Quantifying superluminal signaling in Schrödinger–Newton model

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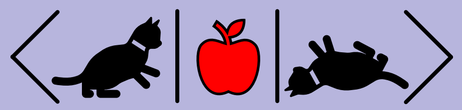


Plan of the presentation

INTRODUCTION SCHRÖDINGER–NEWTON MODEL

MOTIVATION OF STUDY

THEORETICAL BACKGROUND

RESULTS

SUMMARY

Schrödinger equation

- Describes behavior of a quantum system in a potential V

$$i\hbar\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar}{2m}\nabla^2 + V\right)\psi(t,\vec{x})$$

Schrödinger–Newton equation

$$i\hbar\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + \mathrm{V} - Gm^2\int\frac{|\psi(t,\vec{x}')|^2}{|\vec{x}-\vec{x}'|}d\vec{x}'\right)\psi(t,\vec{x})$$

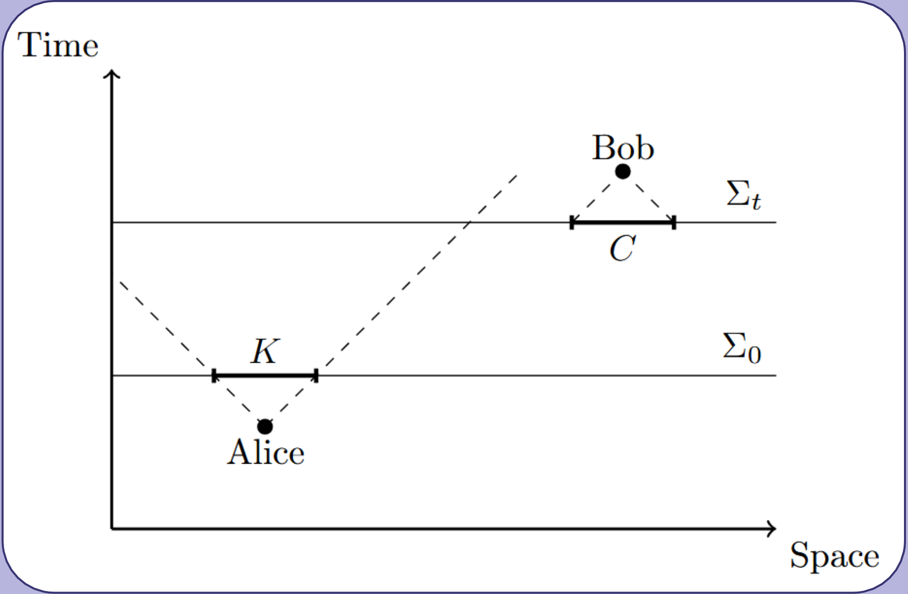
Diósi (1987), Penrose (1996)

- Quantum self-gravitating massive particle
- Non-linear modification, deterministic
- Coupled Schrödinger-Poisson equations / semi-classical grav. / …
- Gravity is fundamentally classical
- Matter fields are quantized

Motivation to Schrödinger–Newton study

$$i\hbar\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V - Gm^2\int\frac{|\psi(t,\vec{x}')|^2}{|\vec{x}-\vec{x}'|}d\vec{x}'\right)\psi(t,\vec{x})$$

- Quantum-to-classical transition
- Boson stars, Cold Dark Matter
- Operational superluminal signaling



Motivation to Schrödinger–Newton study

$$i\hbar\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + \mathrm{V} - Gm^2\int\frac{|\psi(t,\vec{x}')|^2}{|\vec{x}-\vec{x}'|}d\vec{x}'\right)\psi(t,\vec{x})$$

- Violation of causality
  - Schrödinger does it too
  - Reason to disfavor the model?

- Non-relativistic limit of Einstein-Dirac equations

Einstein-Dirac system

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}(\psi)$$

$$\left(\not{D} - \frac{mc}{\hbar}\right)\psi = 0$$

$G_{\mu\nu}$ - Einstein tensor

$\not{D}$ - Dirac operator

$T_{\mu\nu}(\psi)$ - stress-energy tensor for the Dirac field ψ

We have proven also:

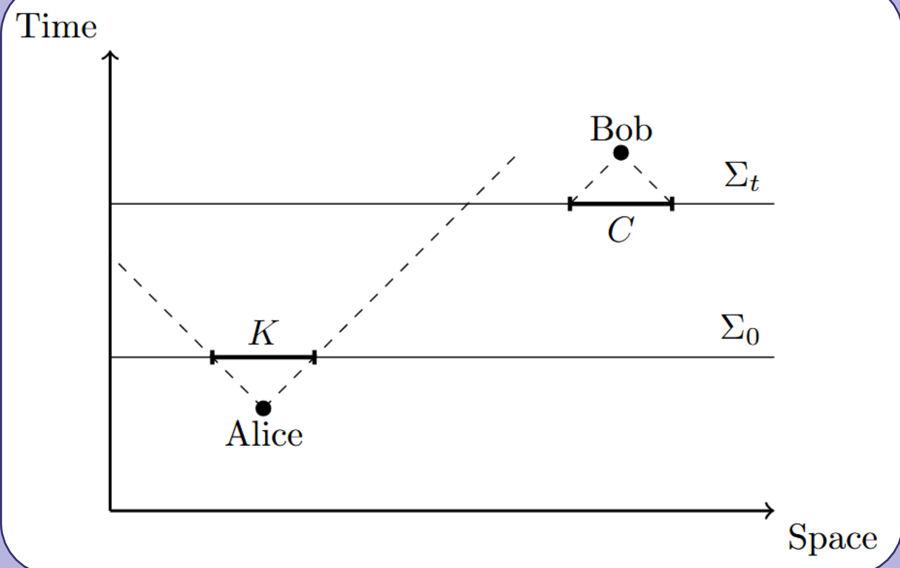
Theorem. *The Einstein–Dirac equations induce a causal evolution of measures for any sensible initial data and any space–time splitting.*

