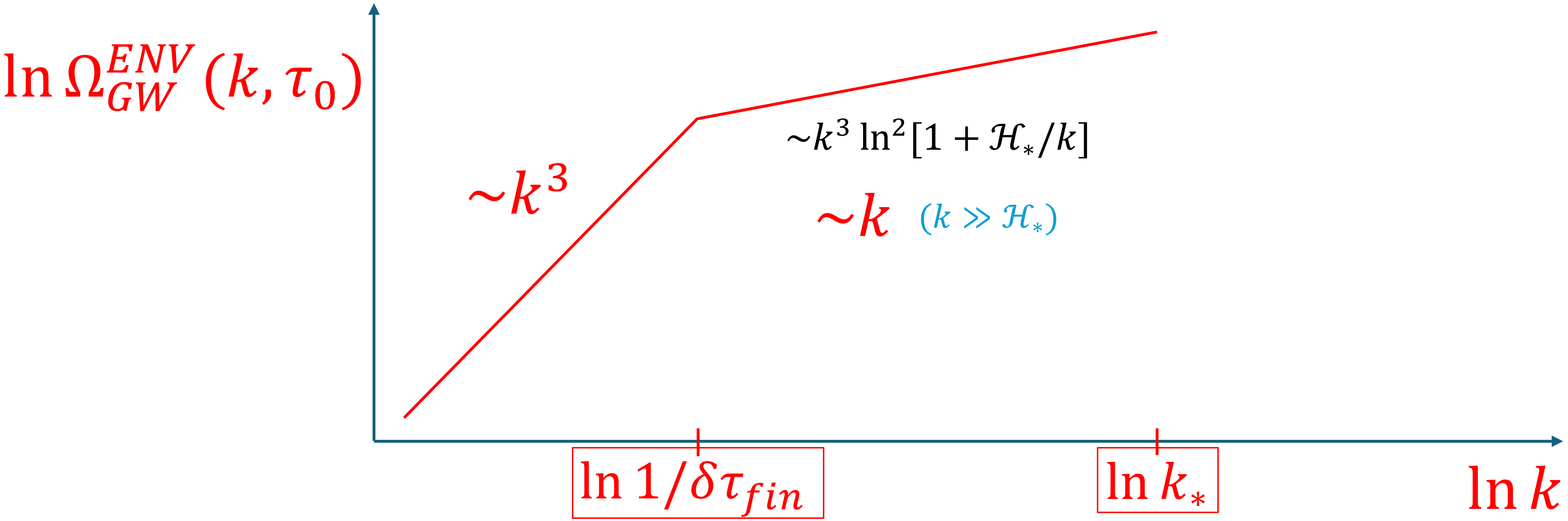

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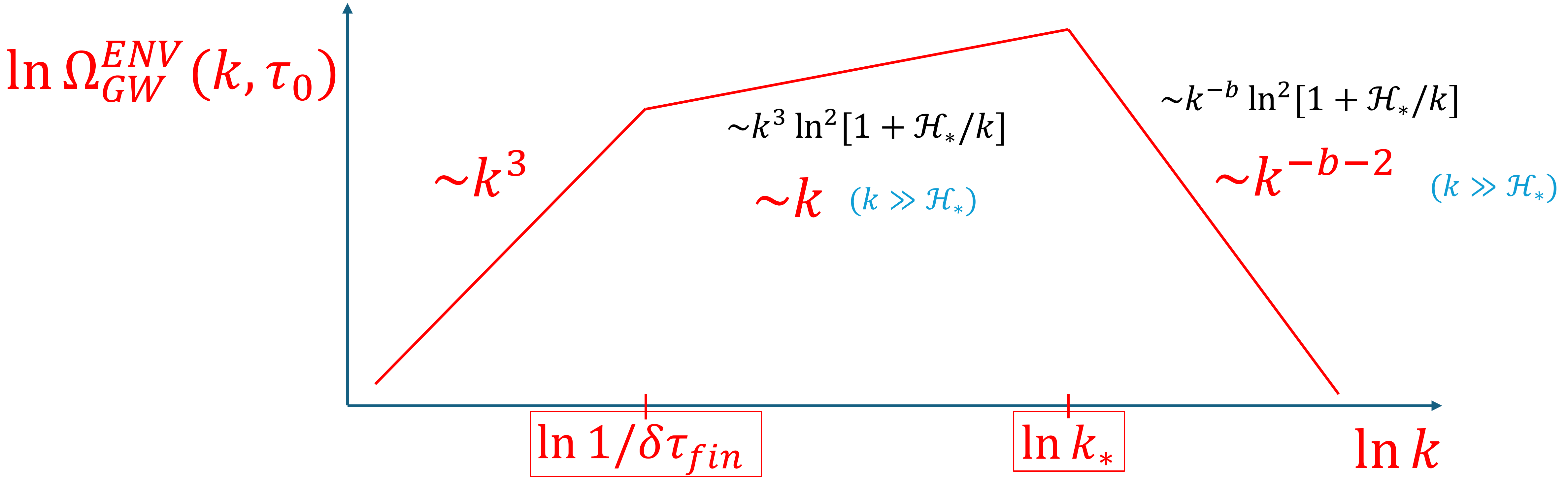


