CONSTRAINING THE GRAVITATIONAL WAVE SPEED IN THE EARLY UNIVERSE VIA GRAVITATIONAL CHERENKOV RADIATION

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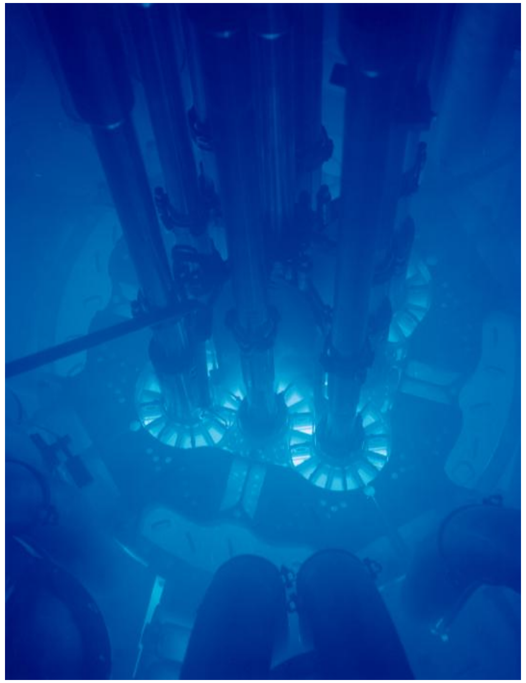
BASED ON: P.C.M. DELGADO, A. GANZ, C. LIN AND RT, JCAP07(2025)088

INTRODUCTION

- Current bounds from GW170817 around $(c_T - 1) \sim 10^{-15}$
- Scalar particles travelling faster than a subluminal gravitational wave will emit a graviton through gravitational Cherenkov radiation
- BBN gives us a bound on the graviton contribution to the total energy density in the early Universe:

$$h_0^2 \Omega_{\rm g} \lesssim 1.12 \times 10^{-6}$$

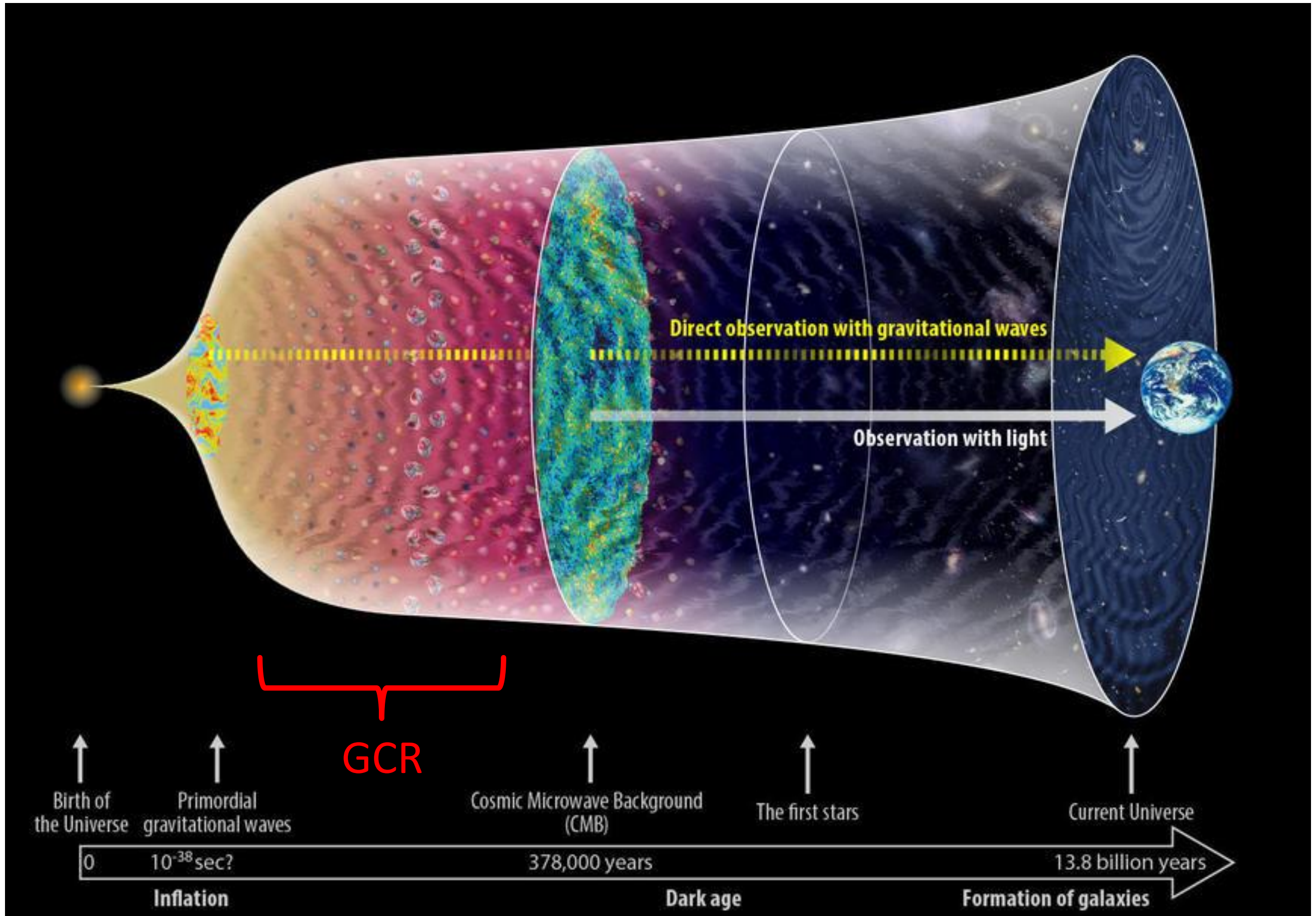
Can we use this bound to place a constraint on the speed of gravitons in the early universe?