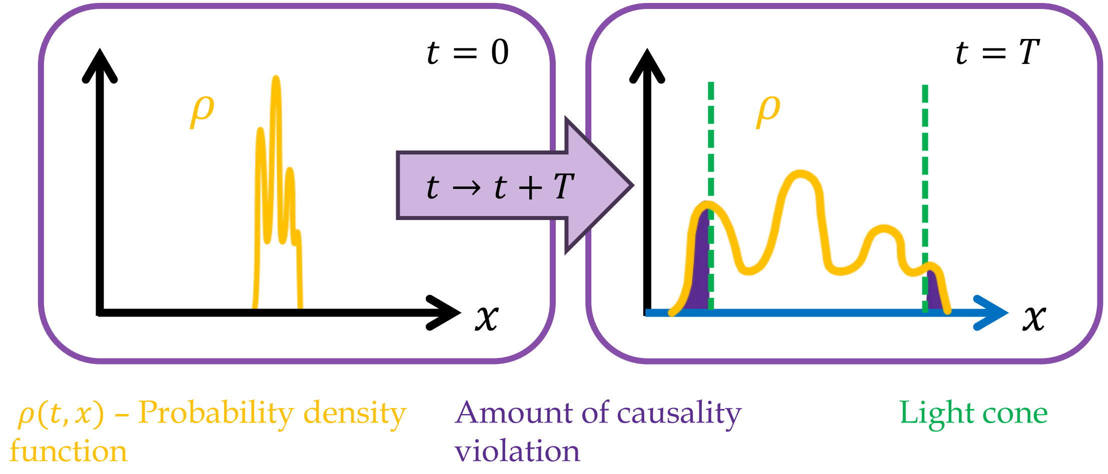
Motivation to Schrödinger–Newton study

$$i\hbar\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + \mathrm{V} - Gm^2\int\frac{|\psi(t,\vec{x}')|^2}{|\vec{x}-\vec{x}'|}d\vec{x}'\right)\psi(t,\vec{x})$$

- Non-relativistic limit of Einstein-Dirac equations
- Is violation of causality smaller or bigger than in Schrödinger?

Quantification of superluminal signaling



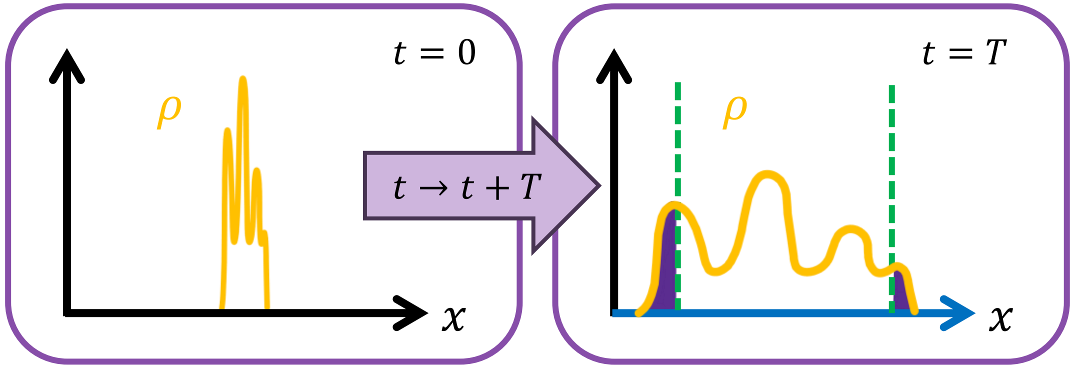


Measure of Causality violation

$$M(t,x,R) = \int_{-R}^{R} \rho(0,x)dx - \int_{-R-ct}^{R+ct} \rho(t,x)dx$$

Eckstein, Miller (2017)

