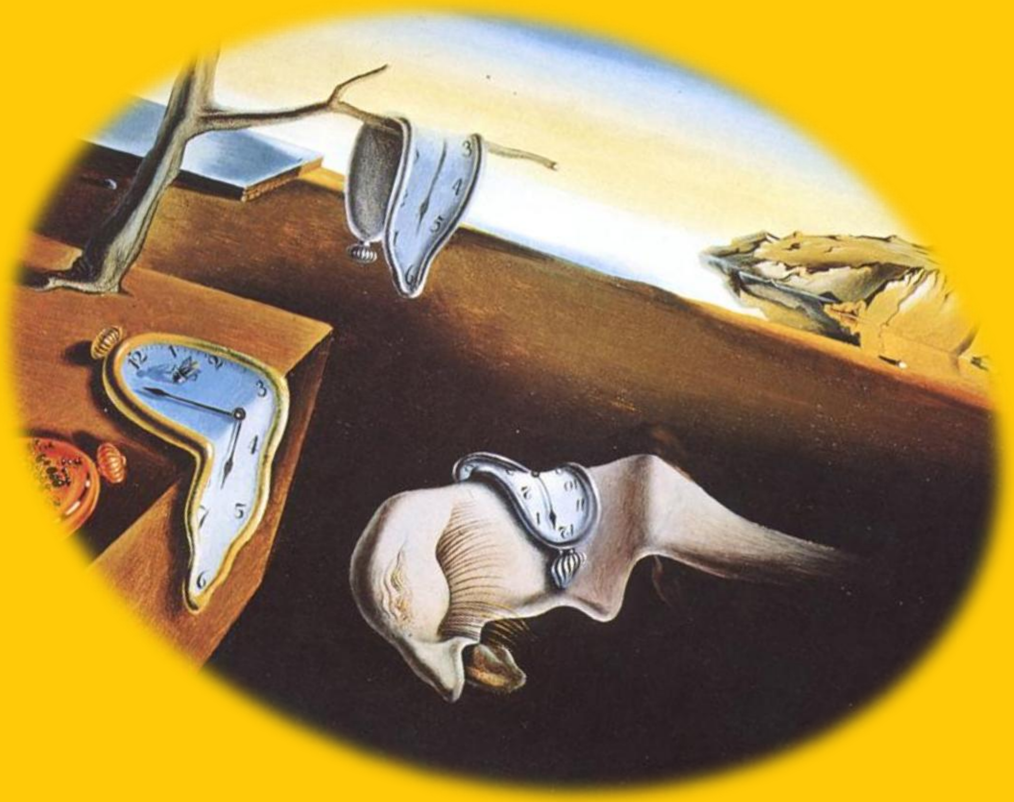
Gravitational Wave Memory from Primordial Black Holes



SILVIA GASPAROTTO

Seminar on the Interface of Particle Physics and Gravitational Waves

12/2/2025

CERN

"The Persistence of Memory"
(also known as "The Soft Watches")
Salvador Dalí, 1931

(Zeldovich & Polnarev'74, Brangisky & Grischchuk '85, Brangisky & Thorne '87)

Linear/Ordinary Memory

Linear memory generated by unbound energy flux (e.g. EM radiation, matter)

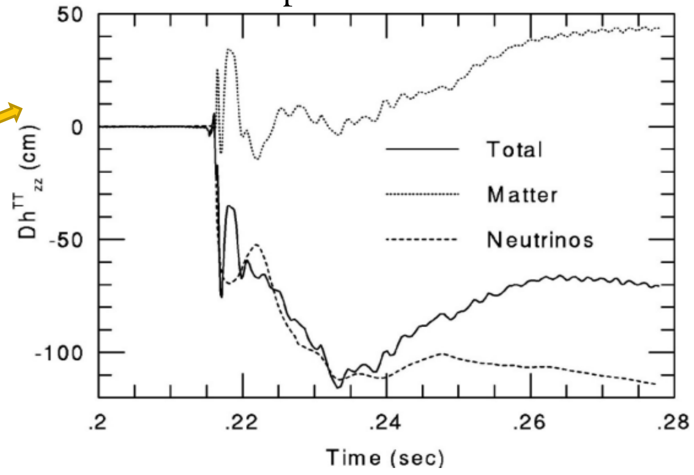
Examples:

- Asymmetric neutrino emission in SN
- Black holes hyperbolic encounters

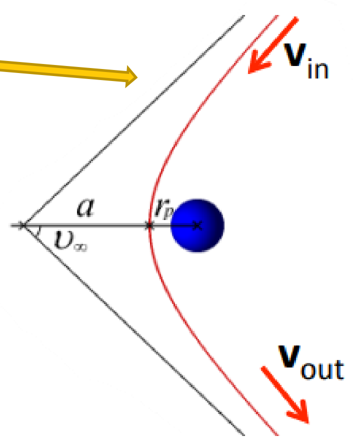
$$\Delta h_{ij}^{TT} = \frac{4\mu}{R}\Delta\left(v_{\infty,i}v_{\infty,j}\right)^{TT}$$

Linear/Ordinary vs Non-linear/Null

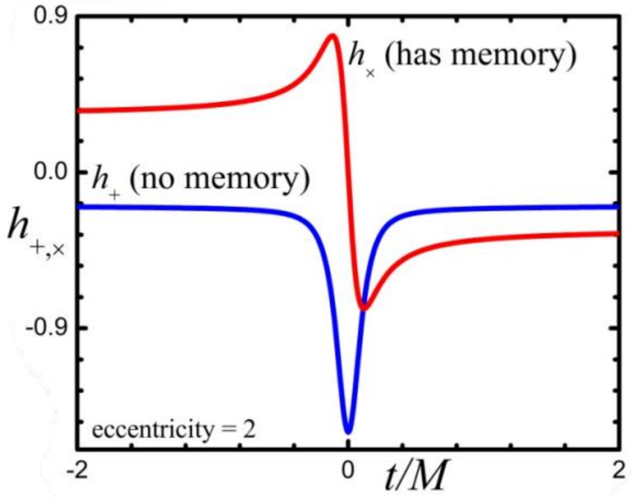
Core-Collapse SN



Credit: Burrows & Hayes '96



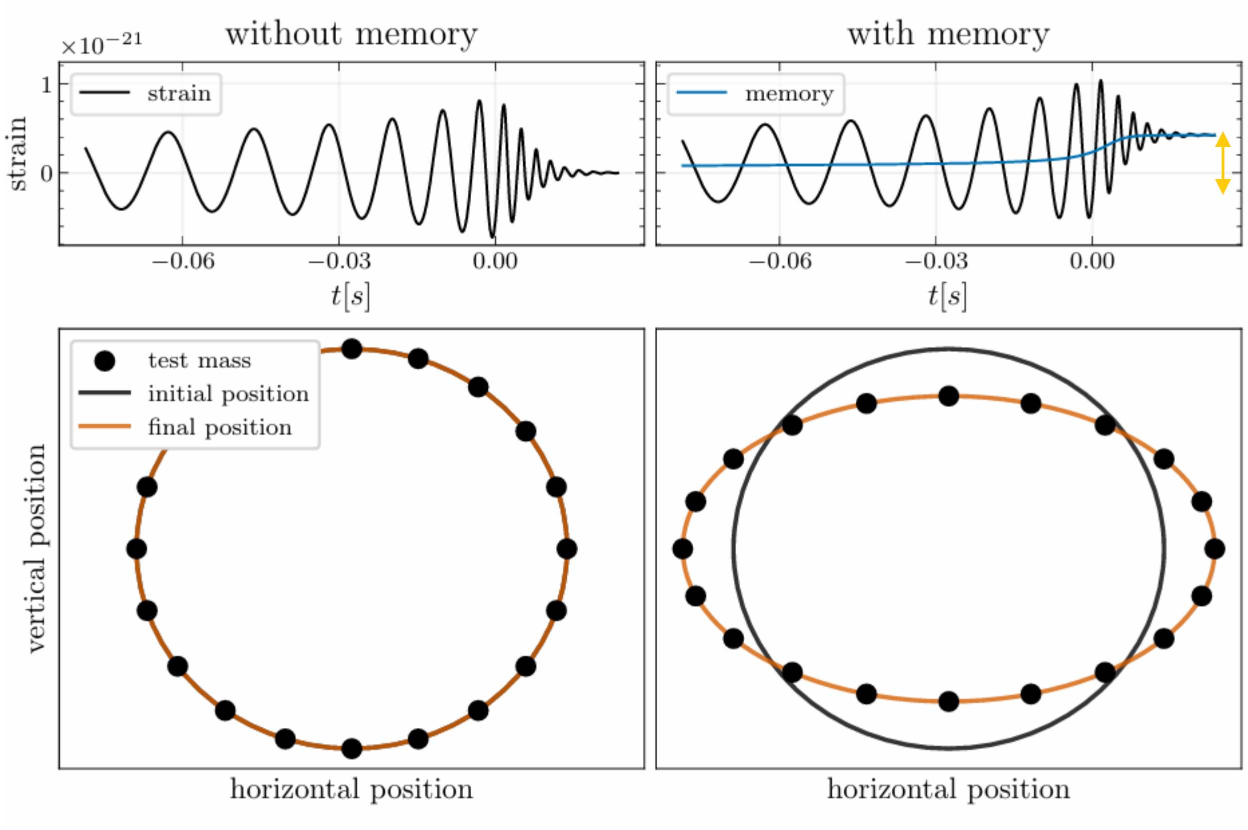
Credit: Favata



Non linear GW memory

Christodoulou '91, Blanchet & Damour '92,Wiseman & Will '91…

GW of a binary system:



Credit: Mitman 2024

Persistent off-set of the GW strain: net displacement between two comoving observers

The GW itself sources GWs!

$$\partial^\mu \partial_\mu \bar{h}^{\ j,k} = 16\pi\left(T^{jk}_{matter} + T^{jk}_{GW}\right)$$

$$T^{jk}_{GW} = \frac{1}{R^2}\frac{dE_{GW}}{dt d\Omega} n_j n_k \sim \frac{c^3}{16\pi G}\left|\dot{h}_0(t,\Omega)\right|^2$$

Thorne Formula:

$$\delta\bar{h}^{\ TT}_{ij}(T_R) = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt' d\Omega'} \frac{n'_j n'_k}{|1 - n' \cdot N|} d\Omega'\right]^{TT}$$

Null memory: generated by **unbound**, **outgoing radiative energy flux** to null infinity

GW Memory and Infrared Diagram

Strominger and A. Zhiboedov 2016
Strominger lectures

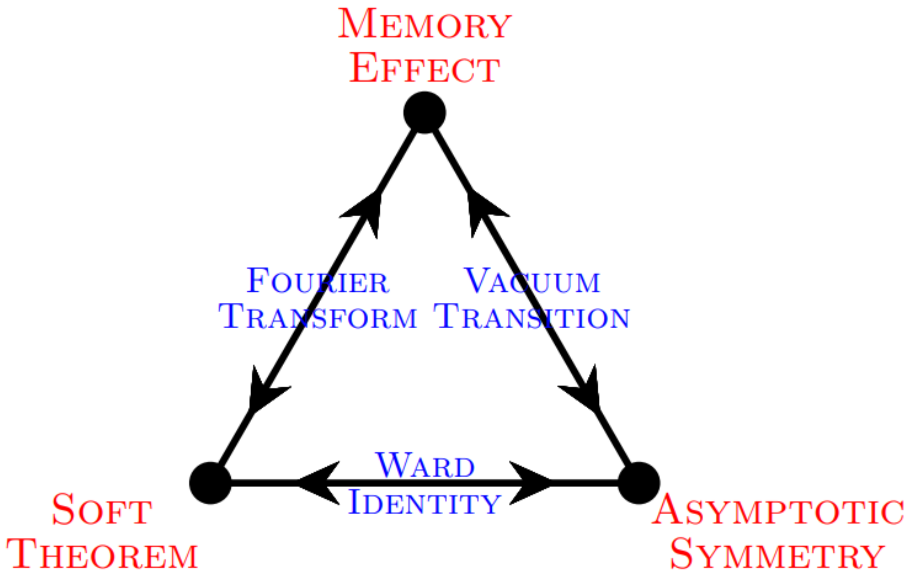
The GW memory represents on of the corners of the infrared triangular diagram

BMS symmetry group: Symmetries of asymptotically flat spacetime at null finite

Gravitational memory: Physical consequence of the shift between different vacuum

Soft theorem: statement about the quantum scattering amplitudes, related to the behavior of low energy (soft) gravitons

$$\Delta h_{mem} = h(t \to \infty) - h(t \to -\infty) \neq 0$$



$$\frac{\mathcal{M}_{n+1}}{\mathcal{M}_n} \xrightarrow{\omega \to 0} \sum_{k=1}^{n} \frac{p_k^{\mu} p_k^{\nu} \epsilon_{\mu\nu}}{p_k q}$$

$$\dot{m} = -\frac{1}{4}\dot{h}\dot{\bar{h}} + \frac{1}{4}\mathrm{Re}\left[\bar{\eth}^2 \bar{h}\right]$$

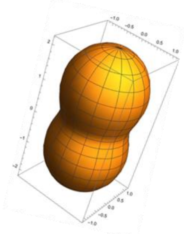
The memory signal for BH binaries

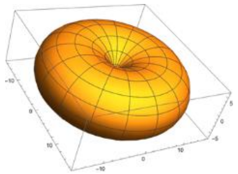
$$h_{+,0PN} = \left[-(1+\cos^2\iota)\cos 2\Phi(t) + \frac{1}{96}\sin^2\iota(17+\cos^2\iota)\right]\frac{2\eta M(M\omega(t))^{2/3}}{R} \qquad \eta = \frac{m_1 m_2}{M^2}$$

Oscillatory mode (2,2)

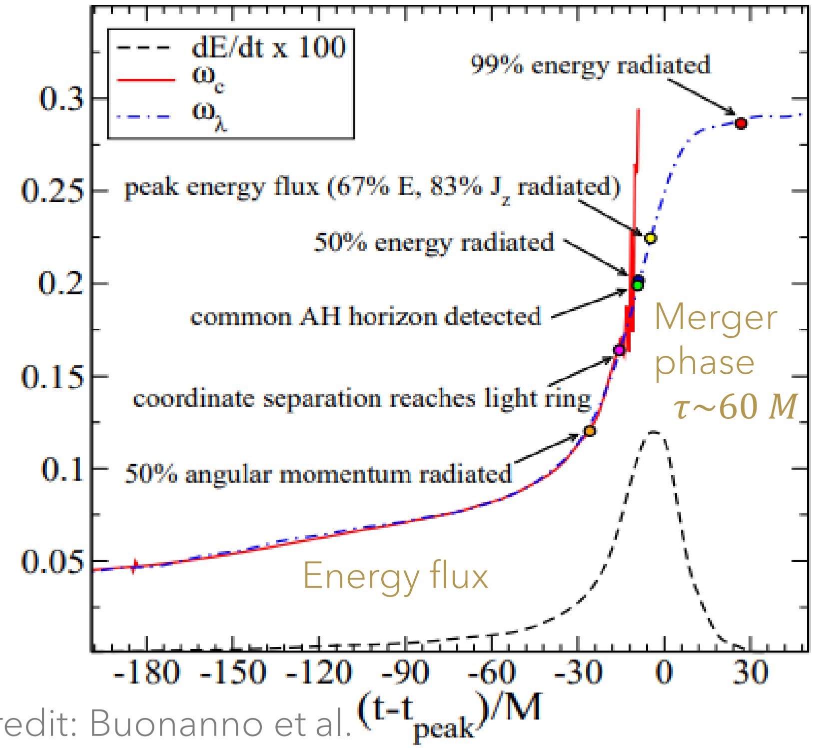
Memory mode (2,0)

inclination angle ι



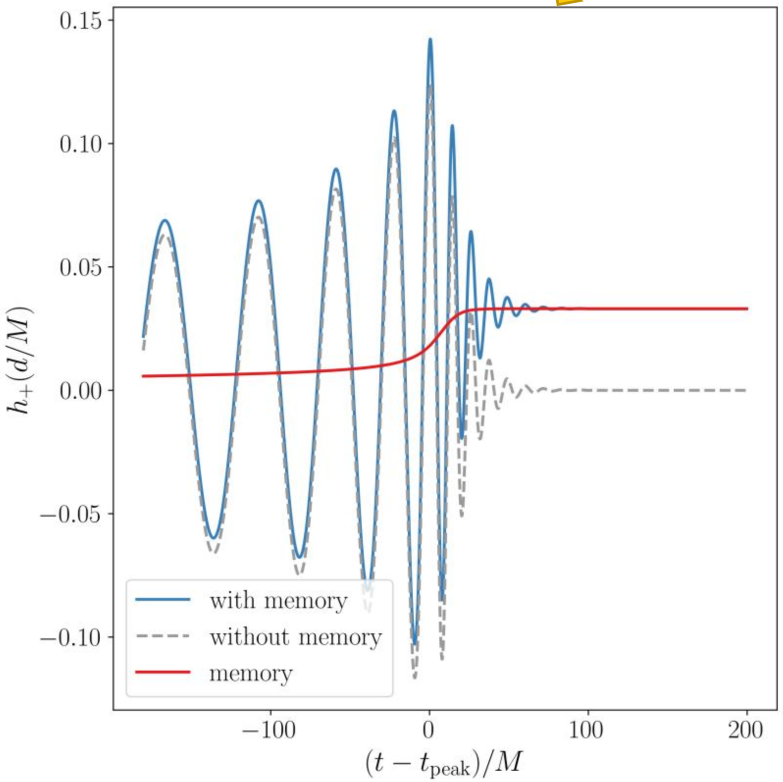


Evolution of the GW frequency



Credit: Buonanno et al.