EM Cherenkov radiation in a nuclear reactor (source: Wikipedia)

GRAVITATIONAL WAVE BACKGROUND

- Most models predict a GW background of the order $h_0^2\Omega_{GWB} \sim 10^{-10}$ for $T_{max} \sim 10^{15}$ GeV
- Can be enhanced if we lower the speed of GWs
- We assume a distribution of hot scalar particles present after inflation

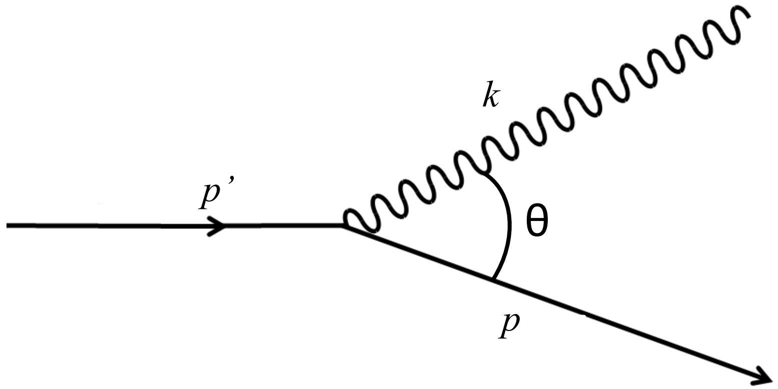


Source: NAOJ

GRAVITATIONAL CHERENKOV INTERACTION

- Gravitational Cherenkov radiation occurs when a particle travels faster than the phase velocity of gravity and emits a graviton
- Analogous to EM Cherenkov radiation
- Interaction $X \to X + h$, where X is a scalar particle traveling faster than c_T
- Matrix element squared:

$$|\mathcal{M}|^2 = 16\pi G \left(p^2 - \frac{(\vec{p} \cdot \vec{k})^2}{k^2} \right)^2 = 16\pi G p^4 \sin^4 \theta$$



MINIMAL SCENARIO

- To compute the graviton background, we use the Boltzmann equation

$$\frac{\mathrm{d}f_h(t,k)}{\mathrm{d}t} = \Gamma\left(f_\phi - f_h\right)$$

- Keeping only leading-order terms:

$$\frac{\mathrm{d}\delta f_h(t,k)}{\mathrm{d}t} \simeq \Gamma f_\phi = \frac{1}{4\omega}\int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E}\frac{2\pi}{2E'}\delta(E'-E-\omega)n_B(t,\vec{p}+\vec{k})(1+n_B(t,\vec{p}))|\mathcal{M}|^2$$