***How much* is *causality* violated?**



$\rho(t,x)$ – Probability density function

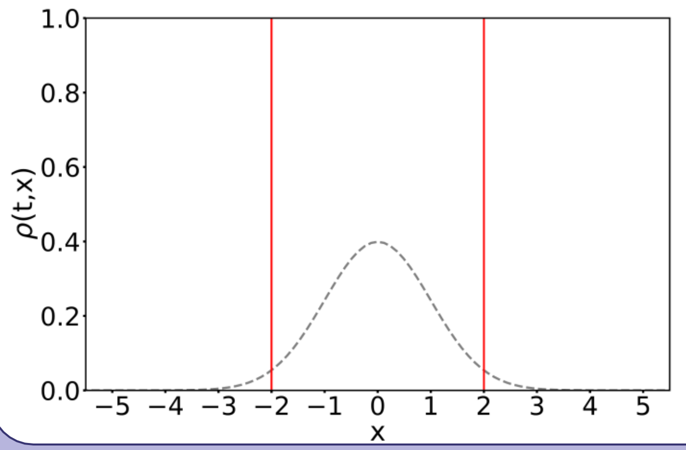
Amount of causality violation

Light cone

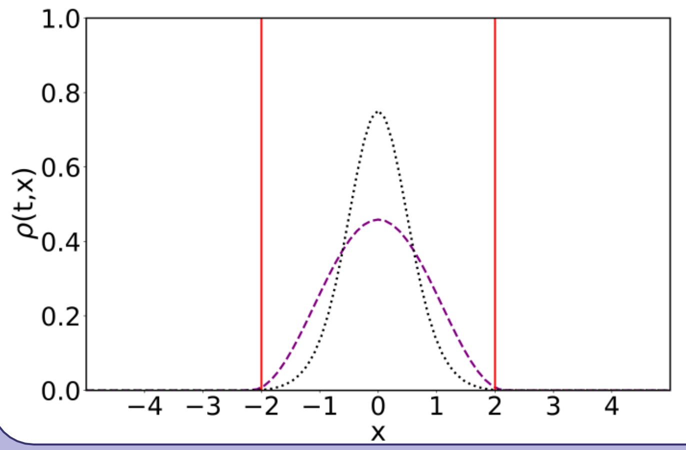
Realization: 2 types of initial states in 1D

Gaussian state: $\boldsymbol{\psi(0,x) = \frac{1}{\sqrt[4]{2\pi}} e^{-x^2/4}}$

Ground state in a trap: $\boldsymbol{V_0 = -20}$



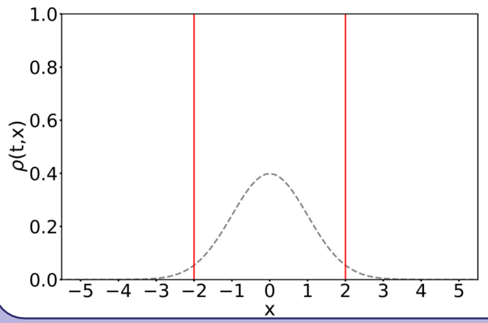
$\kappa = 1$ example



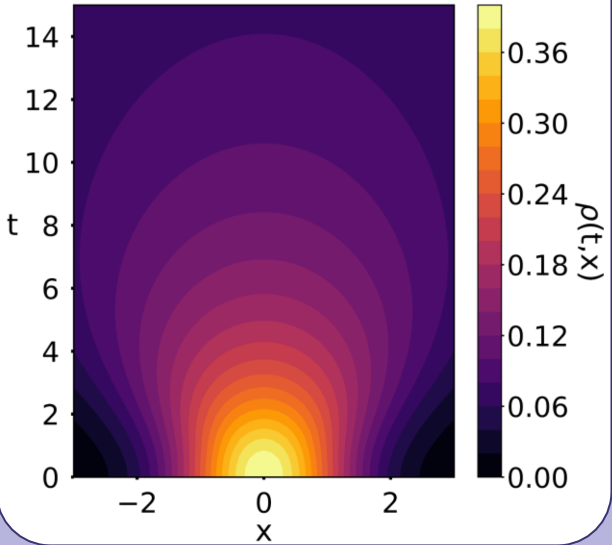
$\kappa = 1$ and $\kappa = 0$ example

$$i\partial_t \psi(t,x) = \left(-\frac{1}{2}\partial_x^2 - \kappa \int \frac{|\psi(t,x')|^2}{|x-x'|} dx'\right)\psi(t,x)$$