Gravitational Waves from decaying turbulence

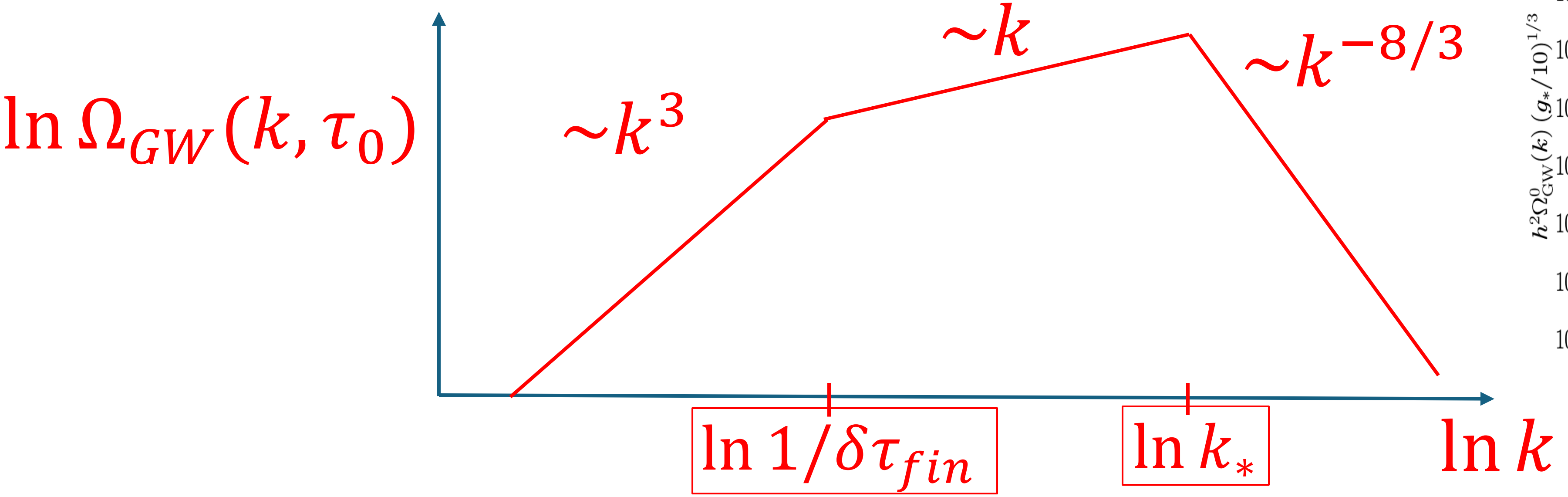
$$\delta\tau_{fin} = \tau_f - \tau_*$$

For a purely vortical velocity field with a Von Kármán spectrum

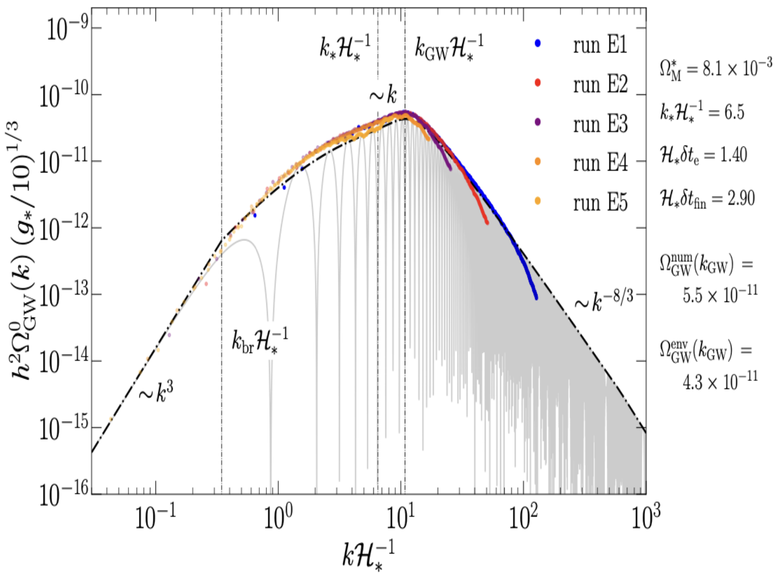
$$E_N^v(k) \sim \begin{cases} k^5 & (k/k_{peak} \to 0) \quad \textit{Batchelor} \\ k^{-2/3} & (k/k_{peak} \to \infty) \quad \textit{Kolmogorov} \end{cases}$$

$$E_\Pi(k) \sim \begin{cases} k^3 & (k/k_* \to 0\,) \\ k^{-2/3} & (k/k_* \to \infty\,) \end{cases}$$

GW spectrum envelope for vortical turbulence in the constant-in-time model (flat spacetime)



Roper Pol et al. [2201.05630]



Conclusions

GW spectrum from sound waves (in the sound shell model) can be understood from the properties of the self-similar profiles and of the bubble nucleation history

For hybrids the GW peak scale is related to the distance between discontinuities instead of the sound-shell thickness (broader spectrum around the peak)

The contribution from slowly decaying MHD turbulence can be described with a constant-in-time UETC of the anisotropic stresses of the source

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How well do these models describe the results from numerical simulations?

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The contribution from slowly decaying MHD turbulence can be described with a constant-in-time UETC of the anisotropic stresses of the source

How well do these models describe the results from numerical simulations?

Strong First-Order Phase Transitions cannot be treated with the linear sound wave phase approximation

For turbulence also the generation phase (not only the decaying part) can be relevant in the final GW spectrum

THANKS FOR YOUR ATTENTION!