- We adopt the relativistic limit where $p \gg m$ such that $E \cong p$, insert the dispersion relation $\omega = c_T k$, and take the integration range $p_{min} = k(1+c_T)/2$, $p_{max} \to \infty$. Then,

$$\frac{d\delta f_h(t,k)}{dt} \simeq \Psi(x,y)$$

where $x = c_T k/T$ and $y = p_{\min}/T$

MINIMAL SCENARIO

- For a flat background, the differential graviton energy density is

$$d\rho_g(t,k) = 2\omega f_h(t,k)\frac{d^3k}{(2\pi)^3}.$$

- We rewrite this as an expression for an expanding universe, then covert time dependence to temperature dependence and redshift all quantities to today, so the fractional energy density is

$$h_0^2\Omega_g(k) \simeq \frac{15\sqrt{45}}{4\pi^{11/2}} M_{\rm pl} g_{\star,s}(T_{\rm today})^{1/3} h_0^2\Omega_\gamma \left(\frac{k}{T_{\rm today}}\right)^3 \times \int_{T_{\rm min}}^{T_{\rm max}} \frac{{\rm d}T}{T^4} \frac{g_{\star,c}(T)}{g_{\star\rho}(T)^{1/2} g_{\star s}(T)^{4/3}} R\left(T, \frac{kT}{T_{\rm today}}\left(\frac{g_{\star s}(T)}{g_{\star s}(T_{\rm today})}\right)^{1/3}\right)$$

where $R(t,k) \equiv 2\omega\delta\dot{f}_h(t,k)$

MINIMAL SCENARIO

- Noticing that the variables *x* and *y* are not time-dependent and assuming the effective relativistic degrees of freedom are constant in the integration range, $R \propto T^4$ and we get

$$h_0^2\Omega_g(k) \simeq \frac{15\sqrt{45}}{4\pi^{11/2}} \frac{g_{\star s}(T_{\rm today})^{4/3}}{g_{\star s}(T_{\rm max})^{11/6}} h_0^2\Omega_\gamma \frac{T_{\rm max}}{M_{\rm pl}} \psi(x_0, y_0)$$

with

$$\begin{aligned}\psi(x_0,y_0) \simeq &\frac{(1-c_T^2)^2 x_0^2}{16 c_T^6}\Big[(1-c_T^2)^2 x_0^4\left(y_0+x_0-\log(-1+e^{x_0+y_0})\right)-8c_T^2 y_0(y_0+x_0)\\ &\times(2c_T^2y_0^2+2c_T^2y_0x_0+(-1+c_T^2)x_0^2)\log(1-e^{-x_0-y_0})+8c_T^2\Big((2y_0+x_0)\\ &\times(2c_Ty_0+(-1+c_T)x_0)(2c_Ty_0+(1+c_T)x_0)\mathrm{Plog}(2,e^{-x_0-y_0})\\ &+2\Big((-x_0^2+3c_T^2(2y_0+x_0)^2)\mathrm{Plog}(3,e^{-x_0-y_0})+12c_T^2(2y_0+x_0)\mathrm{Plog}(4,e^{-x_0-y_0})\\ &+24c_T^2\mathrm{Plog}(5,e^{-x_0-y_0})\Big)\Big)\Big]\end{aligned}$$

$$x_0 = \frac{c_T k}{T_{\rm today}}\left(\frac{g_{\star s}(T_{\rm max})}{g_{\star s}}\right)^{1/3}, \qquad y_0 = \frac{(1+c_T)}{2c_T}x_0$$