Numerical integrating the energy flux



- Hereditary effect
- Small but present in all GW sources
- It scales with energy radiated
- Main support close to merger phase

Detectability of the memory

Fourier Transform of a **step function** → $-\frac{i\Delta h_{mem}}{2\pi f}$

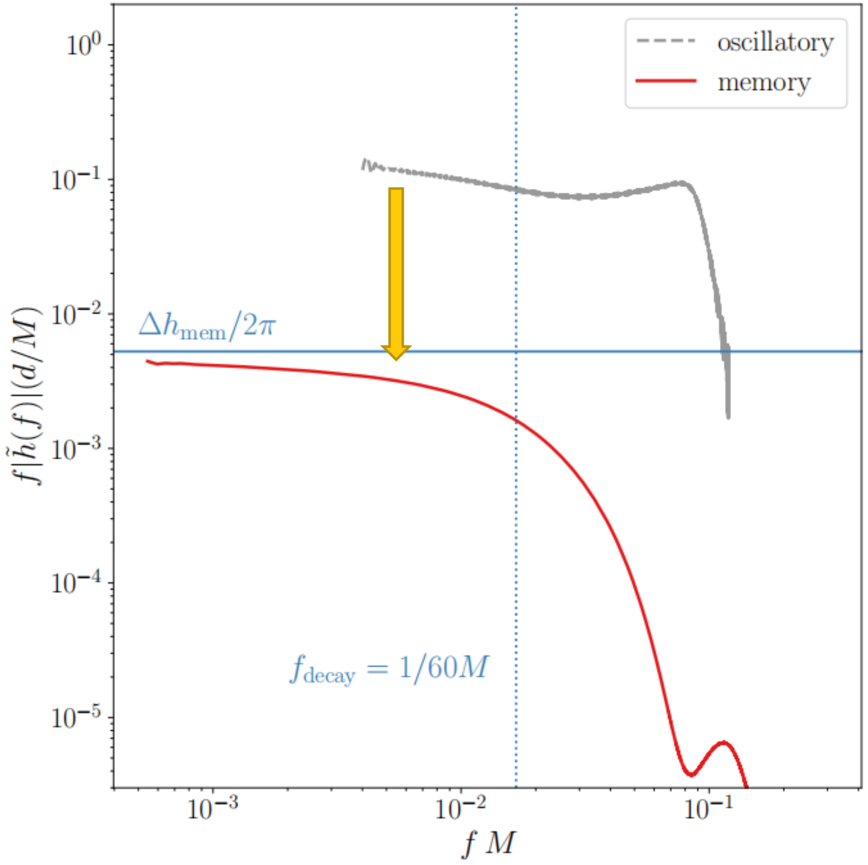
Characteristic strain for $f \ll f_{merger}$, $f\left|\tilde{h}(f)\right| \to \Delta h_{mem}/2\pi$

Current status of GW-memory searches: no detection so far by ground-based detectors (LVK) or PTA experiments.

LVK: detection expected after **O(2000) accumulated events**
PTAs: upper bound on memory amplitude $h_{mem} < 3 \times 10^{-14}$

Next generation prospects:

- Einstein Telescope & Cosmic Explorer: $O(1)\, yr^{-1}$
- LISA: **Inchauspé, SG et al** *PRD* 111 (2025) 4, 044044



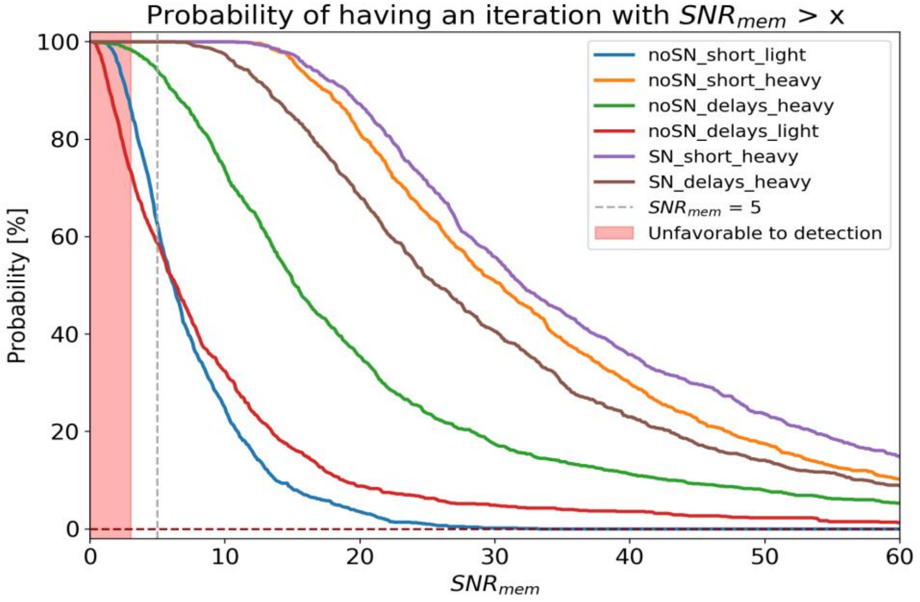
Probing GW memory with LISA

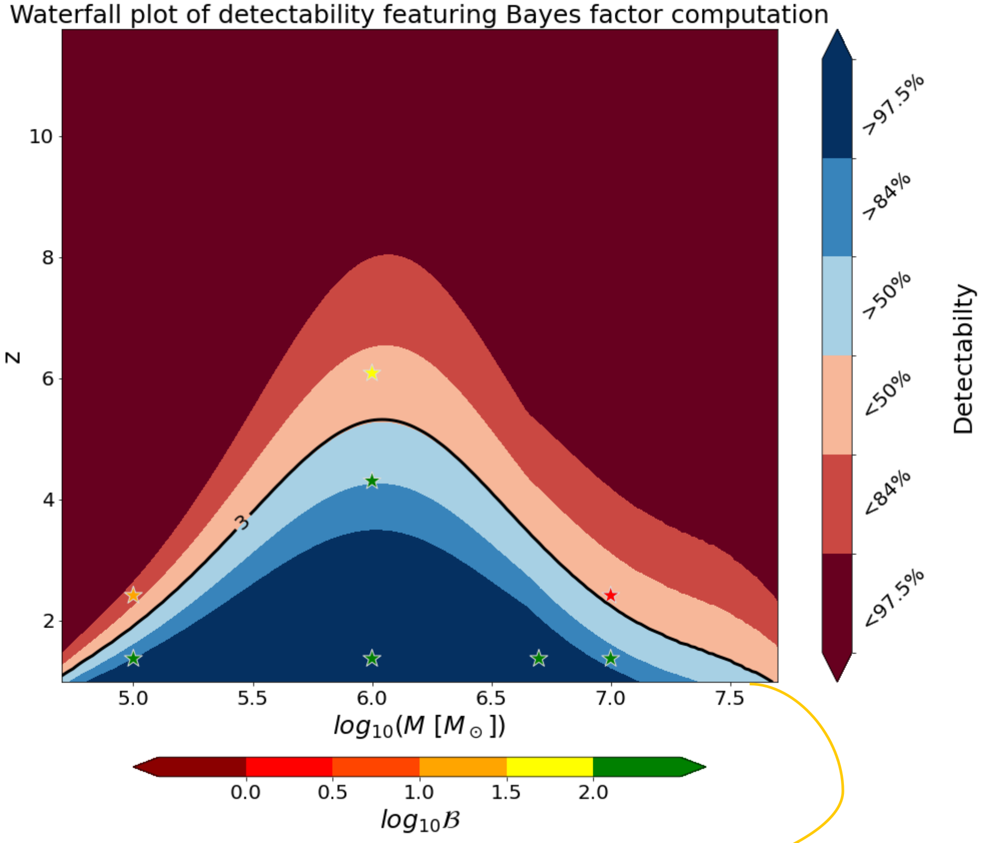
LISA fundamental Physics WG
A.Cogez, **SG** et al (to appear)

Evidence of memory in SMBH mergers

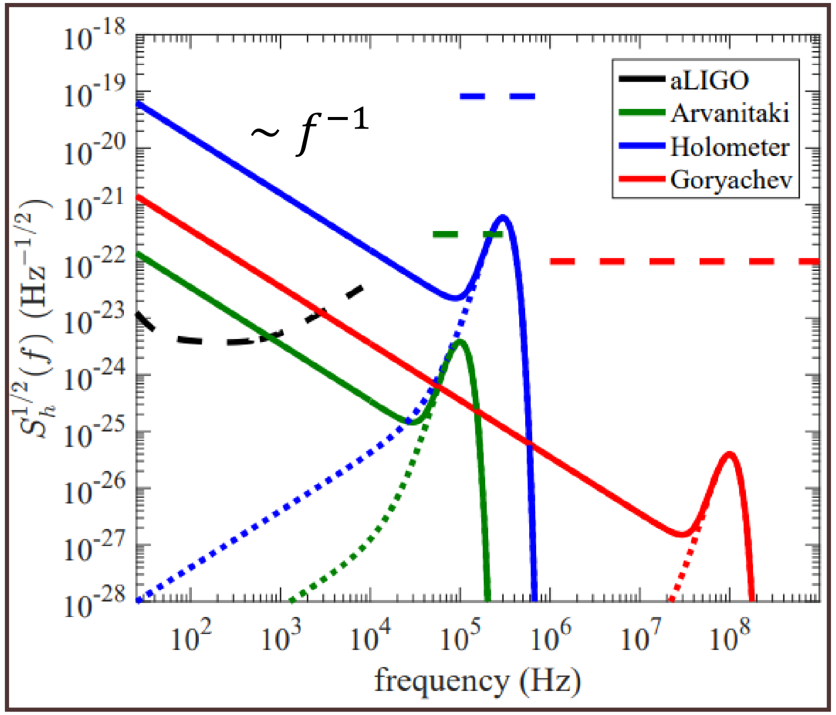
Parameter estimation through Bayesian analysis

Population study





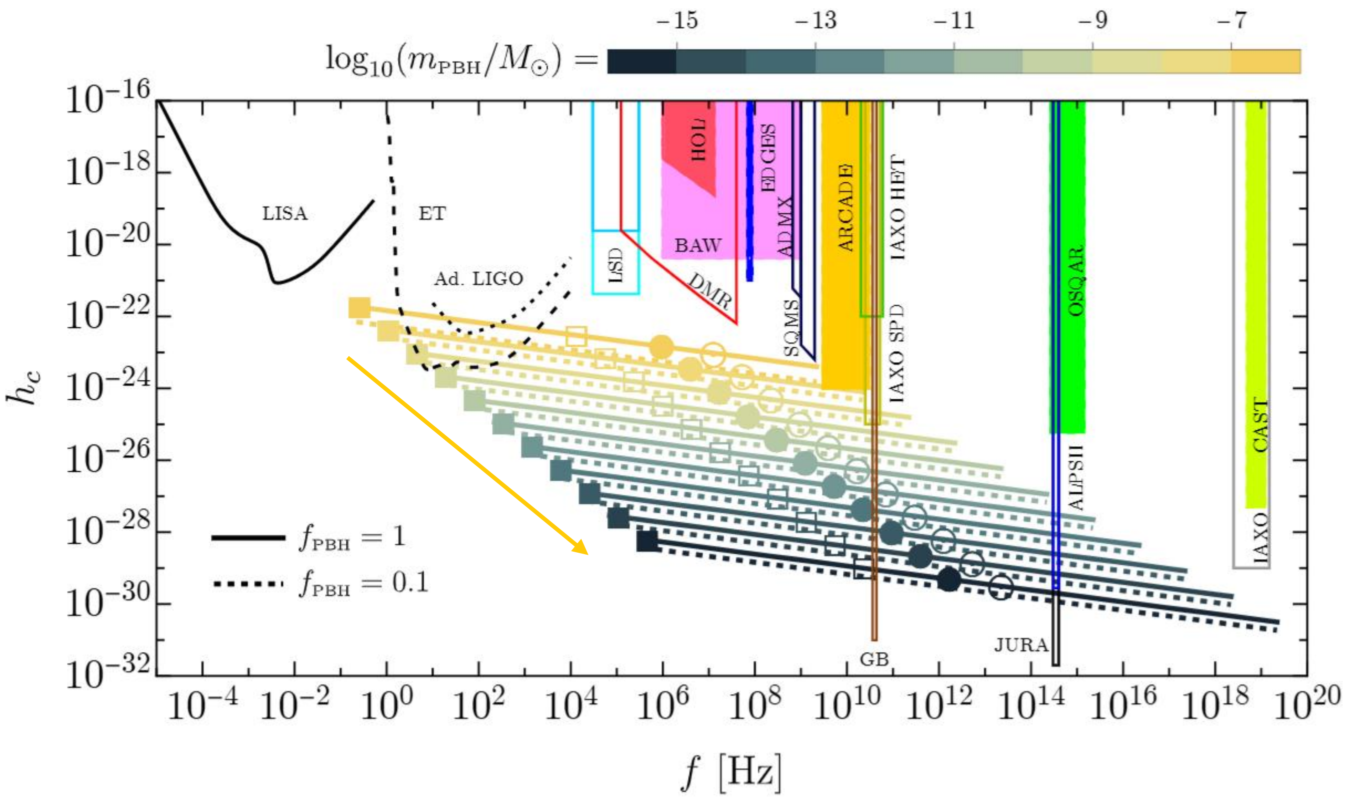
$SNR_{th} \simeq 3$



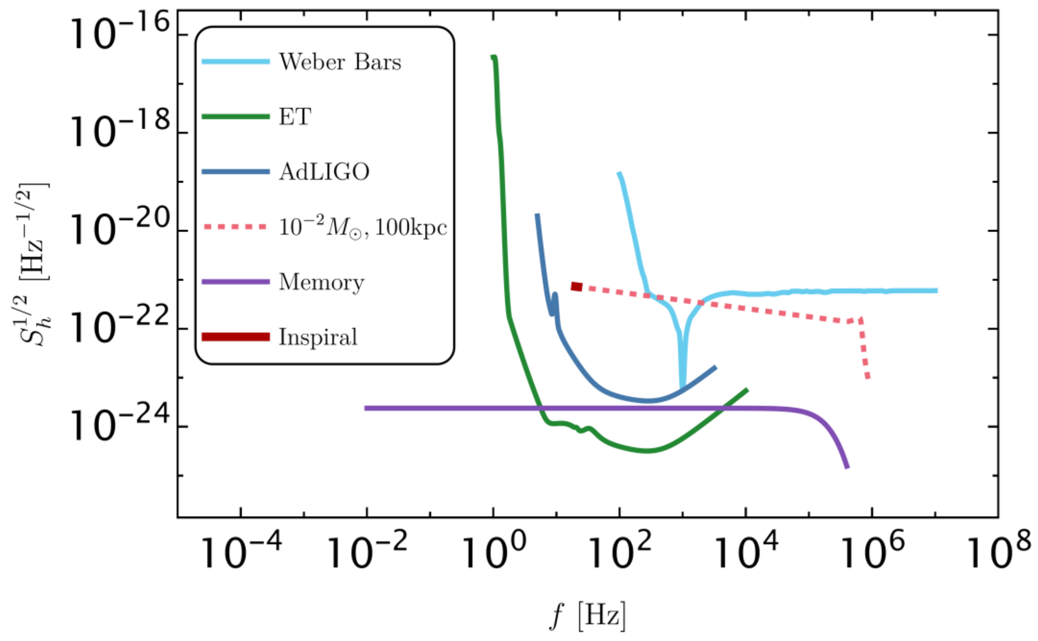
Detecting Gravitational Wave Memory without Parent Signals (Orphan memory signal)
PRL118 (2017) 18, 181103 O.McNeill, Thrane & Lasky

Ground-based detectors outperform proposed HF detectors for high-frequency sine-Gaussian GW signals