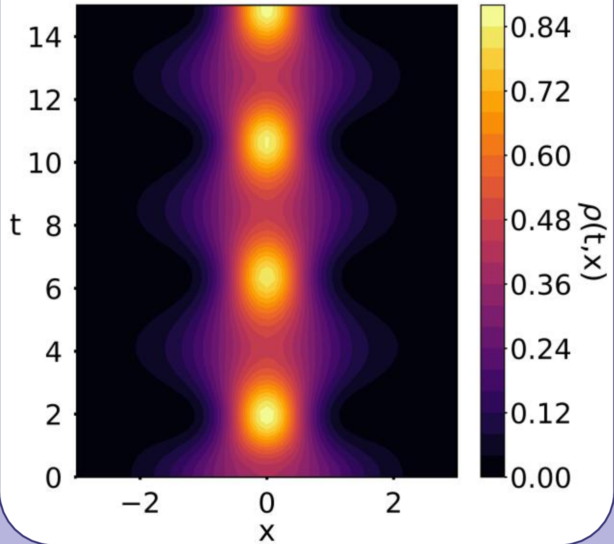
Gaussian initial state evolution



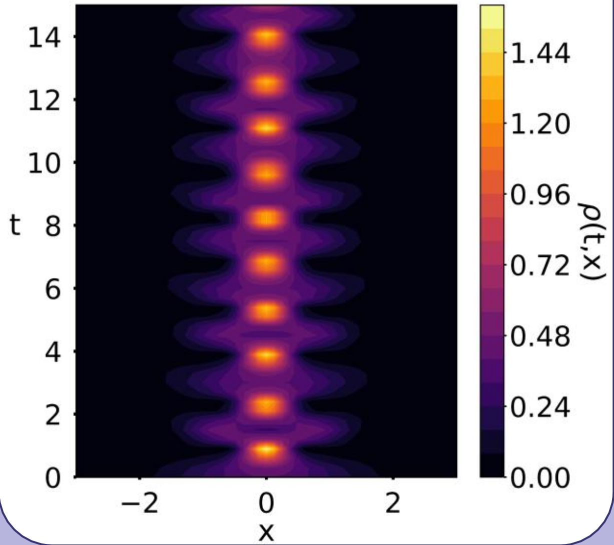
$\kappa = 0.1$



$\kappa = 1$



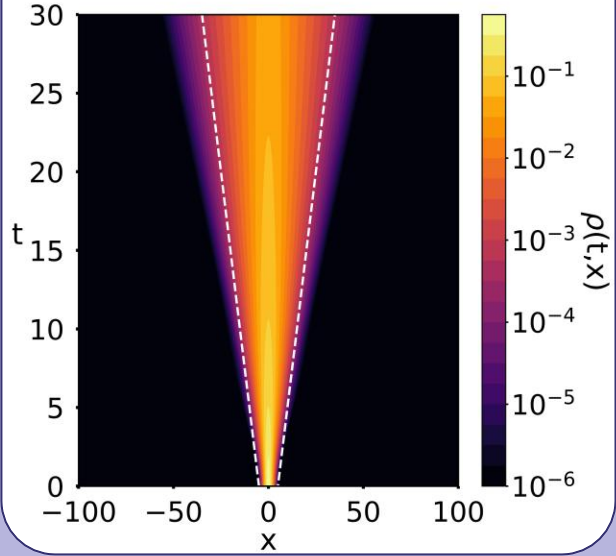
$\kappa = 3$



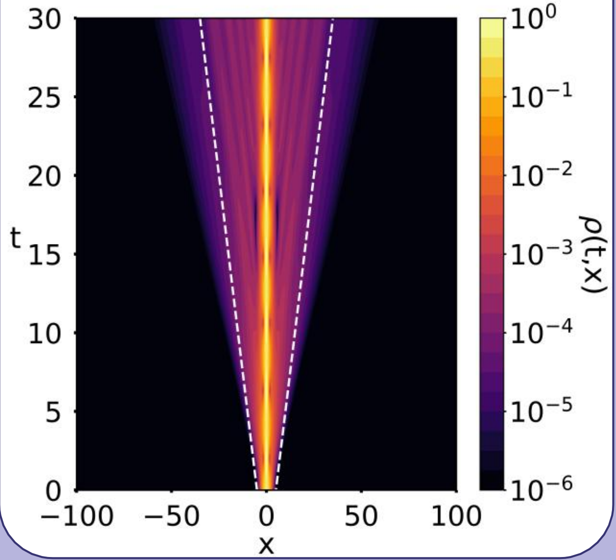
$$i\partial_t \psi(t,x) = \left(-\frac{1}{2}\partial_x^2 - \kappa \int \frac{|\psi(t,x')|^2}{|x-x'|} dx'\right)\psi(t,x)$$

Gaussian initial state evolution (Log scale)

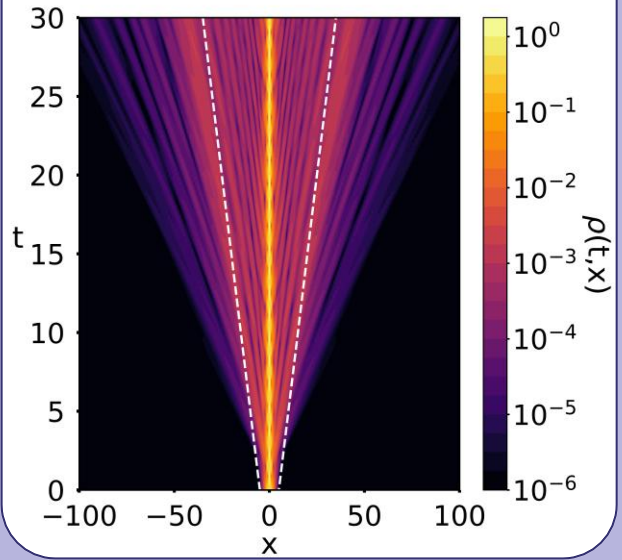
$\boldsymbol{\kappa = 0.1}$



$\boldsymbol{\kappa = 1}$



$\boldsymbol{\kappa = 3}$???



$$i\partial_t\psi(t,x) = \left(-\frac{1}{2}\partial_x^2 - \kappa\int\frac{|\psi(t,x')|^2}{|x-x'|}dx'\right)\psi(t,x)$$

Gaussian initial state evolution (Log scale)