MINIMAL SCENARIO

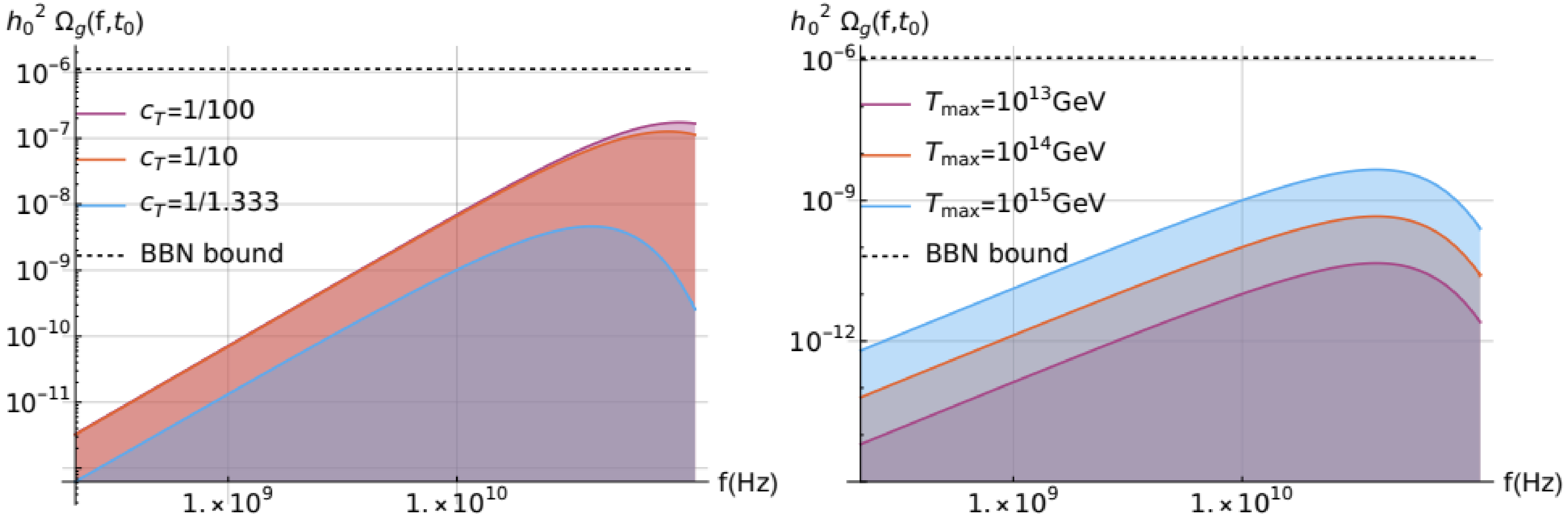


FIG. 1: Graviton background for the minimal scenario, i.e. assuming a gravitational wave speed modification only. In these plots, $m = 0.1\text{GeV}$, $g_{\star s}(T_{\text{today}}) = 3.931$ and $g_{\star s}(T_{\max}) = 1$. In the first plot $T_{\max} = 10^{15}\text{GeV}$, and in the second $c_T = 1/1.333$. The BBN bound is plotted for comparison.

HORNDESKI THEORY

- We now consider Horndeski gravity,

$$\mathcal{L} = \sqrt{-g}\Big[G_2 - G_3\Box\phi + G_4 R + G_{4,X}\left((\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right)$$
$$+G_5 G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}\left((\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right)\Big],$$

where the free functions G_i depend on ϕ and $X = -1/2\partial_\mu\phi\partial^\mu\phi$.

- In unitary gauge, $\delta\phi = 0$ and

$$\mathrm{d}s^2 = -(1+\delta N)^2\mathrm{d}t^2 + a^2 e^{2\xi}(e^\gamma)_{ij}(N^i\mathrm{d}t + \mathrm{d}x^i)(N^j\mathrm{d}t + \mathrm{d}x^j)$$

where $N_i = \partial_i\beta$ and

$$\beta = \frac{1}{a\mathcal{G}_t}\left(a^3\mathcal{G}_s\Delta^{-1}\dot{\zeta} - \frac{a\mathcal{G}_t^2}{\Theta}\zeta\right)$$

HORNDESKI THEORY

- We switch to Newtonian gauge

$$h_{ij} \to h_{ij} - a^{-2}\left(\partial_i\beta\partial_j\beta\right)^{TT}$$
$$\zeta \to \zeta + \frac{1}{4}\Delta^{-1}\left(\dot{h}_{ij}\partial_i\partial_j\beta\right) + \Delta^{-1}\left(h_{ij}\partial_i\partial_j\left(\zeta + \frac{\Theta}{\mathcal{G}_T}\beta\right)\right)$$