Memory from GWs at high frequencies



Credit: Franciolini et al. 2205.02153

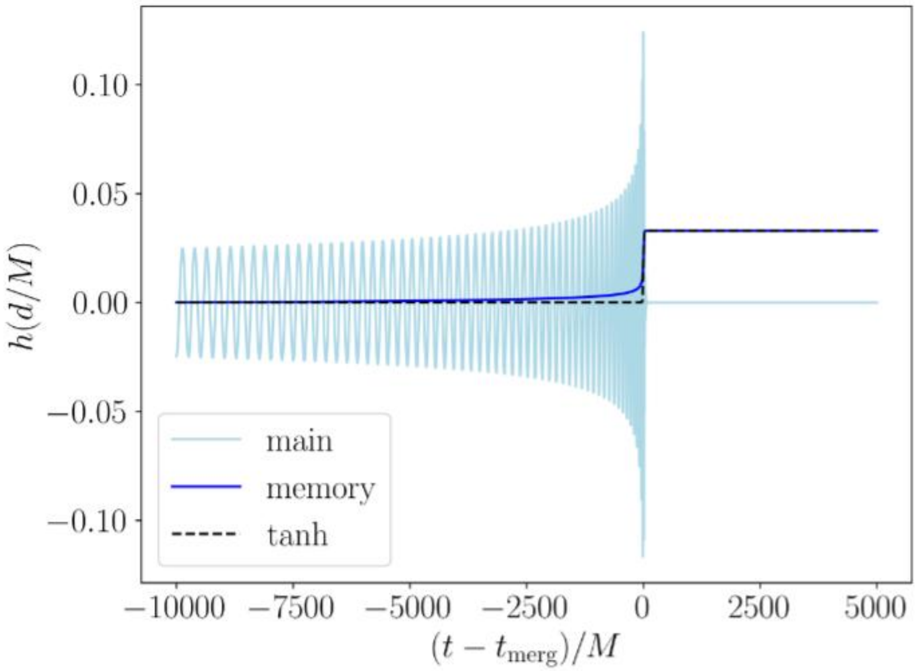


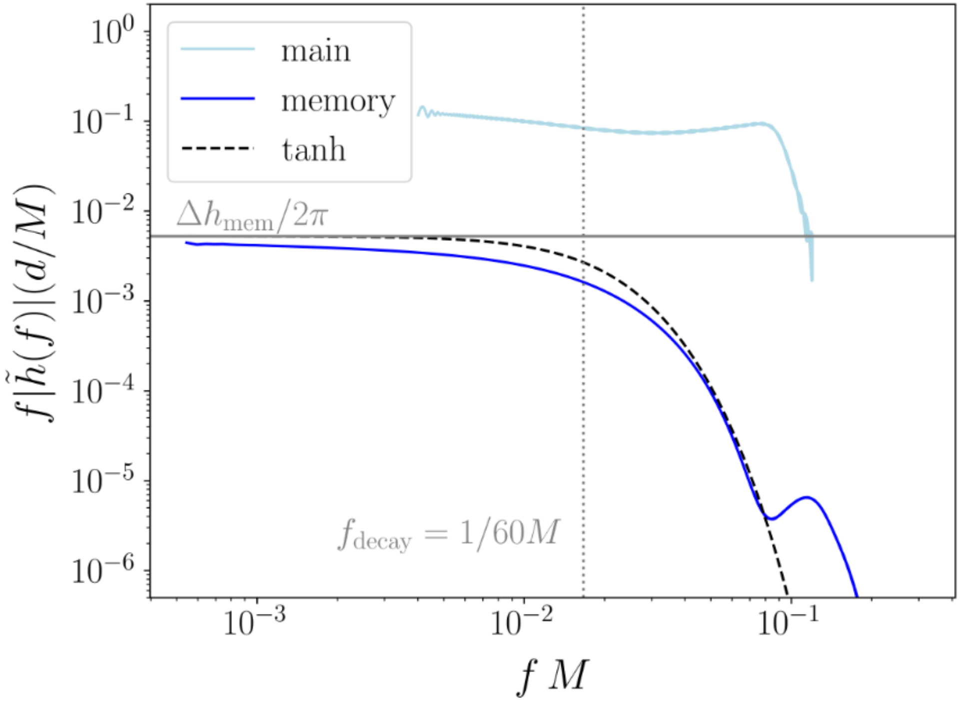
Memory and early inspiral low-frequency counterpart of a high-frequency merger

The case of PBH

SG, G. Franciolini, V. Domcke
PRD 112 (2025) 10, 103021

Memory signal approximation





$$h_{tanh}(t) = \frac{\Delta h_{mem}}{2}\left[\tanh\left(\frac{t-t_{merg}}{\tau}\right) + 1\right] \quad \text{with } \tau = 13\ M$$

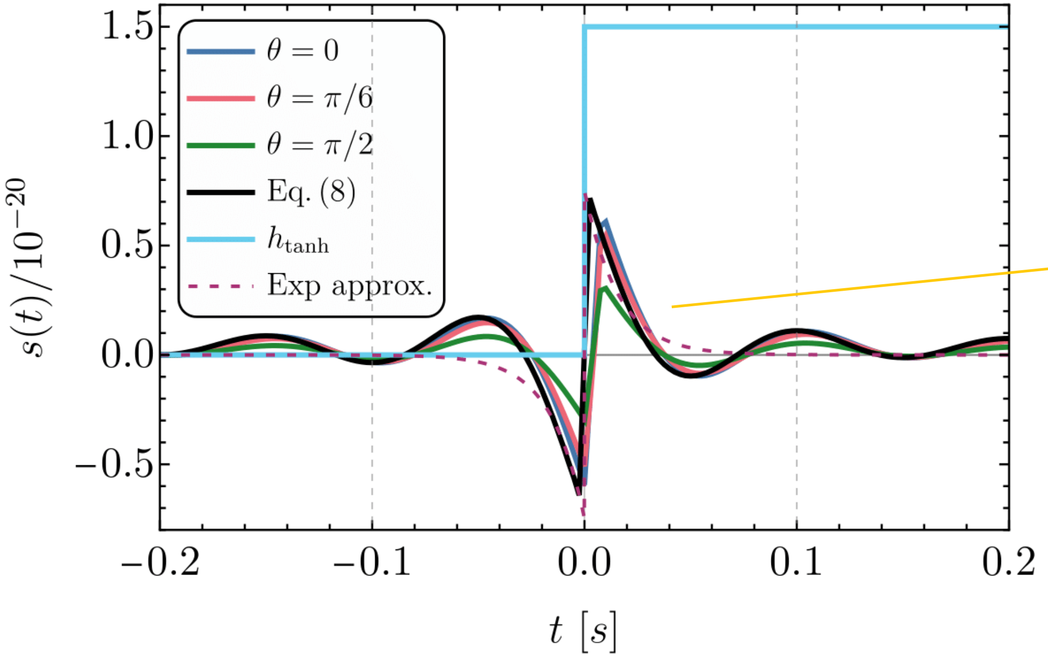
$$\tilde{h}_{tanh}(t) \xrightarrow{f \ll \Delta\tau^{-1}} -i\Delta h_{mem}\left[\frac{1}{2\pi f} - \delta(f)\right]$$

$$\Delta h_{mem} \simeq 10^{-18}\sin^2(\iota)\left(\frac{M}{M_\odot}\right)\left(\frac{kpc}{d}\right)$$

Almost universal memory signal

Band passed signal: $M_{tot} = 10^{-2} M_{\odot}, d = 1\ kpc$

LIGO



Detector response

$$s(t) = \int_{f_{min}}^{f_{max}} df\ R(\boldsymbol{k})\widetilde{h(f)}e^{i2\pi f t}$$

Oscillation frequency given by $f_{min}^{LIGO} \sim 10\ Hz$

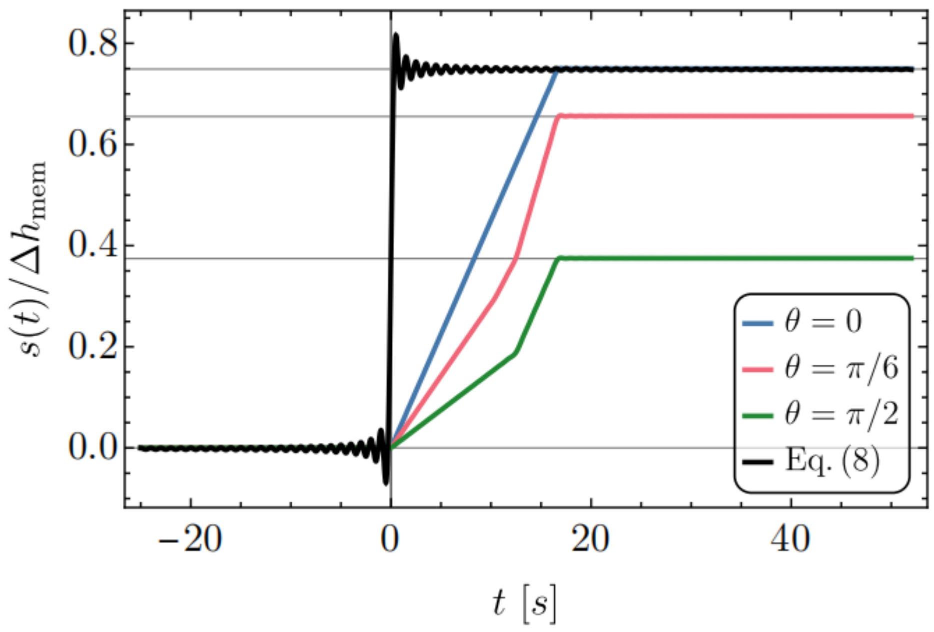
Signal depends only on **amplitude, merger time, and sky position**, the memory emerges as a simple and robust target for detection

One can do a **match-filter search** for such a universal signal with **few free parameters**!

Almost universal memory signal

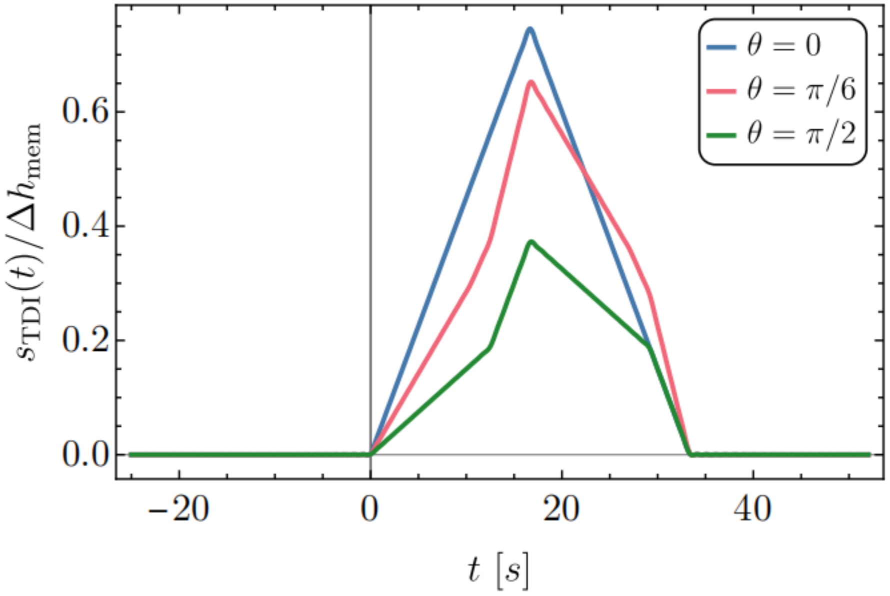
Space-based interferometer (LISA)

Simple interferometer response



Response with no low-frequency cut-off

After TDI post-processing



First-generation TDI noise $s_{TDI} = s(t) - s(t-2L)$

PBH Merger Rate

PBH merger rate in the **early-universe scenario** for **monochromatic mass** function

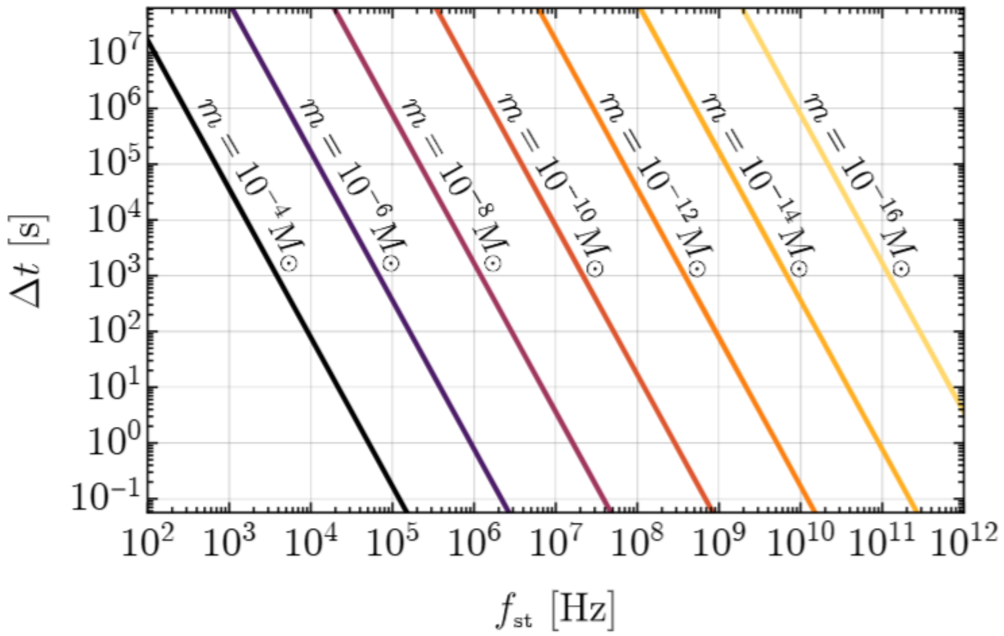
Local DM overdensity over the cosmological density

$$\frac{d^2\mathcal{R}(t,d)}{d\ln m_1 d\ln m_2} = \left(\frac{0.038}{\mathrm{kpc}^3\,\mathrm{yr}}\right)[1+\delta(d)] f_{\mathrm{PBH}}^{\frac{53}{37}}\left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \times \left[\frac{m_1 m_2}{(m_1+m_2)^2}\right]^{-\frac{34}{37}}\left(\frac{m_1+m_2}{10^{-12}M_\odot}\right)^{-\frac{32}{37}} \times S(t, M, f_{\mathrm{PBH}}, \psi)\,\psi(m_1)\psi(m_2), \quad (17)$$

Suppression factor from the interaction with the environment in the early and late universe

Normalized mass distribution

Fraction of PBH $f_{PBH} = \frac{\Omega_{PBH}}{\Omega_{DM}}$



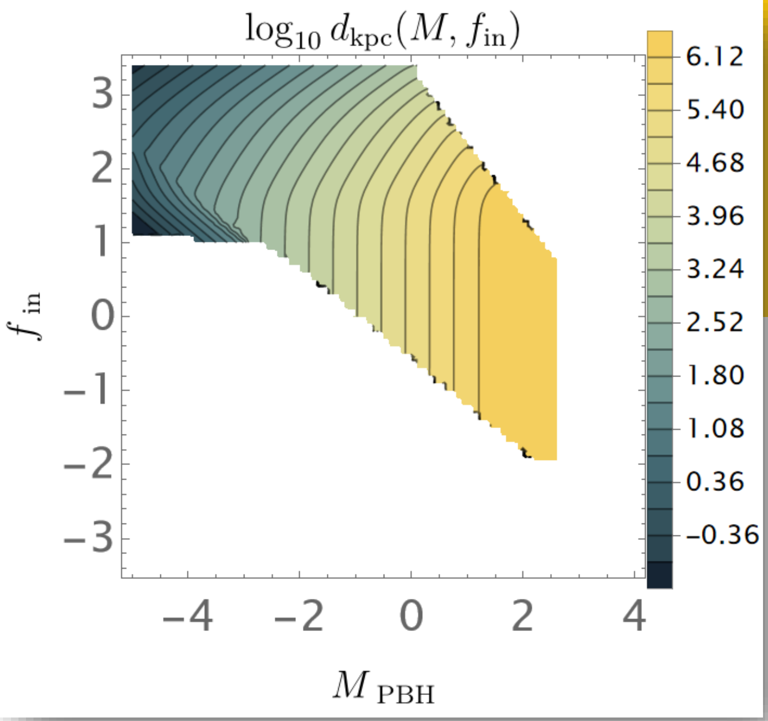
Number of detectable events

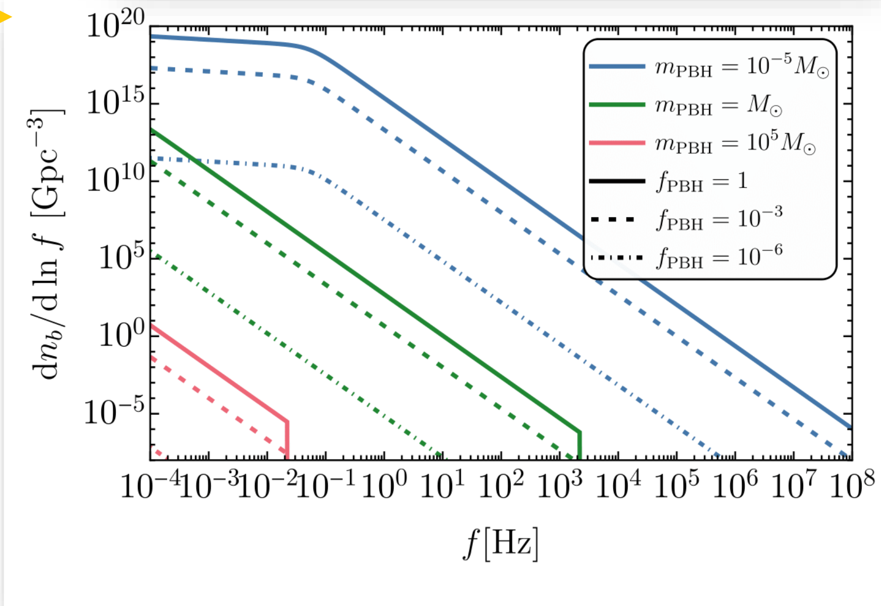
The total **number of detectable events** for a given detector during a certain observation window is

$$N_{\rm det} = \int df_{\rm in} \int dz \frac{1}{(1+z)} \frac{dV_c}{dz} \frac{dn_b(t(z), d(z), f_{\rm in})}{df_{\rm in}} \times \Theta(\hat{\rho}(f_{\rm in}, z) - \hat{\rho}_{\rm th})$$