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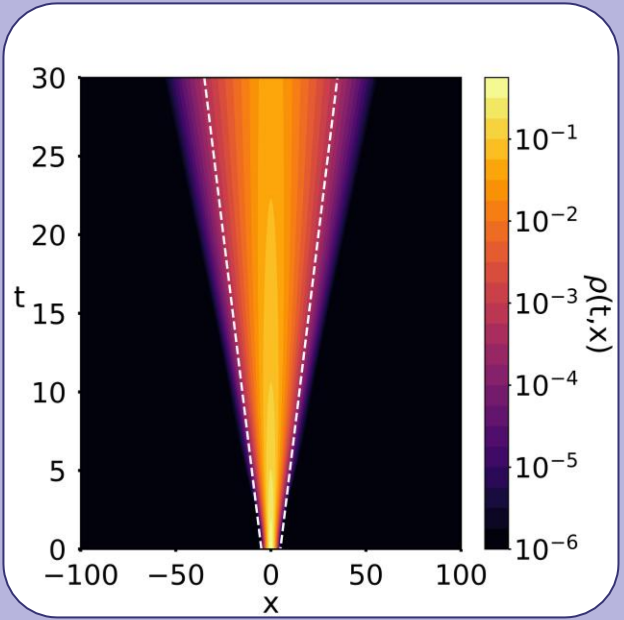
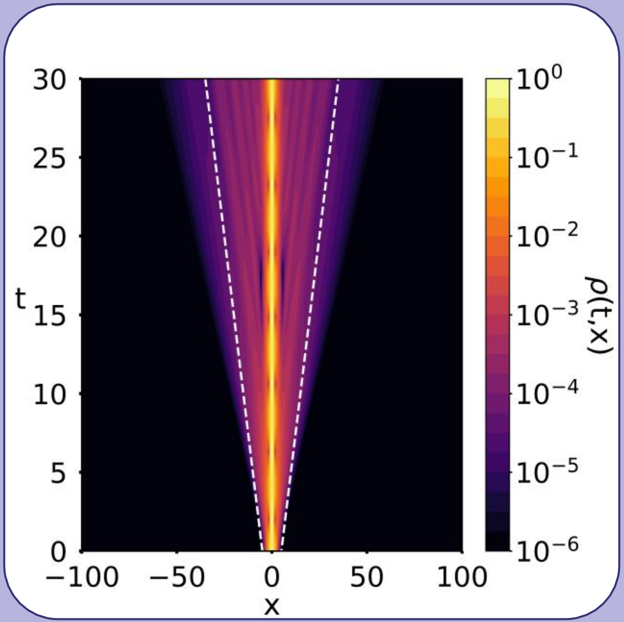
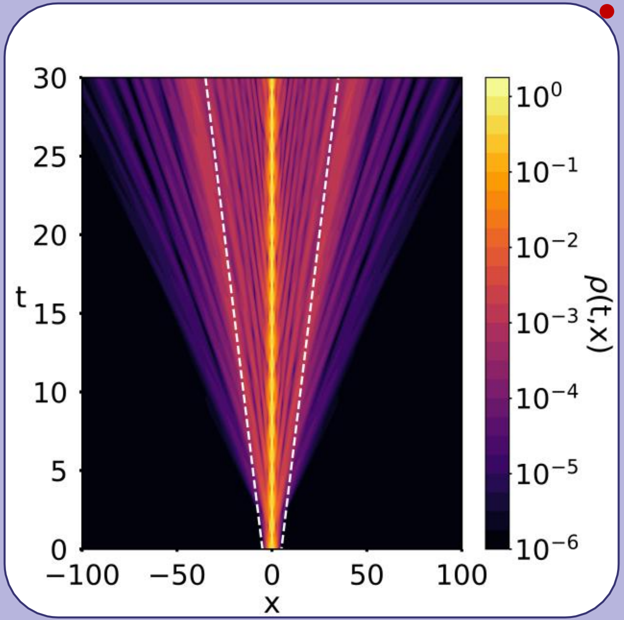


$\kappa = 1$



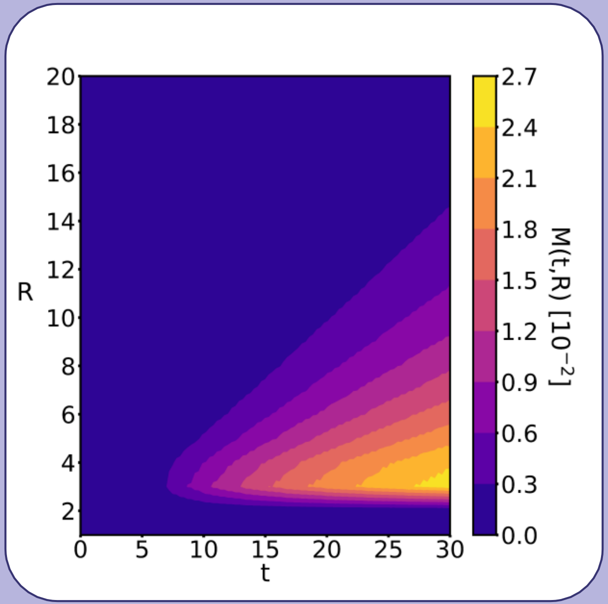
$\kappa = 3$???



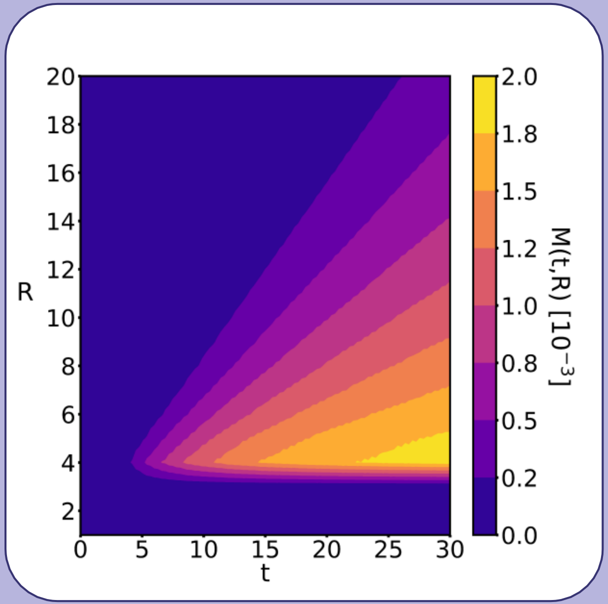
Self-interaction → Compression → Broadens momentum distribution → superluminal escape velocity of high momentum modes