And use the relation $\beta = \delta\phi/\dot{\phi} \equiv \pi$

- Then the scalar second order action is

$$S_s^{(2)} \simeq \int \mathrm{d}^4x\, a^3 \frac{\Theta^2\mathcal{G}_s}{\mathcal{G}_T^2}\left(\dot{\pi}^2 - a^{-2}c_s^2(\partial_k\pi)^2\right) \simeq \int \mathrm{d}^4x\, \frac{a^3}{2}\left((\dot{\pi}^{(c)})^2 - a^{-2}c_s^2(\partial_k\pi^{(c)})^2\right)$$

where

$$\pi^{(c)} = \sqrt{\frac{2\mathcal{G}_s\Theta^2}{\mathcal{G}_T^2}}\,\pi \qquad\qquad \zeta \simeq -\frac{\Theta}{\mathcal{G}_T}\pi - \frac{\mathcal{G}_s\Theta^2}{\mathcal{G}_T^3}\frac{a^2}{\Delta}\dot{\pi}$$

HORNDESKI THEORY

- At leading order, the cubic interaction term is now

$$\mathcal{L}_{ssh} \simeq \frac{\mathcal{G}_T^2}{2\Theta}\left(1-\frac{\Gamma}{\mathcal{G}_T}\right)\dot{\gamma}_{ij}\partial_i\pi\partial_j\dot{\zeta} + \frac{\mathcal{G}_T}{4}(1-c_T^2)\Delta(\partial_i\pi\partial_j\pi) + \mu\left(-\gamma_{ij}\Delta(\partial_i\dot{\pi}\partial_j\pi) - \frac{\mathcal{G}_T}{\Theta}\gamma_{ij}\Delta(\partial_i\dot{\zeta}\partial_j\pi)\right)$$
$$\simeq -\frac{\mathcal{G}_T}{4}\left(1-\frac{\Gamma}{\mathcal{G}_T}\right)\gamma_{ij}\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\partial_i\pi\partial_j\pi) + \frac{\mathcal{G}_T}{4}(1-c_T^2)\gamma_{ij}\Delta(\partial_i\pi\partial_j\pi) + \frac{\mu\mathcal{G}_T}{\Theta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Theta}{\mathcal{G}_T}\right)\gamma_{ij}\Delta(\partial_i\pi\partial_j\pi)$$
$$+\frac{\mathcal{G}_s\Theta}{\mathcal{G}_T^2}\gamma_{ij}\Delta\left(\frac{\partial_i\ddot{\pi}}{\Delta}\partial_j\pi\right).$$

- Using the linear equations of motion and dispersion relation in the $k \gg H$ limit, we can further simplify all the terms into one interaction:

$$\mathcal{L}_{ssh} \simeq \left[-\frac{\mathcal{G}_T c_T^2}{4}\left(1-\frac{\Gamma}{\mathcal{G}_T}\right) + \frac{\mathcal{G}_T}{4}(1-c_T^2) + \frac{\mu\mathcal{G}_T}{\Theta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Theta}{\mathcal{G}_T}\right) + \frac{\mathcal{G}_s c_s^2\Theta}{\mathcal{G}_T^2}\right]\frac{\mathcal{G}_T^{3/2}}{\mathcal{G}_s\Theta^2 c_T^2}\ddot{\gamma}_{ij}^{(c)}\partial_i\pi^{(c)}\partial_j\pi^{(c)}$$

- And the corresponding interaction amplitude is

$$i\mathcal{M} = \frac{2i}{\Lambda_\star^3}k^2 p_m p'_n \epsilon^\sigma_{mn}(\vec{k}),$$

where

$$\Lambda_\star^3 \equiv \frac{\mathcal{G}_s\Theta^2}{\mathcal{G}_T^{3/2}}\left[-\frac{\mathcal{G}_T c_T^2}{4}\left(1-\frac{\Gamma}{\mathcal{G}_T}\right) + \frac{\mathcal{G}_T}{4}(1-c_T^2) + \frac{\mu\mathcal{G}_T}{\Theta}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Theta}{\mathcal{G}_T}\right) + \frac{\mathcal{G}_s c_s^2\Theta}{\mathcal{G}_T^2}\right]^{-1}$$

HORNDESKI THEORY

- As before, we sum over all polarizations, which leads to

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\sum_\sigma |i\mathcal{M}|^2 = \frac{2}{\Lambda_\star^6}k^4\left(p^2 - \frac{(\vec{p}\cdot\vec{k})^2}{k^2}\right)^2$$