Selection factor

$$\frac{{\rm d}n_b(t,d,f)}{{\rm d}f} = \left|\frac{{\rm d}f_{\rm coal}(\tau(f))}{{\rm d}\tau}\right|^{-1} \mathcal{R}(t+\tau(f), d)$$





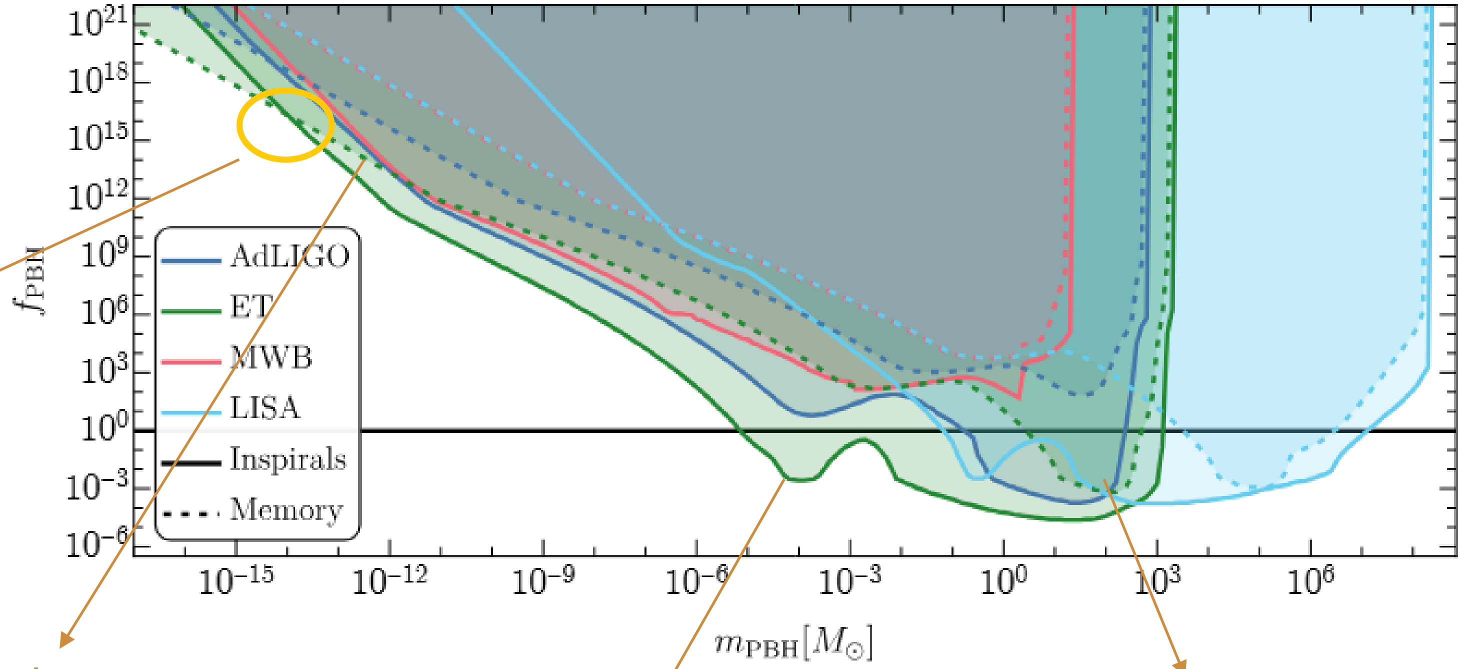
Many binary emitting **quasi-monochromatic GWs** in their **early inspiral**

Inspiral and memory probe **different stages** of the binary evolution

GW memory and PBH Bound

Constraints on the **fraction of PBH** from GW detection for one year of observation time requiring $N(f_{PBH}) > 1$

The **memory effect** becomes **more sensitive for very light PBH** but detection requires $f_{PBH} \gg 1$



High frequency detectors do not perform better than low frequency detectors, but the signal is **complementary**

Effect of the **over density** of dark matter in the solar system

The memory can also probe value $f_{PBH} < 1$

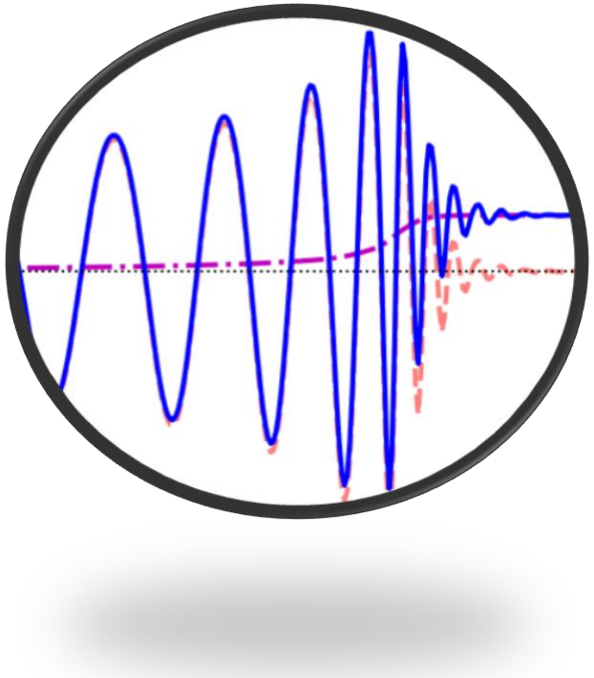
Match-filter search with a **few free parameters**

Memory and **inspiral** probe **different stages** of the PBH evolution

Inspiral signals, in general, outperform memory in SNR for early universe production of PBH

Memory **becomes competitive only for extreme, out-of-band mergers**, though detection would require **unrealistically high PBH abundances**.

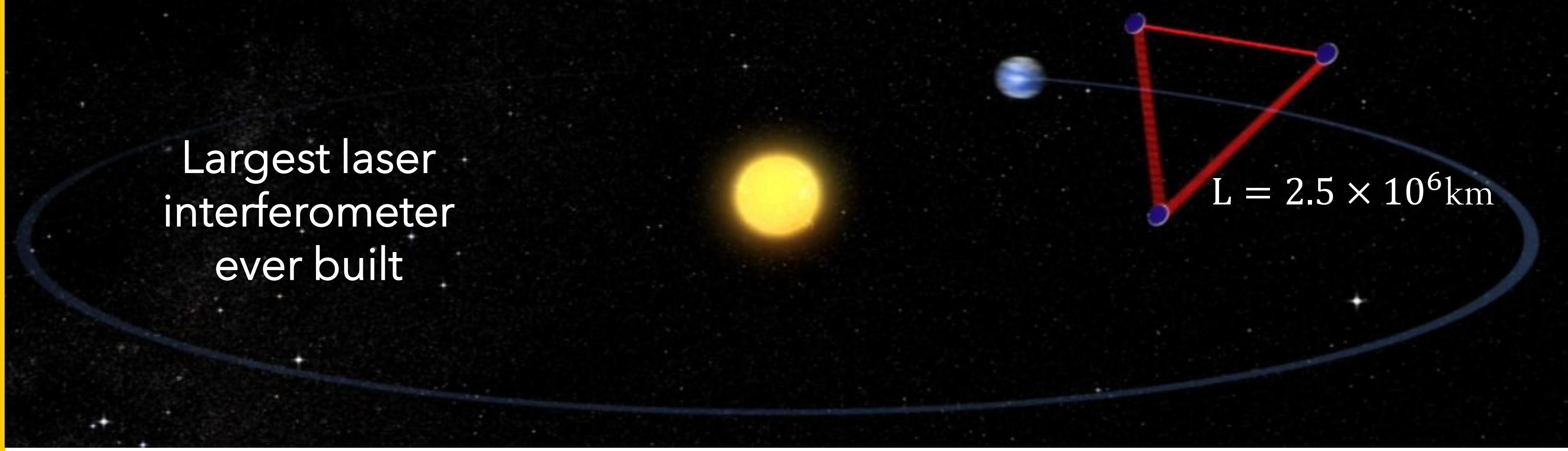
Memory is a promising target for sources lacking a strong inspiral phase (e.g., PBH encounters, hyperbolic flybys, EMRIs, cosmic strings)



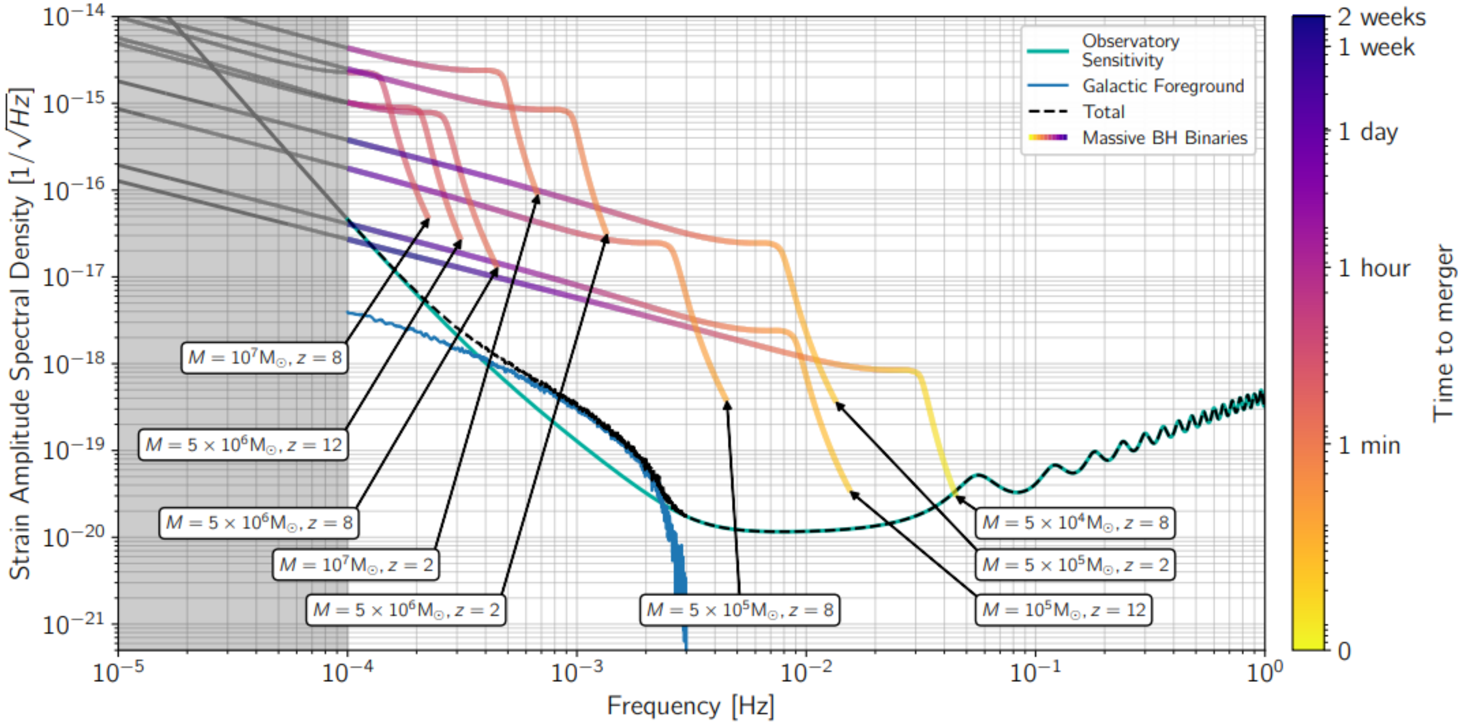
Conclusions

Thanks!

LISA : Laser Interferometer Space Antenna



LISA golden sources:
Massive Black Hole Binaries (MBHBs)



Impact on Parameter Estimation

S.G .et al [2301.13228]

$$h(\vec{\theta},t) = h_{osc}(\vec{\theta},t) + \delta h_{mem}(\vec{\theta},t)$$

<u>Forecasts for LISA:</u>
Fisher and covariance matrix:

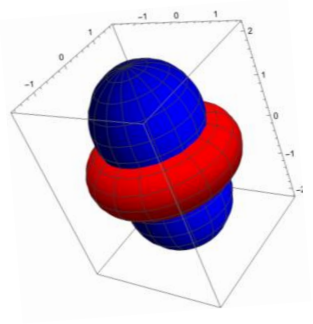
$$\Gamma_{ij} = \left(\frac{dh}{d\theta_j}\middle|\frac{dh}{d\theta_j}\right), \qquad \Sigma_{ij} = \Gamma_{ij}^{-1}$$

$$(a|b) = 4\int_{f_{min}}^{f_{max}} \frac{\mathcal{R}[a^*(f)b(f)]}{S_n(f)}\,df$$

Signal-to-noise-ratio SNR:

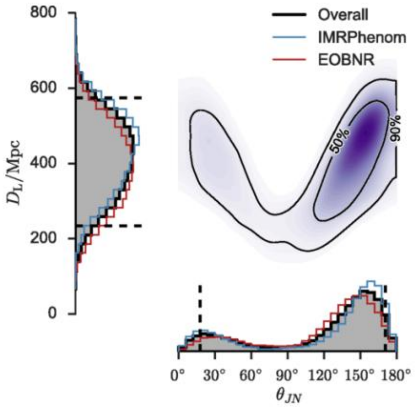
$$\rho = \sqrt{(h|h)}$$

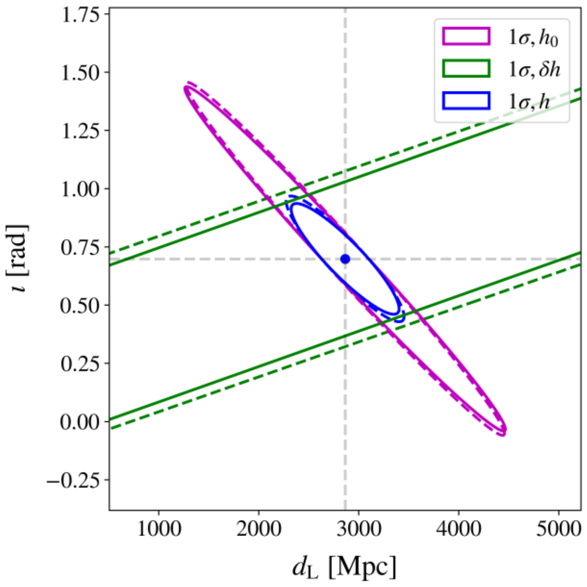
Opposite inclination dependence



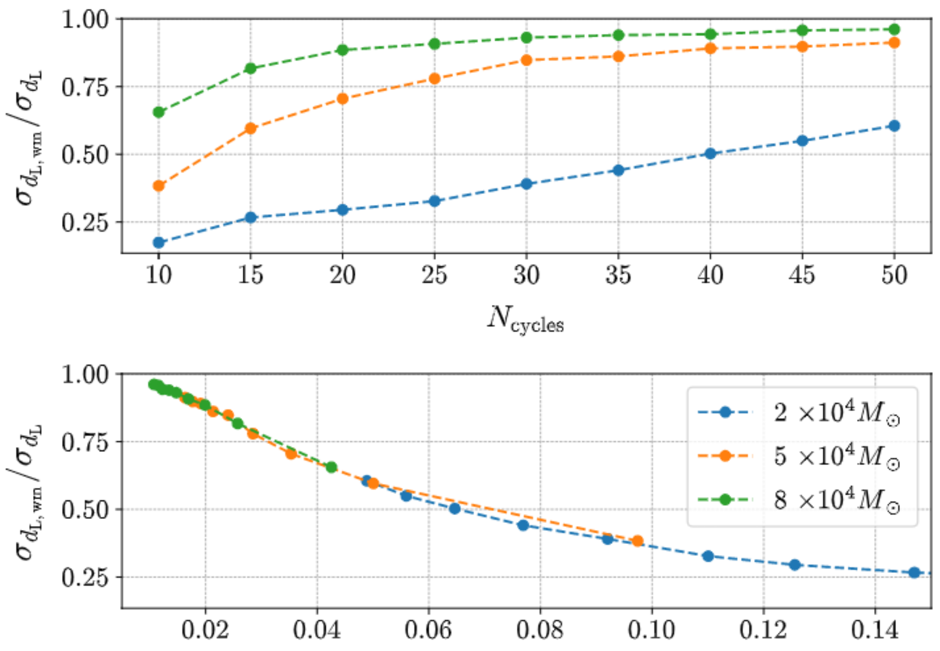
Can it break the distance-inclination degeneracy?