Causality violation: Gaussian initial state

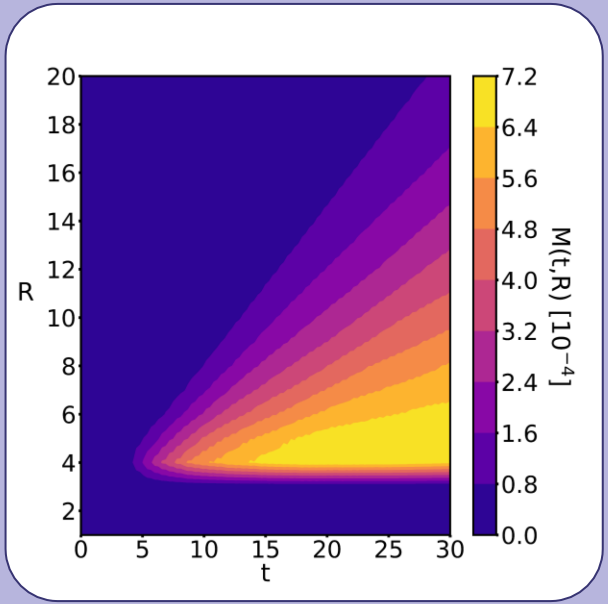
$\kappa = 0$



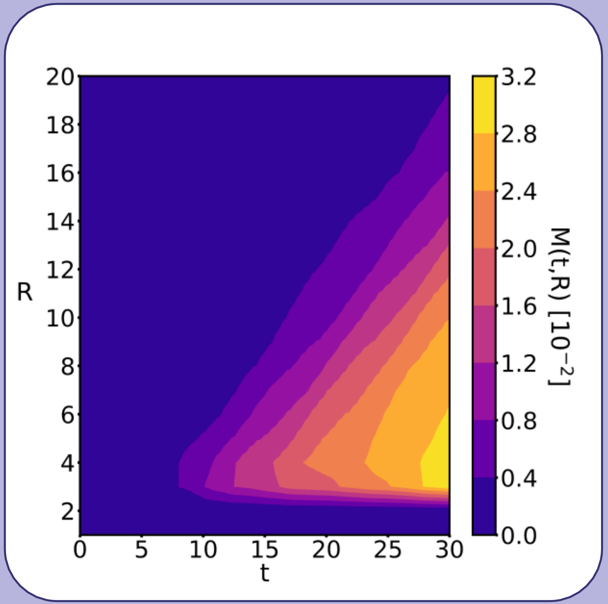
$\kappa = 0.5$



$\kappa = 1$

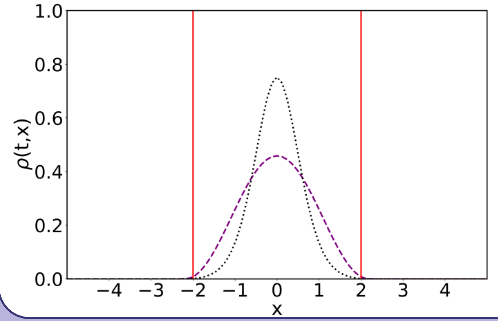


$\kappa = 3$

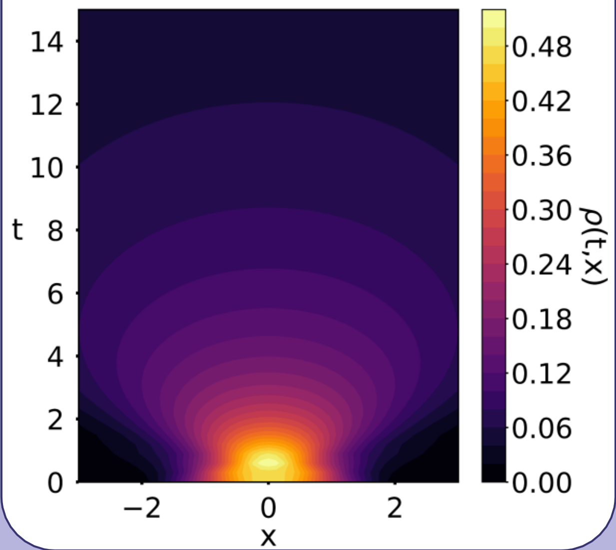


$$M(t,R,\kappa) = \int_{-R}^{R} \rho(0,x)dx - \int_{-R-ct}^{R+ct} \rho(t,x)dx$$

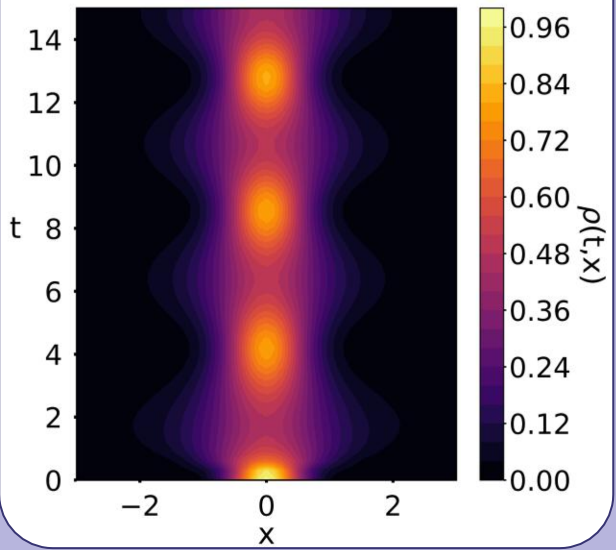
Ground state evolution for $R = 2$



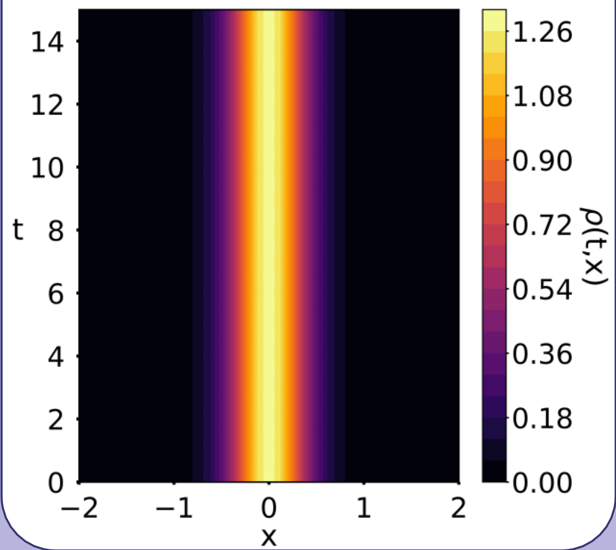
$\kappa = 0.1$