- Comparing with the minimal scenario:

$$\frac{\langle|\mathcal{M}|^2\rangle_{\mathrm{H}}}{\langle|\mathcal{M}|^2\rangle_{\mathrm{min}}} \simeq \frac{1}{8\pi G}\frac{k^4}{\Lambda_\star^6}$$

- For the relic graviton background:

$$\frac{\Omega_{\mathrm{g,H}}}{\Omega_{\mathrm{g,min}}} \simeq \frac{1}{5}\frac{T_{\mathrm{max}}^4 M_{\mathrm{pl}}^2}{8\pi\Lambda_\star^6}\tilde{x}_0^4,$$

HORNDESKI THEORY

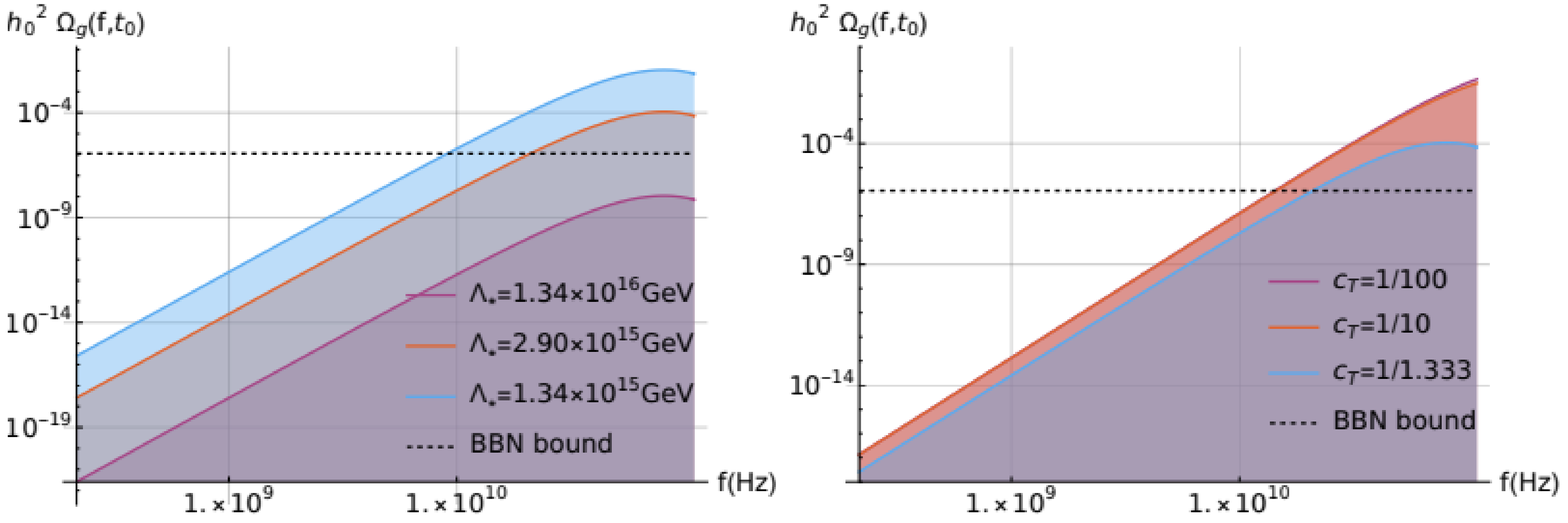


FIG. 2: Graviton background in a general Horndeski theory with $g_{\star s}(T_{\rm today}) = 3.931$, $g_{\star s}(T_{\rm max}) = 1$, $T_{\rm max} = 10^{15}$GeV, $m = 0.1$GeV. In the first plot we fix $c_T = 1/1.333$, while in the second $\Lambda_* = 2.90 \times 10^{15}$GeV.