- In extreme cases, it can help
- Complementary effect of Higher modes





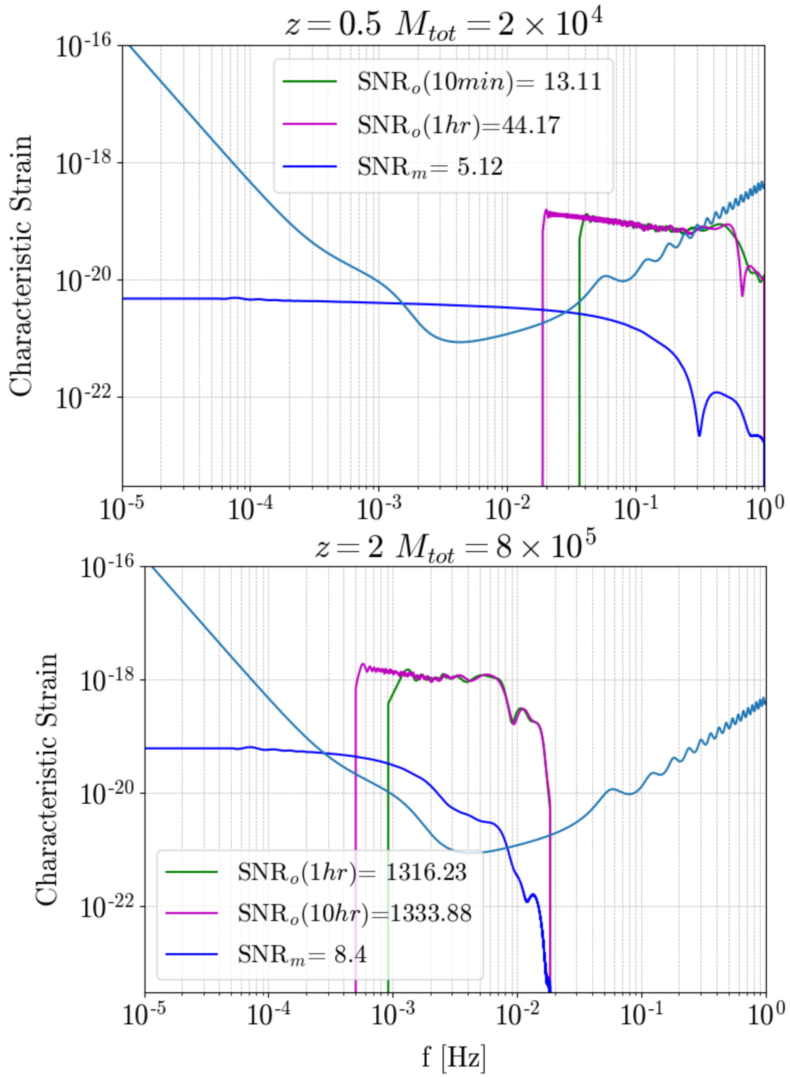
The memory helps for "short" and "light" signals



SNR-ratio

Dependence on the **mass** and the **duration** of the signal **pre-merger** ⟹ the **SNR-ratio changes!**

Complementary effect to higher modes!



Massive Black Hole Binaries

8 different population models

Barausse & Lapi (2020), Barausse et al (2020)

- Population depends on the primary black holes origins:

  Light seeds: collapse of first-generation stars (PopIII)

  Heavy seeds: direct collapse of giant gas cloud.

- Merger delay (*"Last parsec problem"*)
- Supernovae (SN) feedback in the evolution

N_{th} number of events with detectable memory, i.e. $SNR_{th} \geq 1$ (or $SNR_{th} \geq 5$) , in 4 years from

A relevant improvement of d_L is **statistically unlikely** even including **unscheduled gaps**

$\rho \equiv SNR$

Astrophysical Catalogues

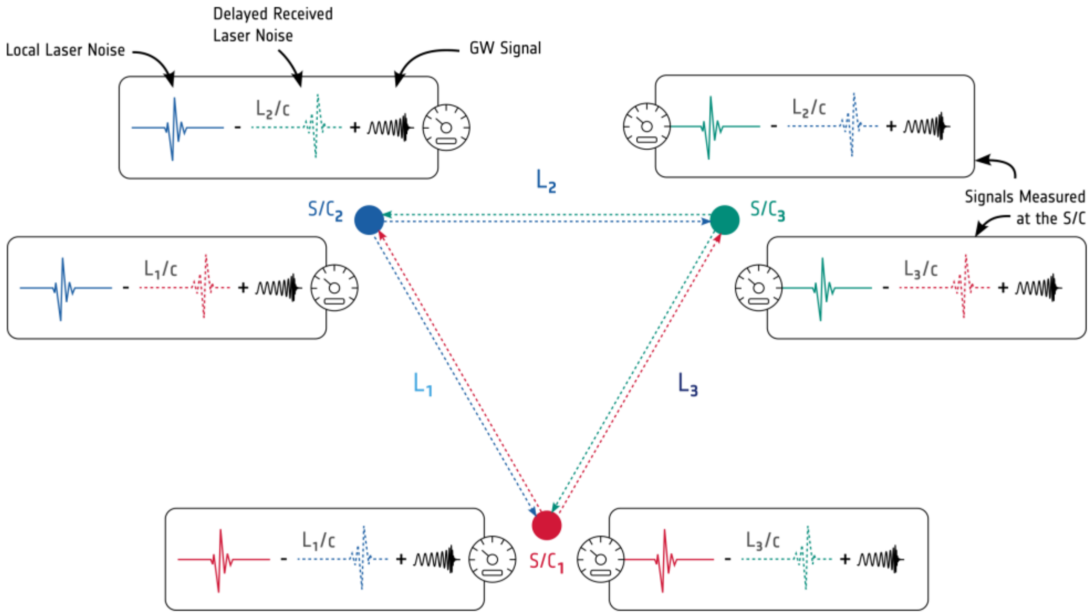
	Light seeds	Heavy seeds
SN-delays	$N_{\rm tot} = 47$ $N_{\rm th} = 0.4\,(0.1)$ $\langle\rho\rangle = 0.04$ $\rho_{\rm max} = 7$	$N_{\rm tot} = 27.3$ $N_{\rm th} = 21.2\,(10)$ $\langle\rho\rangle = 6$ $\rho_{\rm max} = 97$
noSN-delay	$N_{\rm tot} = 191$ $N_{\rm th} = 6\,(1)$ $\langle\rho\rangle = 0.17$ $\rho_{\rm max} = 11.64$	$N_{\rm tot} = 10$ $N_{\rm th} = 7.5\,(4)$ $\langle\rho\rangle = 6.9$ $\rho_{\rm max} = 68.7$
SN-short Delays	$N_{\rm tot} = 149$ $N_{\rm th} = 1\,(1)$ $\langle\rho\rangle = 0.04$ $\rho_{\rm max} = 5.01$	$N_{\rm tot} = 1245$ $N_{\rm th} = 418\,(33)$ ★ $\langle\rho\rangle = 1$ $\rho_{\rm max} = 43$
noSN-short Delays	$N_{\rm tot} = 1203$ $N_{\rm th} = 12\,(2)$ $\langle\rho\rangle = 0.06$ $\rho_{\rm max} = 17$	$N_{\rm tot} = 1251$ $N_{\rm th} = 392\,(29)$ $\langle\rho\rangle = 1.1$ $\rho_{\rm max} = 51$

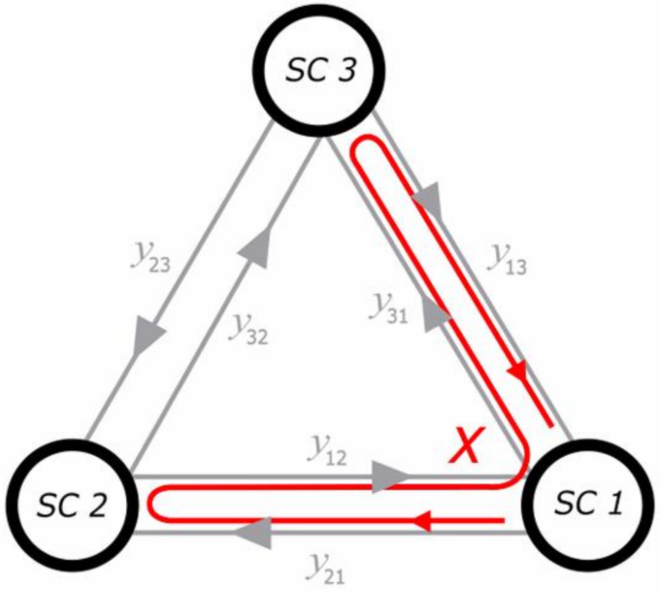
75-78% of events

31-33% of events

LISA GW Response

H. Inchauspé, SG et al [2406.09228]





The actual observable is the *Doppler modulation of the laser between spacecrafts* $y = \left(\frac{\Delta \nu}{\nu_0}\right) = y_{sc} + y_{orb} + y_{GW} (+y_{noise})$

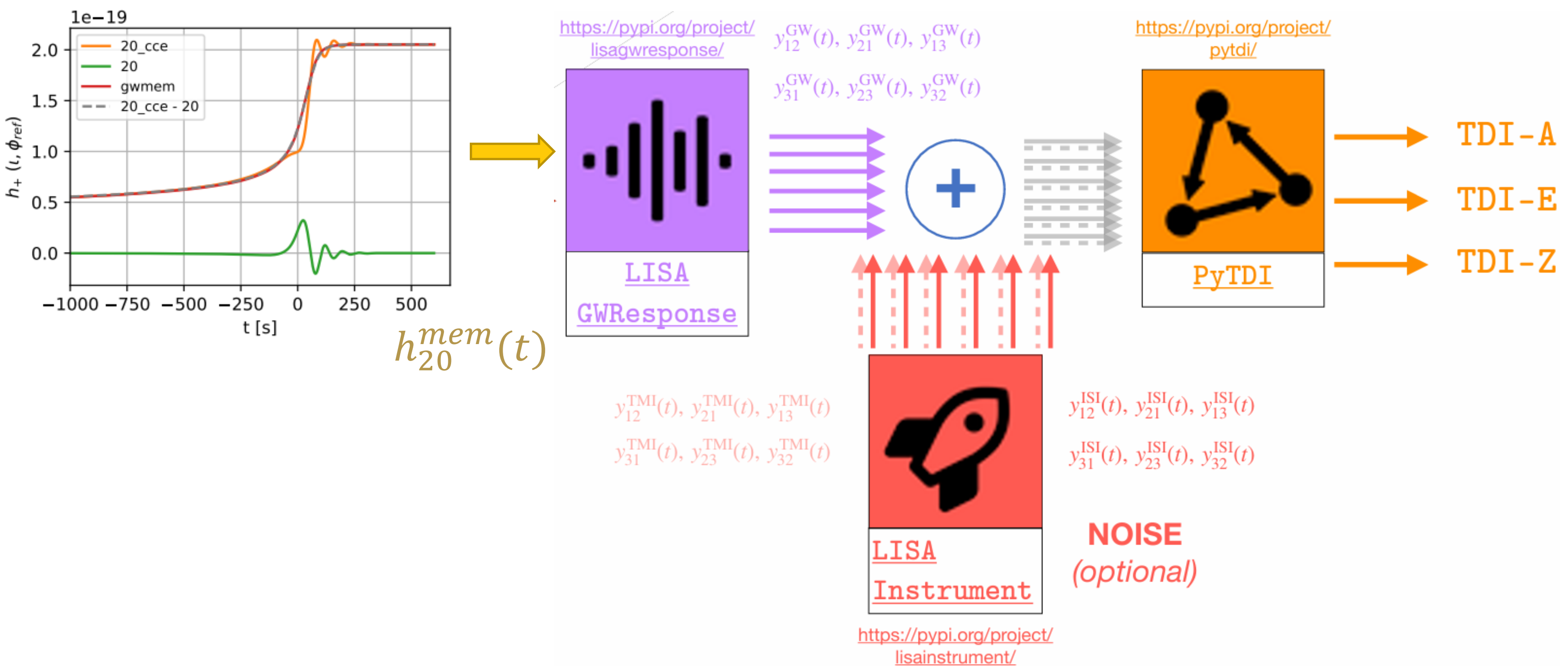
$$y_{ij}^{\mathrm{GW}} = \frac{1}{2}\frac{\epsilon^a \epsilon^b}{1 - \vec{\epsilon}\cdot\vec{k}}\left[h_{ab}\left(ct_r^{(i)} - \vec{k}\cdot\vec{x}_r^{(i)}\right) - h_{ab}\left(ct_e^{(i)} - \vec{k}\cdot\vec{x}_e^{(i)}\right)\right]$$

$\sim \dot{h}$ 1^{st} order transfer function

Time Delay Interferometry (TDI) → combining measurements from across the constellation with appropriate time delays to suppress the laser noise while retaining the GW signal.

Second-generation TDI acts as a high-order differentiator

LISA simulation workflow



Second generation TDI X variable is

$$\begin{aligned} X_2 = X_{1.5} &+ \mathbf{D}_{13121} y_{12} + \mathbf{D}_{131212} y_{21} + \mathbf{D}_{1312121} y_{13} \\ &+ \mathbf{D}_{13121213} y_{31} - \left[\mathbf{D}_{12131} y_{13} + \mathbf{D}_{121313} y_{31} \right. \\ &+ \left. \mathbf{D}_{1213131} y_{12} + \mathbf{D}_{12131312} y_{21}\right], \end{aligned}$$

with

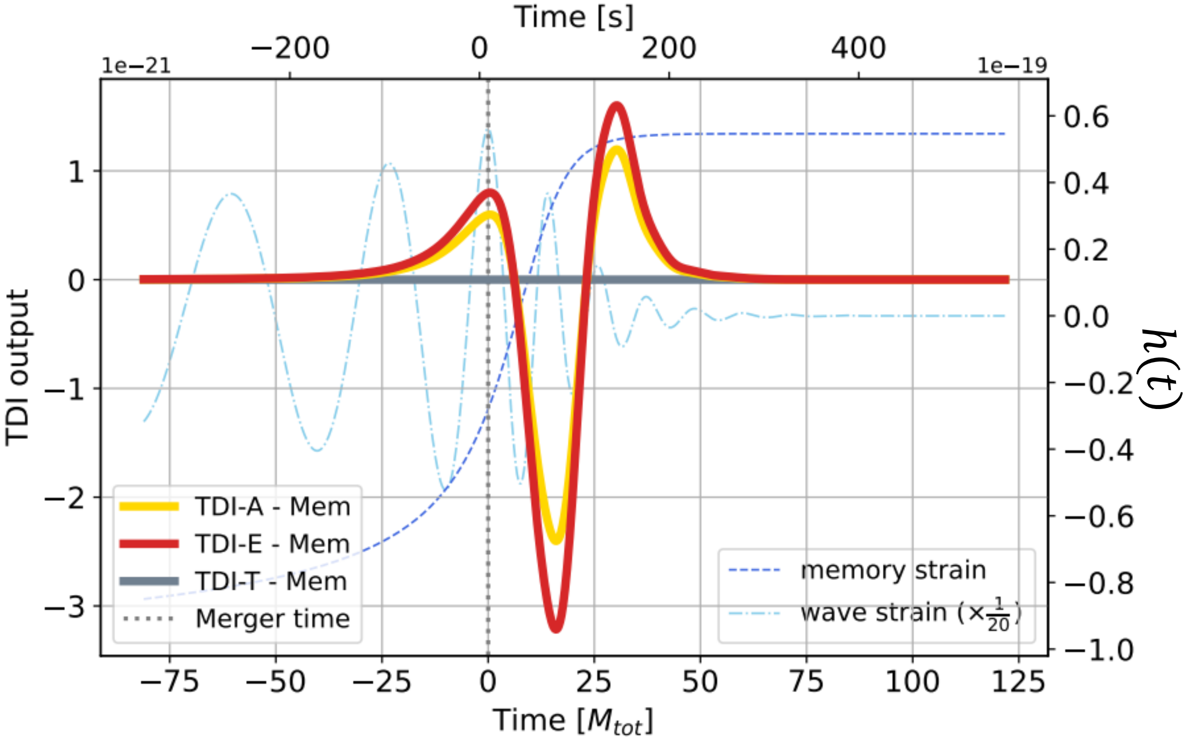
$$\begin{aligned} X_{1.5} = y_{13} &+ \mathbf{D}_{13} y_{31} + \mathbf{D}_{131} y_{12} + \mathbf{D}_{1312} y_{21} \\ &- (y_{12} + \mathbf{D}_{12} y_{21} + \mathbf{D}_{121} y_{13} + \mathbf{D}_{1213} y_{31}). \end{aligned}$$