$\kappa = 1$



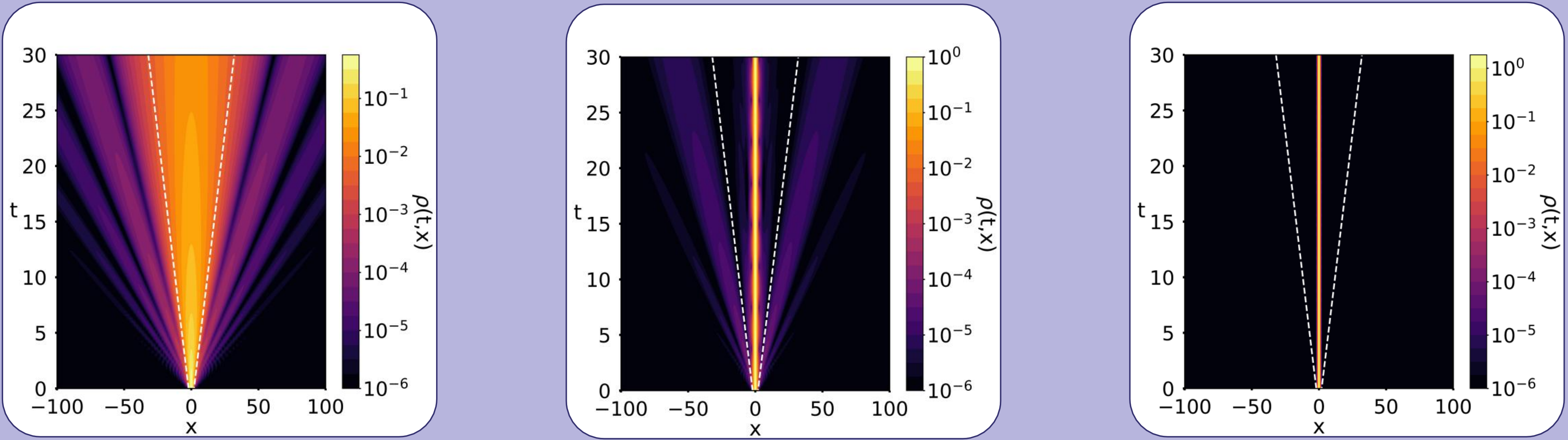
$\kappa = 3$



$$i\partial_t \psi(t,x) = \left(-\frac{1}{2}\partial_x^2 - \kappa \int \frac{|\psi(t,x')|^2}{|x-x'|} dx' \right) \psi(t,x)$$

Ground state evolution for $R = 2$ (Log scale)

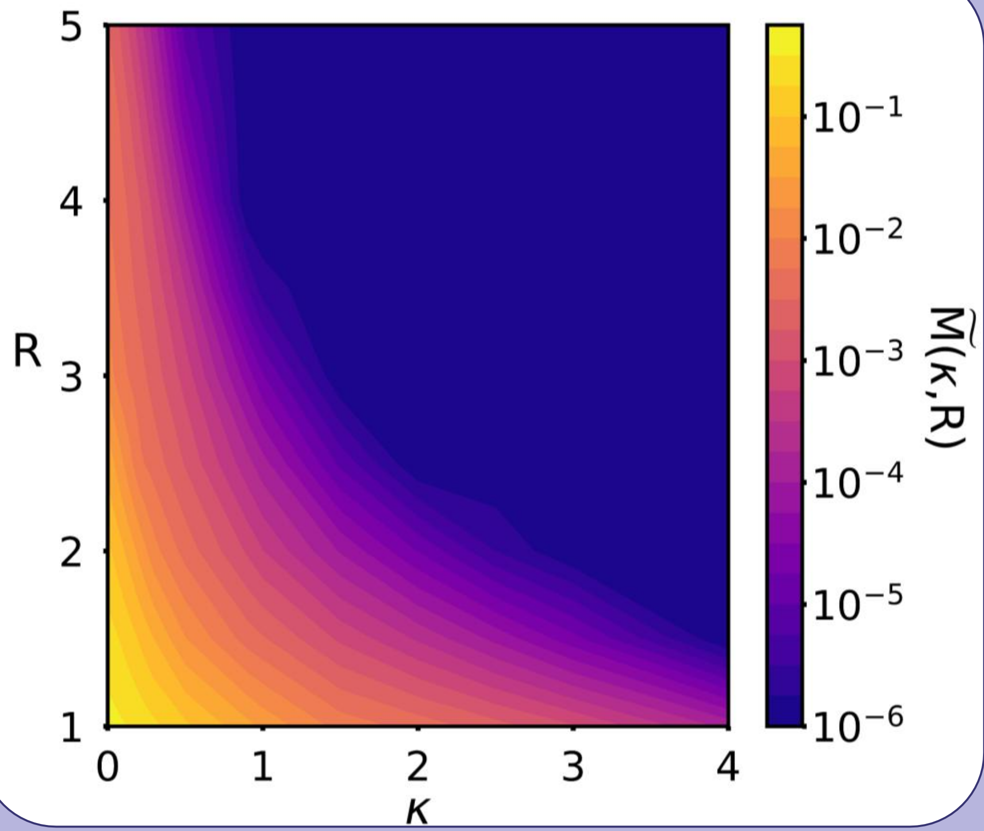
$\boldsymbol{\kappa = 0.1}$ $\boldsymbol{\kappa = 1}$ $\boldsymbol{\kappa = 3}$



$$i\partial_t\psi(t,x) = \left(-\frac{1}{2}\partial_x^2 - \kappa\int\frac{|\psi(t,x')|^2}{|x-x'|}dx'\right)\psi(t,x)$$