HORNDESKI THEORY

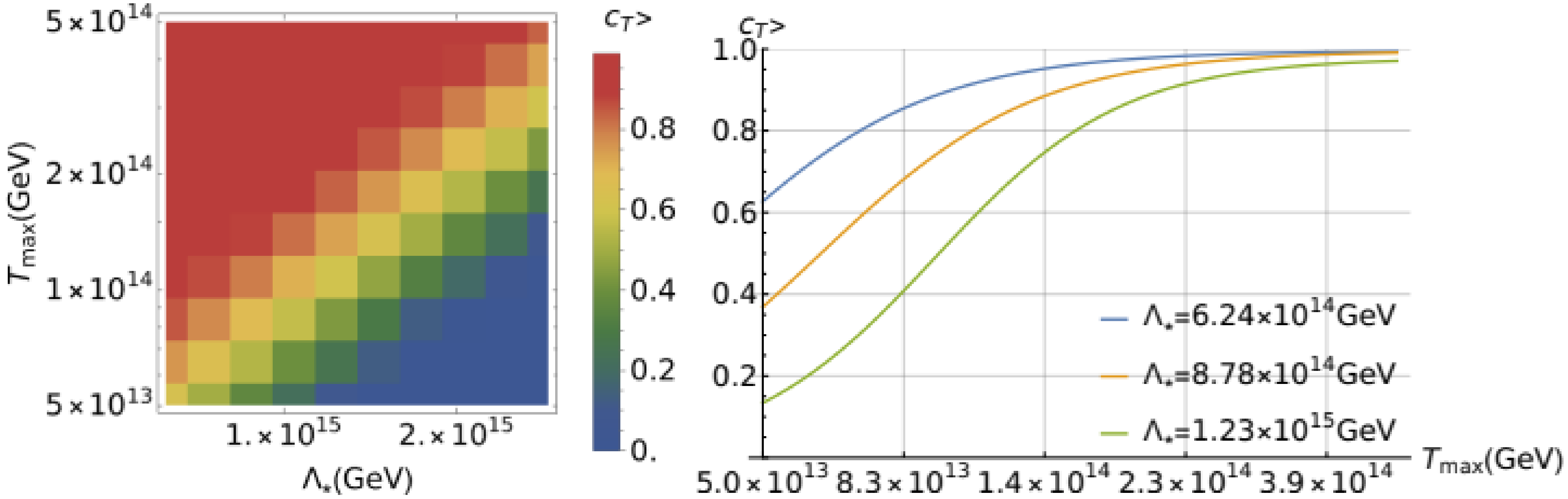


FIG. 3: Lower bound on the gravitational wave speed c_T within Horndeski theories as a function of $\Lambda_\star$ and $T_{\rm max}$. In these plots $g_{\star s}(T_{\rm today}) = 3.931$, $g_{\star s}(T_{\rm max}) = 1$, and $m = 0.1$GeV. The bound is set by requiring that the background contribution to the number of effective relativistic species $N_{\rm eff}$ is allowed by BBN. We consider 10 values of $\Lambda_\star$ and 10 values of $T_{\rm max}$ in the first plot, which are then fitted for three values of $\Lambda_\star$ and depicted in the second plot.

GALILEON THEORY

- We now consider a weakly-broken Galilean symmetry. At leading order for the free functions, we have

$$\mathcal{L} = \sqrt{-g}\Big[X - V(\phi) - c_3 X\frac{\Box\phi}{\Lambda_3^3} + \frac{1}{2}M_{\rm pl}^2 R + \frac{c_4 X^2}{\Lambda_3^6}R + \frac{2c_4 X}{\Lambda_3^6}\left((\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right)$$
$$+ c_5\frac{X^2}{\Lambda_3^9}G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{c_5}{3}\frac{X}{\Lambda_3^9}\left((\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right)\Big],$$

- We consider $c_5 = 0$ and $c_3 \sim c_4 \sim O(1)$. The free functions simplify to

$$\mathcal{G}_T = M_{\rm pl}^2 - 6c_4\frac{X^2}{\Lambda_3^6}, \qquad \Theta = -c_3\dot{\phi}\frac{X}{\Lambda_3^3} + HM_{\rm pl}^2 - \frac{30c_4 X^2 H}{\Lambda_3^6}, \qquad \Gamma = M_{\rm pl}^2 - \frac{30c_4 X^2}{\Lambda_3^6},$$

$$c_T^2 = \frac{1 + \frac{2c_4 X^2}{\Lambda_3^6 M_{\rm pl}^2}}{1 - \frac{6c_4 X^2}{\Lambda_3^6 M_{\rm pl}^2}} \simeq 1 + 8c_4\frac{X^2}{\Lambda_2^8},$$

Where the expansion parameter for the EFT is X/Λ_2^4, while $M_{pl}^2 = \Lambda_2^8/\Lambda_3^6$.