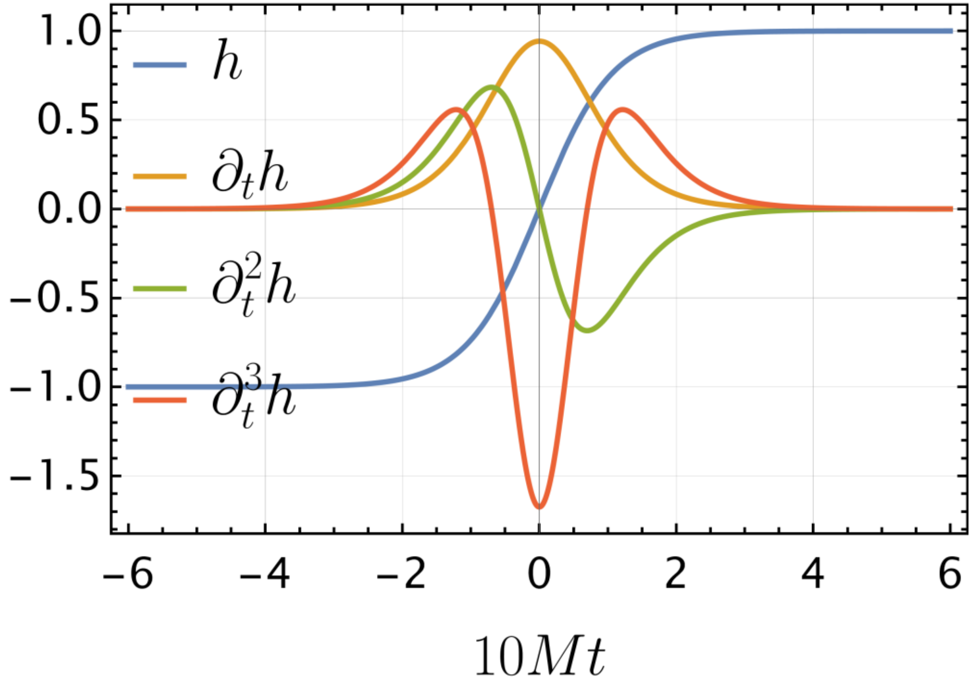
Delay operator:

$$\mathbf{D}_{ij} x(t) = x(t - L_{ij}(t)) \xrightarrow{FT} \tilde{\mathbf{D}}_{ij} \tilde{x}(f) = \tilde{x}(f)\, e^{-2\pi i f L_{ij}(t)}$$

LISA response to GW Memory: time domain

GW memory imprint of a binary merger with $M_{tot} = 10^6 M_{\odot}, q = 1, z = 1, \iota = \frac{\pi}{2}$



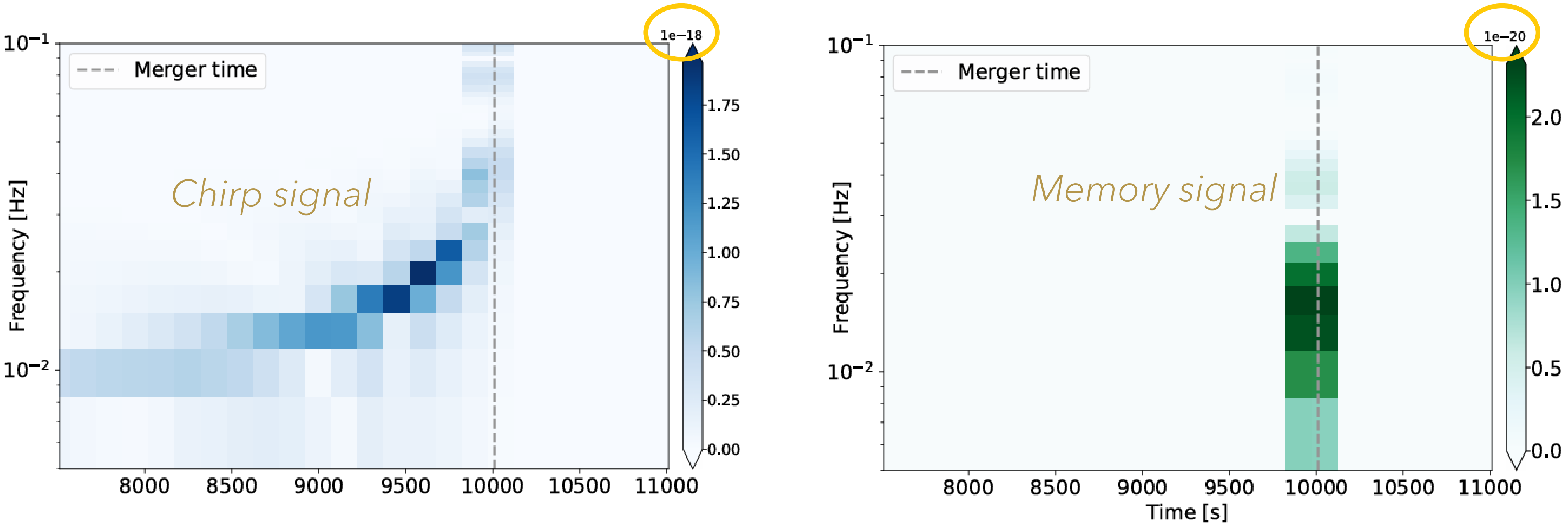


Burst-like signal: We don't observe the persistent off-set of the memory, but just its time-variation $X \propto \partial^3 h$

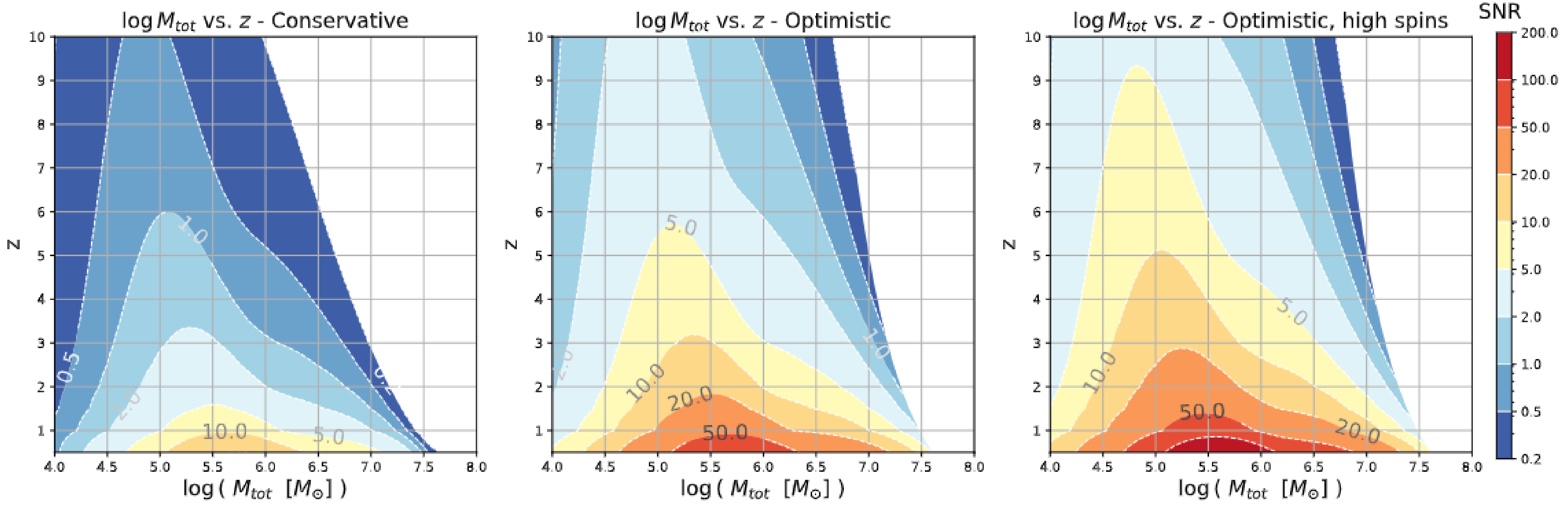
$h^{mem} = \Delta h \tanh\left(\frac{t-t_c}{\Delta T}\right)$ with $\Delta T = 60\ M$

Time-Frequency representation

Oscillatory and memory signals have very separate time-frequency representation.
Can we use this to separate the two?



Scientific Reach of LISA: Memory Waterfall Plots



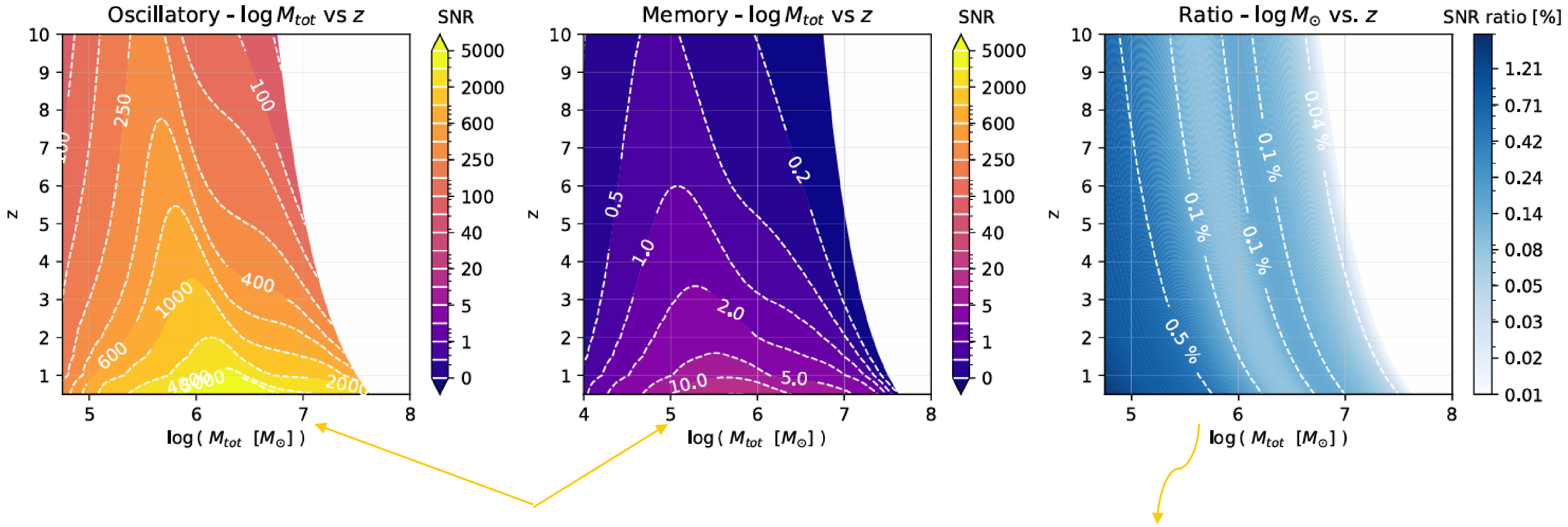
Average situation

Equal mass ratio, edge-on, optimal sky direction

With aligned spins $\chi = 0.8$

SNR Waterfall: Oscillatory vs Memory

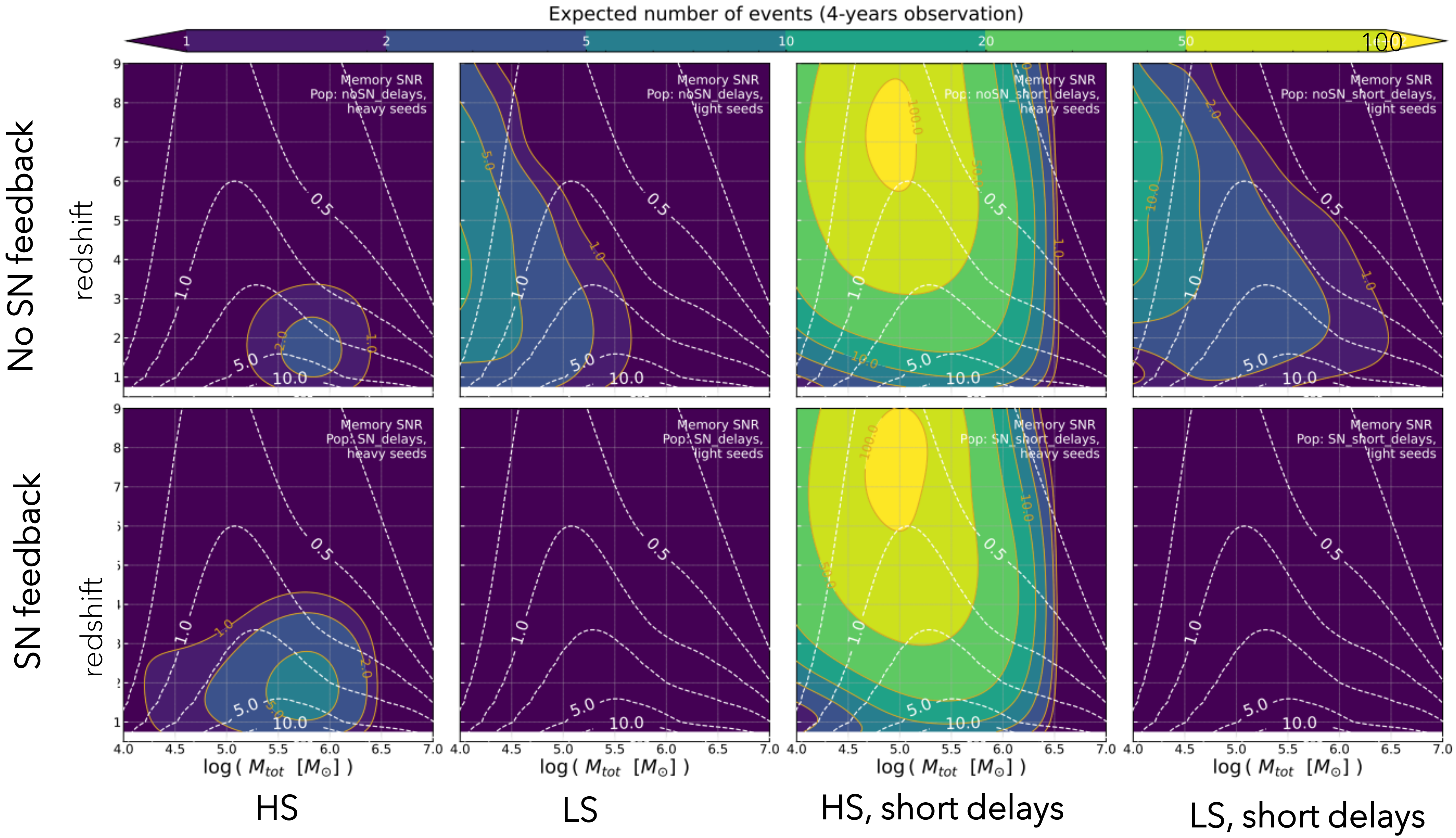
Results for the conservative baseline



SNR peaks at different total masses reflecting the different frequency dependence

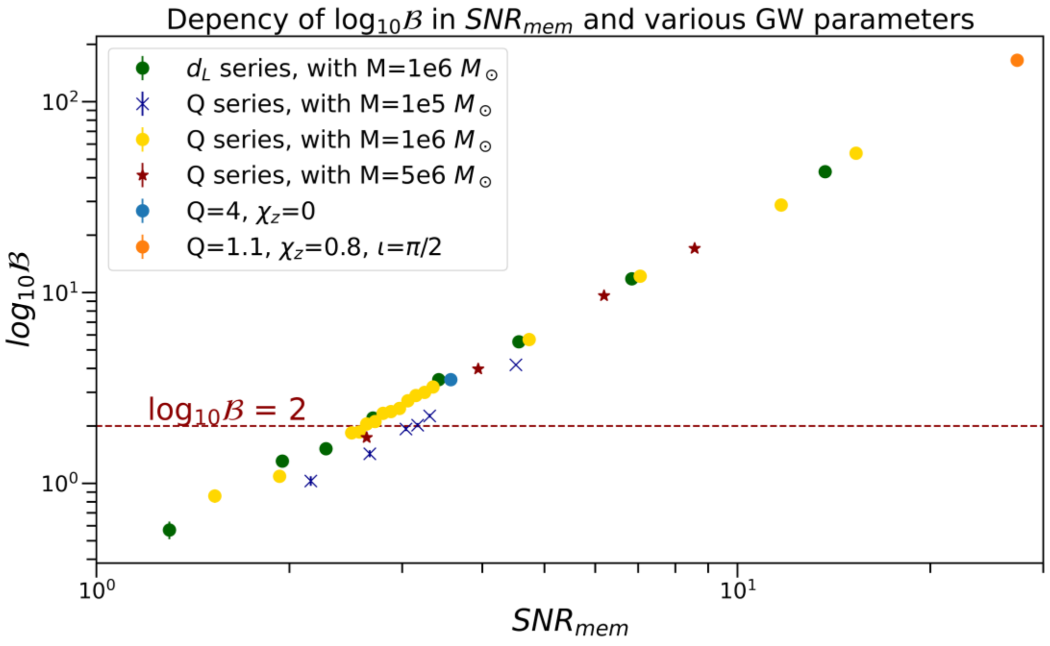
The SNR ratio can be up to a few percent for edge-on systems

SNR vs SMBH population models



Bayes factor computation

LISA fundamental Physics WG
A.Cogez, **SG** et al (to appear)