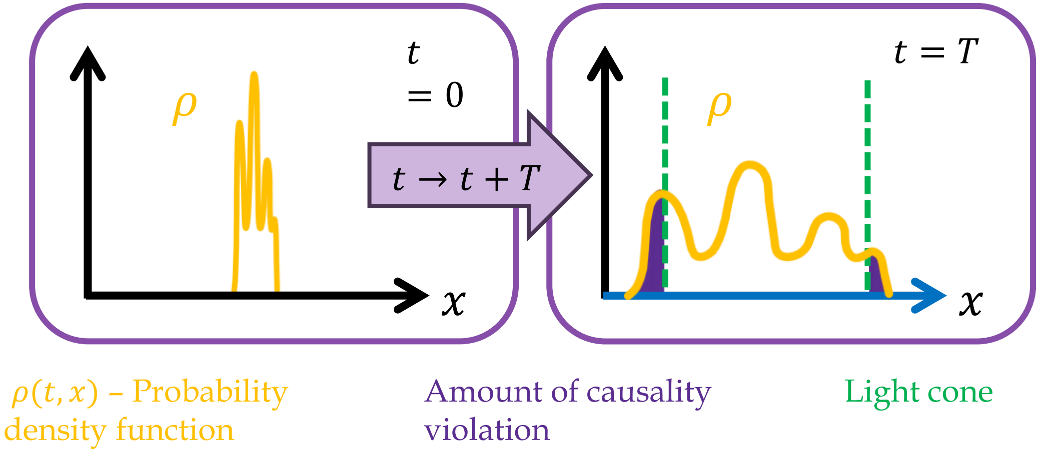
Ground state

Measure of causality violation



$$M(t,R,\kappa) = \int_{-R}^{R} \rho(0,x)dx - \int_{-R-ct}^{R+ct} \rho(t,x)dx$$

$$\widetilde{M}(\kappa,R) = \max_{t\in(0,30)} M(t,R,\kappa)$$



Summary

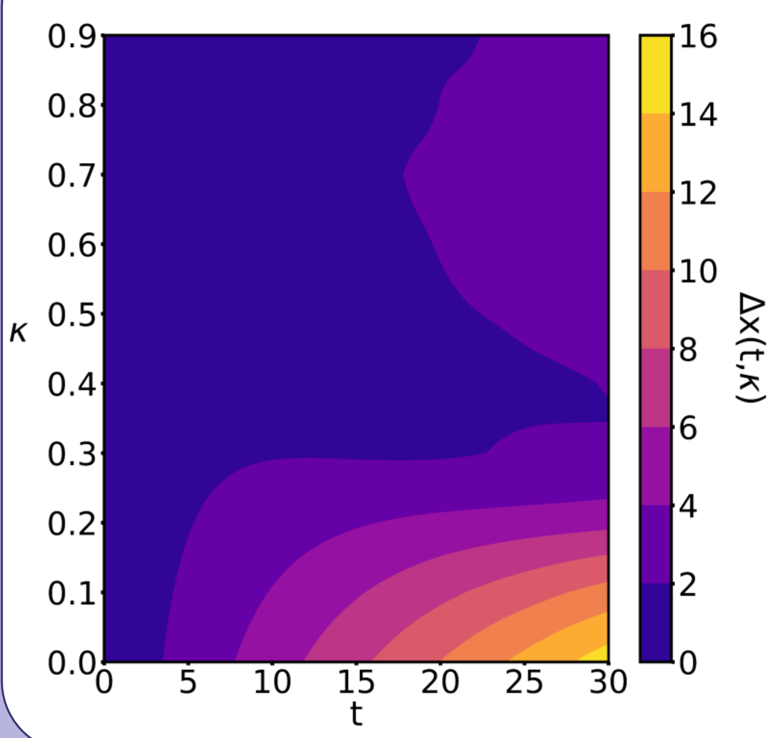
- Schrödinger-Newton equation (1 dim)
- Time evolution of probability density $\rho(x, t)$ – single system
  - Gaussian initial state
  - Ground state of S-N eq. in a trapping potential
- Measure of causality violation $M(t, R, \kappa)$ – quantification of superluminal signal

Self-interaction improves causality properties!

Paper coming soon!

Gaussian initial state

Wave function variation



$$\Delta x = \langle x^2 \rangle - \langle x \rangle^2$$

$$\Delta x(t,\kappa) = \int_{-\infty}^{\infty} x^2 \rho(t,x)dx - \left(\int_{-\infty}^{\infty} x\, \rho(t,x)dx \right)^2$$

Critical $\kappa \approx 0.4$ for localization: wave function is not spreading

$\kappa = 1$ is for a system with a mass $\sim 10^{12}$ A.M.U. (top quark $\approx 2 \times 10^2$ A.M.U.)