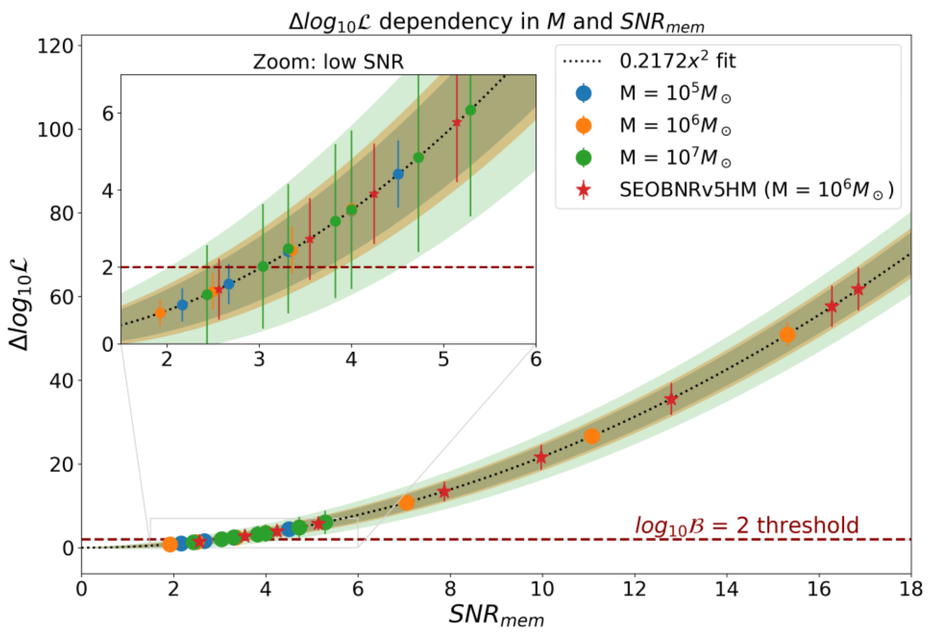
Bayes factor: $\mathcal{B} = \frac{Z_{o+mem}}{Z_o}$ with evidence $Z = p(d|m)$

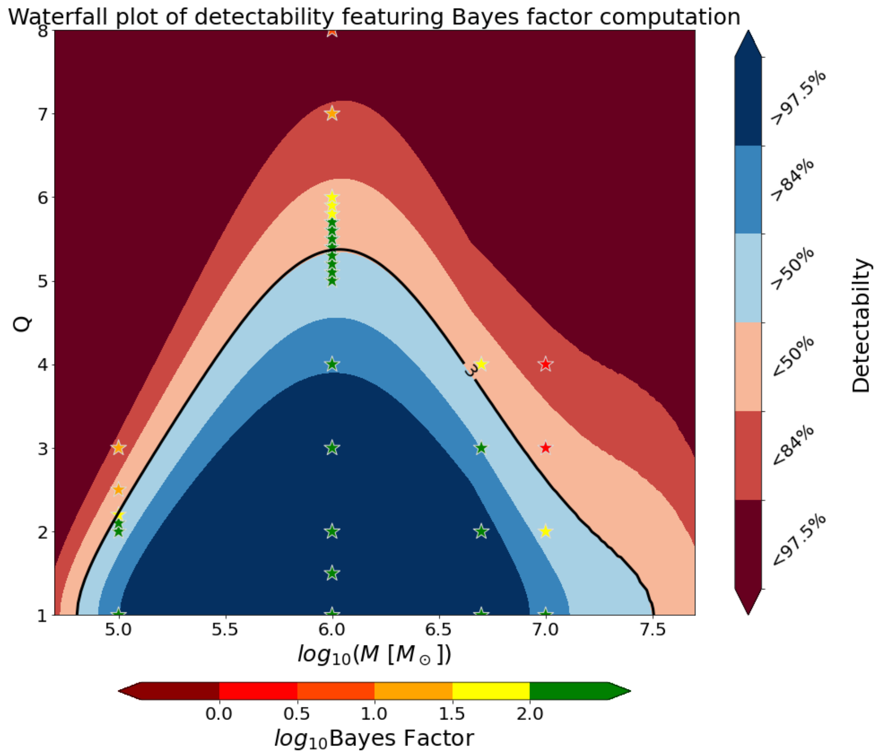
$\mathcal{B}$	$\log_{10}\mathcal{B}$	Interpretation
$[1, 3]$	$[0, \frac{1}{2}]$	Barely worth mentioning
$[3, 10]$	$[\frac{1}{2}, 1]$	Substantial
$[10, 32]$	$[1, \frac{3}{2}]$	Strong
$[32, 100]$	$[\frac{3}{2}, 2]$	Very strong
$[100, +\infty[$	$[2, +\infty[$	Decisive

Decisive evidence for $SNR_{mem} \sim 3$

Full parameter exploration

$$\log_{10} \mathcal{B} \approx \log_{10} \mathcal{L}^{o+mem}(d|\theta_{source}) - \log_{10} \mathcal{L}^{o}(d|\theta_{source}) = \Delta \log_{10} \mathcal{L}$$

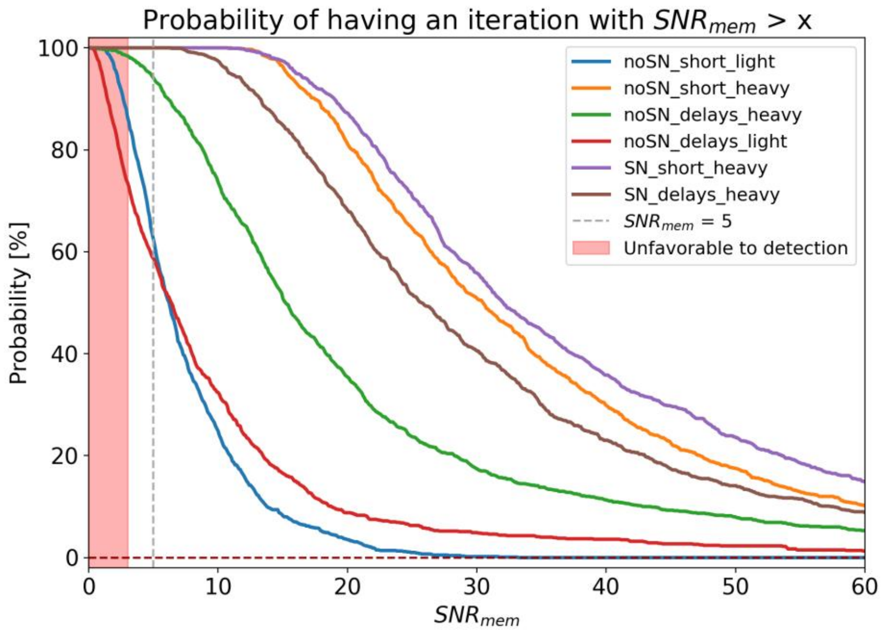




Latest updates on population studies

SN	Delays	Seeds	Nb of events detected	Nb of events with $SNR_{mem} > 3$	Nb of events with $SNR_{mem} > 5$	Nb of events to reach $\log_{10} B^{cumul} > 2$
Yes	Short delays	Light	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	N\A
		Heavy	1024^{+46}_{-47}	87^{+16}_{-15}	37^{+10}_{-10}	$7.0^{+0.7}_{-0.6}$
	Delays	Light	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	$0.0^{+0.0}_{-0.0}$	N\A
		Heavy	$21^{+8.0}_{-6.0}$	$11^{+6.0}_{-5.0}$	$8.0^{+5.0}_{-4.0}$	$1.7^{+0.7}_{-0.4}$
No	Short delays	Light	38.0^{+10}_{-10}	$2.0^{+3.0}_{-2.0}$	$1.0^{+2.0}_{-1.0}$	$10.8^{+8.7}_{-4.5}$
		Heavy	1033^{+48}_{-52}	81^{+16}_{-15}	$32^{+10}_{-9.0}$	$7.3^{+0.8}_{-0.7}$
	Delays	Light	$13.0^{+6.0}_{-6.0}$	$1.0^{+3.0}_{-1.0}$	$1.0^{+2.0}_{-1.0}$	$5.2^{+4.6}_{-2.2}$
		Heavy	$8.0^{+5.0}_{-4.0}$	$4.0^{+3.0}_{-3.0}$	$3.0^{+3.0}_{-3.0}$	$1.8^{+1.4}_{-0.6}$

TABLE V: Table with the expectation of detection for different models, using Barausse's populations. The values are provided as median$^{+2\sigma}_{-2\sigma}$ to reflect the asymmetries of the distributions.



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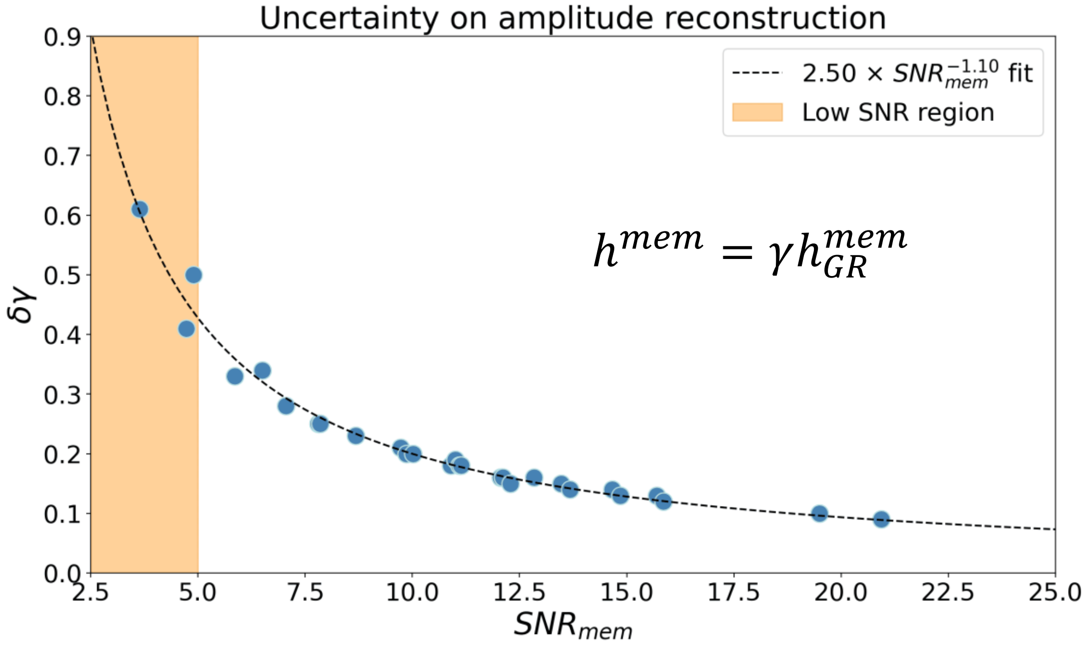


Outcomes

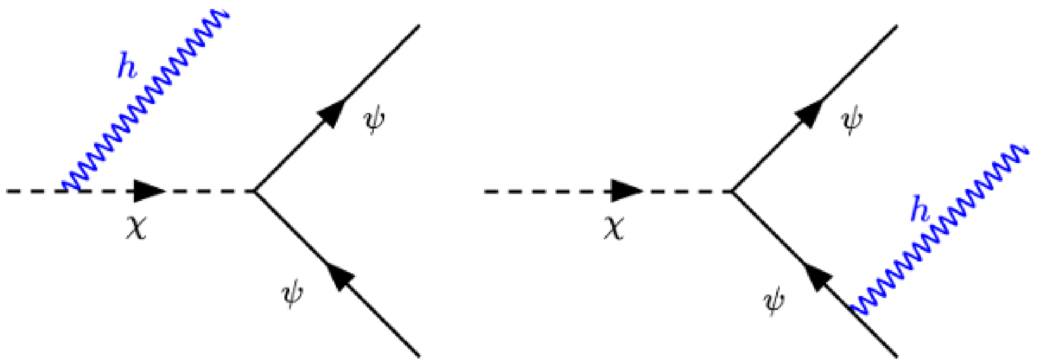
1. GW memory contributes complementary angular information to the waveform
2. After the **LISA response** and **TDI processing**, the memory manifests as a **burst-like event** → not sensitive to zero frequency shift
3. **Extended SNR analysis**: sky map, mass, redshift, spin, mass ratio
4. The SNR ratio is maximized at **low total mass** and **redshift** → relevant of *Intermediate black Hole binaries/ out-of-band mergers (→ Memory from PBH mergers paper)*
5. After Bayesian analysis, we find that for decisive detection $SNR_{mem}^{thres} \approx 3$
6. LISA is likely to detect several memory events even with **high SNR** → **important confirmation for GR**

GW memory from the early universe

Gravitational wave memory modifies the **tail of the GW spectrum** → relevant for detection

The memory and the soft radiation are related through the soft theorem

Are the calculation for Ω_{GW} equivalent?



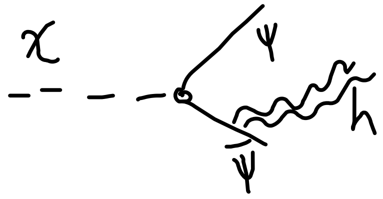
GW bremsstrahlung decays $\chi \to \psi\bar{\psi} + h$

Decay rate $\Gamma_{\chi\to\psi\bar{\psi}} = \frac{Y^2 m_\chi}{8\pi}$

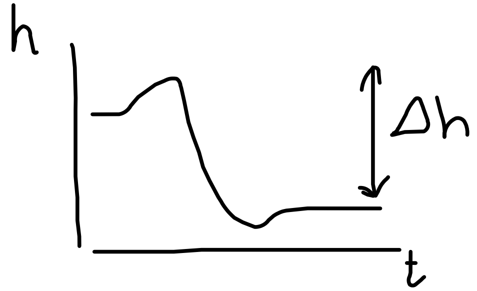
Equivalence of the calculation

Gasparotto, Zosso, Schütte-Engel, Blas…*to appear*

Graviton bremsstrahlung



Memory background



$$\frac{d\Gamma_{br}}{dlnE} = \frac{Y^2 m_\chi^3}{64\pi^3 m_{pl}^2} \qquad E \ll m_\chi$$

$$\delta h_{ij}^{TT}(u, r, \Omega) = 2\frac{GM}{r}\Theta(u)\epsilon_{ij}^{+}$$

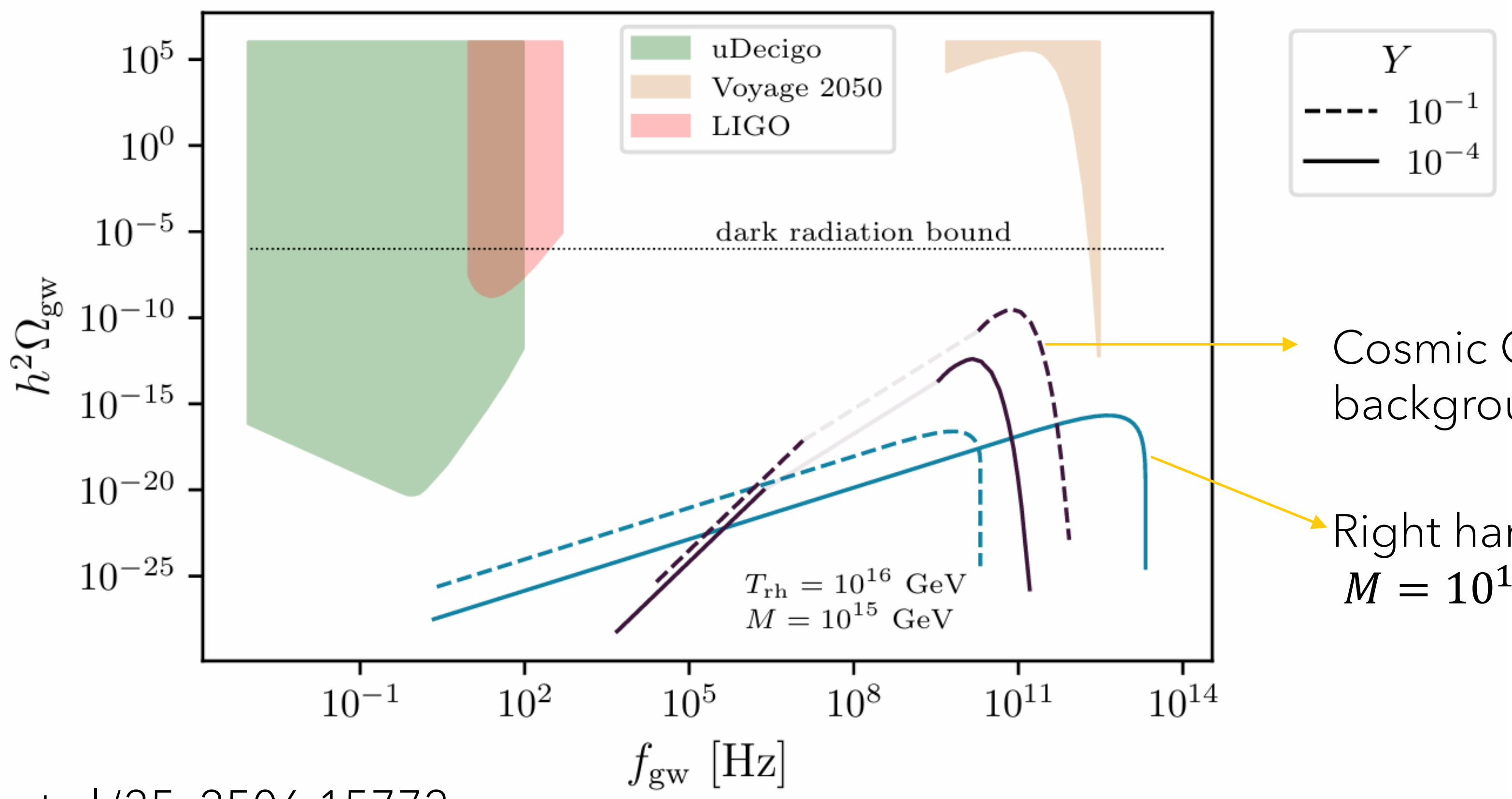
$$\Omega_{GW} = \frac{E}{\rho_c}\int dt\, n_\chi \left(\frac{a}{a_0}\right)^3 \frac{d\Gamma_{br}}{dlnE}$$

$$\Omega_{GW} = \frac{\pi}{2G\rho_c}\int dz\, f^3 \left\langle \left|\tilde{h}_+(f)\right|^2 + \left|\tilde{h}_\times(f)\right|^2 \right\rangle \frac{\mathrm{d}\#}{\mathrm{dtdz}}$$

$$\frac{\mathrm{d}\#}{\mathrm{dtdz}} = \frac{\Gamma}{1+\mathrm{z}} n_\chi \frac{4\pi r^2}{H(z)}$$

$$\Omega_{GW} = \frac{2fG\rho_\chi \Gamma m_\chi}{H}\left(\frac{a_D}{a_0}\right)^3$$

Gravitational wave spectrum

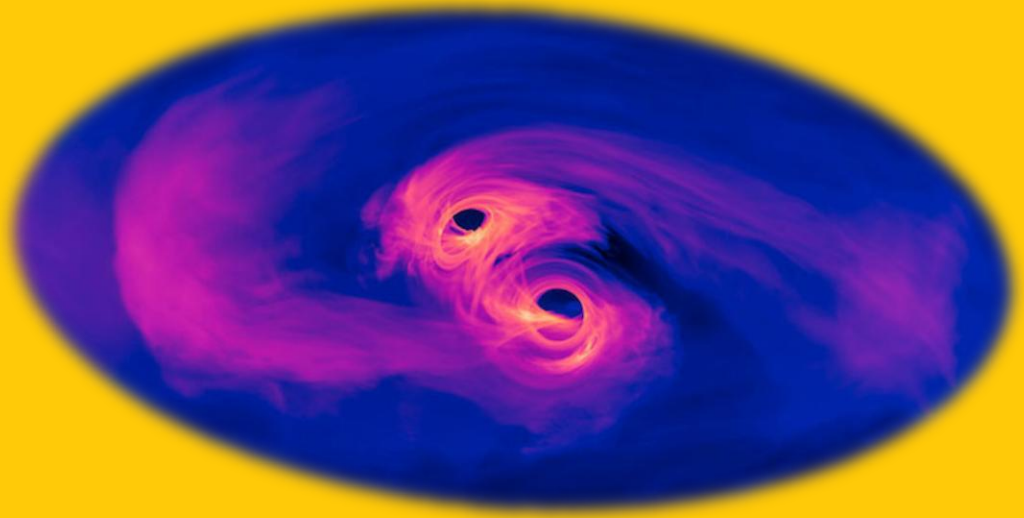


Cosmic Gravitational Microwave background $T_{rh} = 10^{16} GeV$

Right handed neutrino decay $M = 10^{15} GeV$

Murayama et al '25, 2506.15772

Memory for tests of GR?



- Does the memory provide **complementary information**?
- How the memory is **modified in beyond GR theories**?

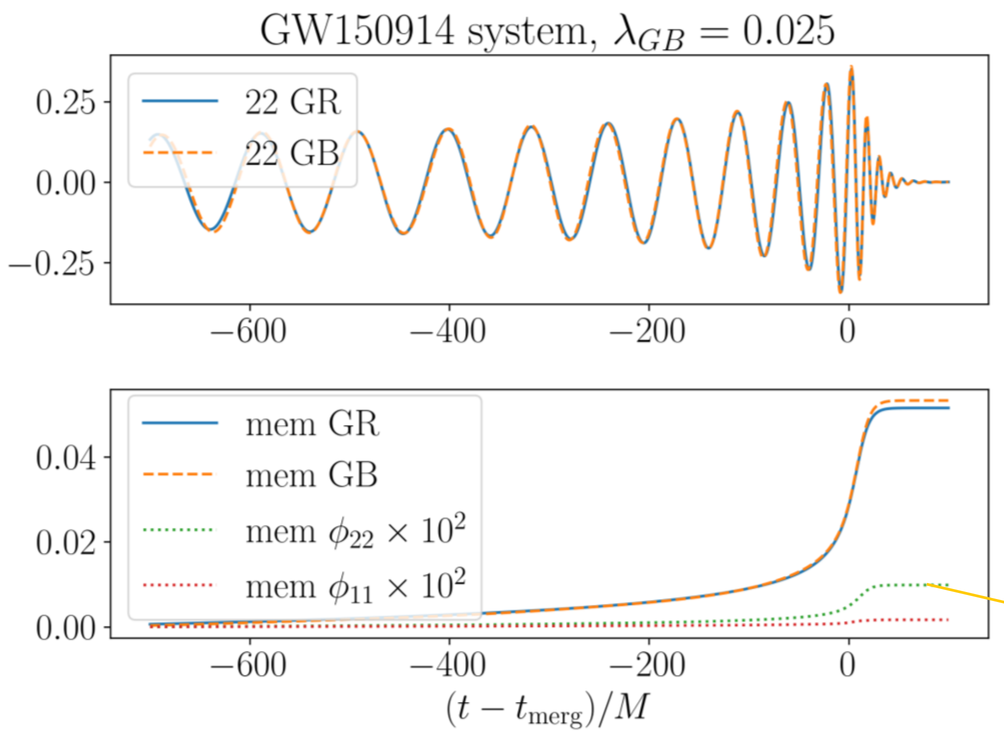
L.Heisenberg et al 2303.02021

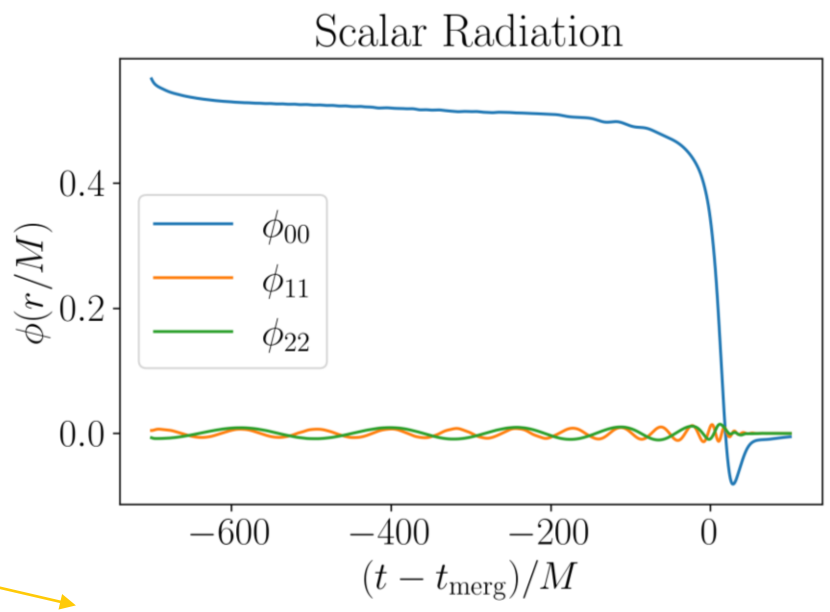
$$\delta h_H^{lm}(u,r) = \frac{1}{r}\sqrt{\frac{(l-2)!}{(l+2)!}} \int_{S^2} \mathrm{d}^2\Omega' \, \bar{Y}^{lm}(\Omega') \times \int_{-\infty}^{u} \mathrm{d}u' \, r^2 \left\langle |\dot{\hat{h}}_+|^2 + |\dot{\hat{h}}_\times|^2 + \sum_{\lambda=1}^{N} |\dot{\hat{\psi}}_\lambda|^2 \right\rangle$$

Memory sourced by every other channel of radiation

Gasparotto, Zosso, Areste Salo…*to appear*

Memory in Gauss-Bonnet

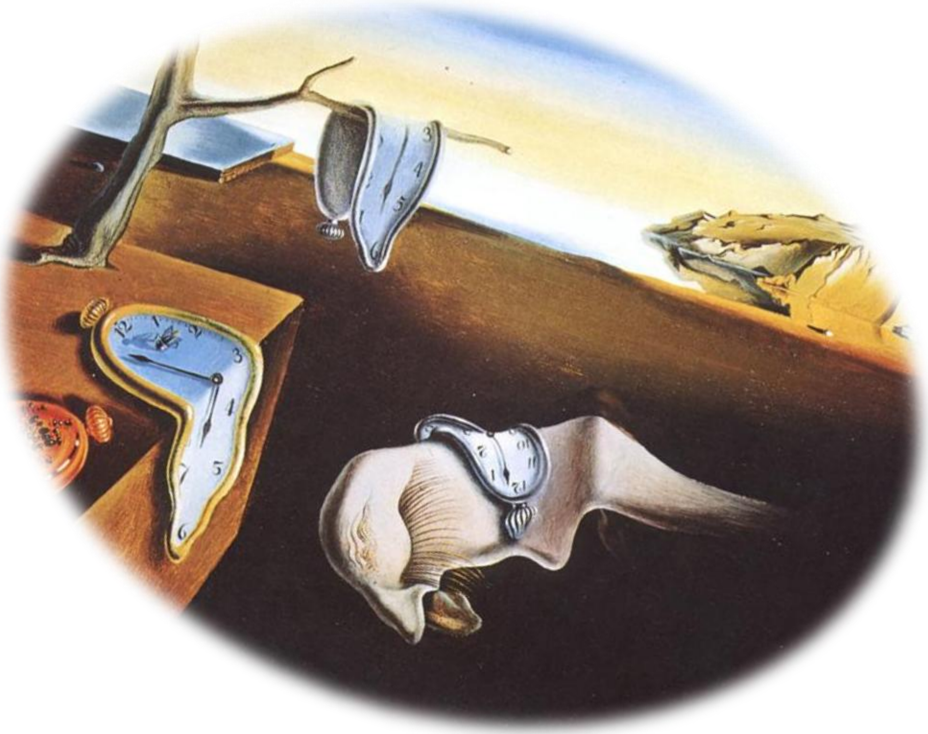




Memory from scalar radiation is suppressed

- Energy radiated in the 00 mode does not directly generate memory
- Merger is louder → higher memory
- Direct coupling with matter → memory in the 00 mode is dominant in the scalar polarization

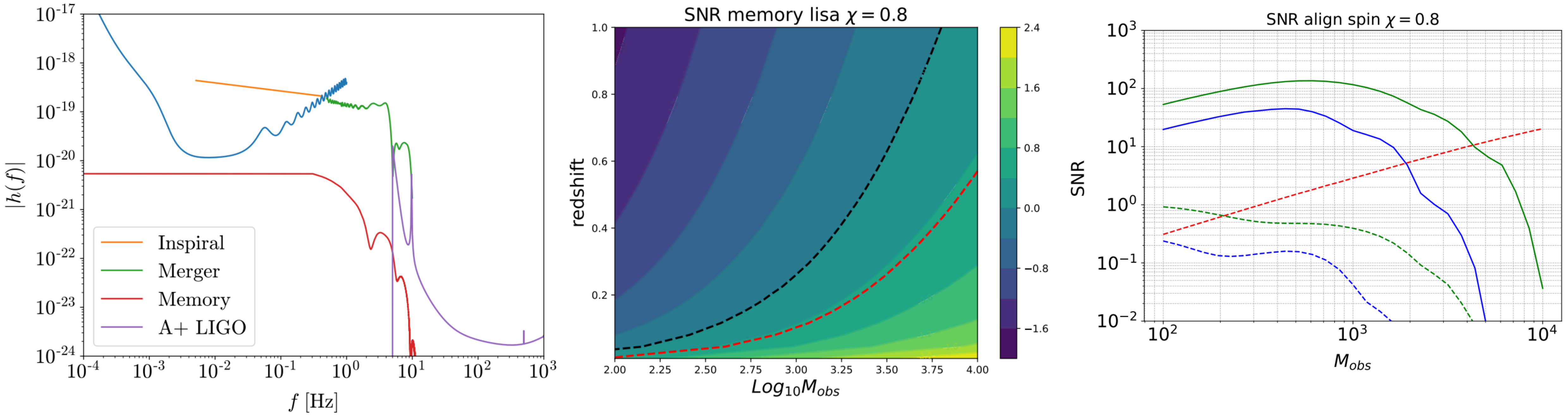
- Relevance of the memory for sources of GW at high frequency → PBH and early universe
- Relation of the memory and soft radiation
- Possible use of the memory as a complementary probe for beyond GR theories



Conclusions

Thank you!

Intermediate Mass Black Holes



Example of multi-band event between LISA and LIGO

$M = 4 \times 10^3 M_{\odot}, z = 0.05, \chi = 0.8, q = 1.2; SNR_{LIGO} = 17, SNR_{LISA} = 20$

The memory decreases by half the uncertainty on d_L

The memory computation

Thorne Formula

$$\delta \bar{h}_{ij}^{TT}(T_R) = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt' d\Omega'} \frac{n'_j n'_k}{|1 - n' \cdot N|} \, d\Omega' \right]^{TT}$$

$$\frac{1}{R^2}\frac{dE_{GW}}{dt d\Omega} n_j n_k \sim \frac{c^3}{16\pi G} \left|\dot{h}_0(t,\Omega)\right|^2$$

Harmonic decomposition of the energy flux

$$h_+ - i h_\times = \sum_{\ell \geq 2} \sum_{|m| \leq \ell} h^{\ell,m}(u,r) \; {}_{-2}Y_{\ell m}(\iota, \phi)$$

$$\delta h^{\ell m}(u) = -R \sum_{\ell', \ell'' \geq 2} \sum_{m', m''} \sqrt{\frac{(\ell-2)!}{(\ell+2)!}}$$

$$\times \int \mathrm{d}\Omega \, Y^*_{\ell m} \, {}_{-2}Y^*_{\ell' m'} \, {}_{-2}Y_{\ell'' m''} \int_{-\infty}^{u} \mathrm{d}u' \dot{h}_0^{*\ell' m'} \dot{h}_0^{\ell'' m''}, \quad (4)$$

Angular integral,

Harmonics decomposition of the oscillatory GW source as input

$$\propto \int e^{i(m' - m'' - m)\phi} \, d\phi,$$

Selection rules

$$\dot{h}_{l'm'} \dot{h}^*_{l''m''} \propto x^n e^{-i(m' - m'')\varphi},$$

$$x = (M\omega)^{2/3}$$

Dominant term is for $m' = m'' = 2$

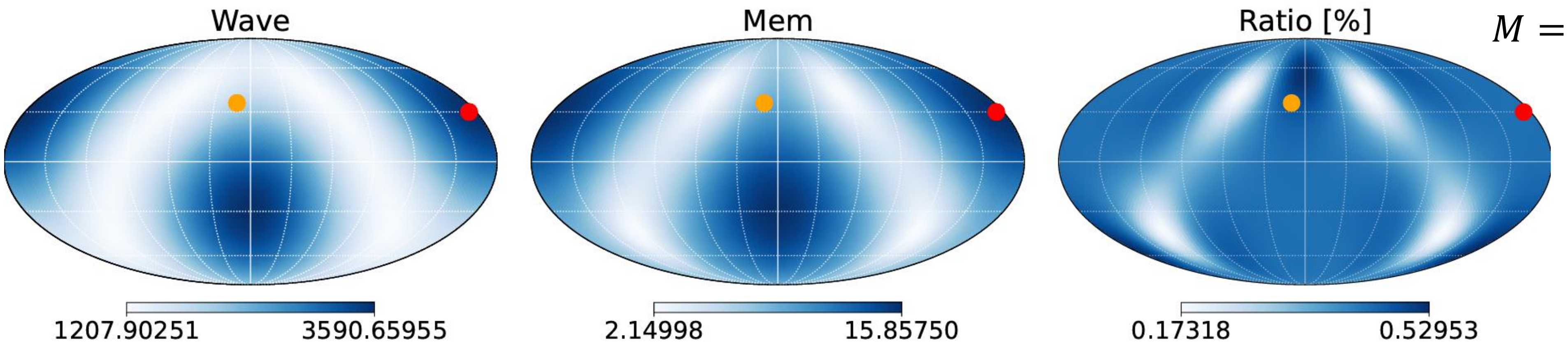
→ dominant memory

$m = 0, l = 2$ mode

Non-oscillating integrand

SNR vs Sky Localization

Merger with
$M = 10^6 M_\odot, \iota = \frac{\pi}{3}$



SNR sky map for the primary wave (left), the memory signal (center) and the percent ratio between the (right). The yellow and red dots indicate the average and maximal memory SNR sky-locations

Baseline	q	χ	inclination ι [rad]	lat. β [rad]	long. λ [rad]	pixel p
1. Conservative	2.5	0.0	1.047	0.62	0.20	145
2. Optimistic	1.0	0.0	1.571	0.52	3.24	192
3. Opt. & Spin.	1.0	0.8	1.571	0.52	3.24	192