GALILEON THEORY

- The prefactor to the modified energy spectrum reads

$$-\left(1-\frac{\Gamma}{\mathcal{G}_T}\right)c_T^2+(1-c_T^2)=8c_4\frac{X^2}{\Lambda_2^8}\frac{-32\frac{X^2}{\Lambda_2^8}}{\left(1-6c_4\frac{X^2}{\Lambda_2^8}\right)^3}\simeq-32c_4\frac{X^2}{\Lambda_2^8}$$

- By using $\mathcal{G}_S \simeq X/H^2 + \mathcal{O}(X/\Lambda_2^4)$ we get

$$\Lambda_\star^3 \simeq 2\frac{X}{\Lambda_2^4}\Lambda_3^3$$

- And the graviton background can be expressed as

$$\frac{\Omega_{\mathrm{g,G}}}{\Omega_{\mathrm{g,min}}} \simeq \frac{32c_4^2}{5\pi}\frac{T_{\mathrm{max}}^4 M_{\mathrm{pl}}^4}{\Lambda_2^8}\frac{X^2}{\Lambda_2^8}\tilde{x}_0^4$$

GALILEON THEORY

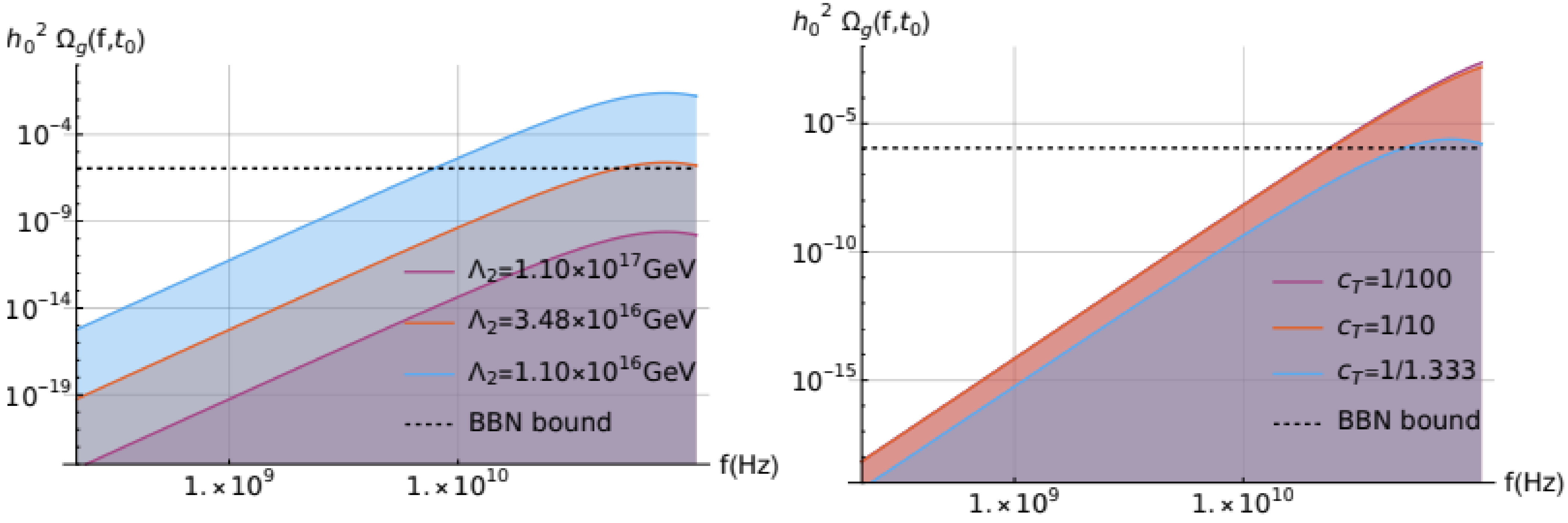


FIG. 4: Graviton background in Galileon theory with $T_{\max} = 10^{15}$GeV, $g_{\star s}(T_{\text{today}}) = 3.931$, $g_{\star s}(T_{\max}) = 1$, $m = 0.1$GeV and $c_4 = -1$. In the first plot we fix $c_T = 1/1.333$, while in the second $\Lambda_2 = 3.48 \times 10^{16}$GeV.

GALILEON THEORY

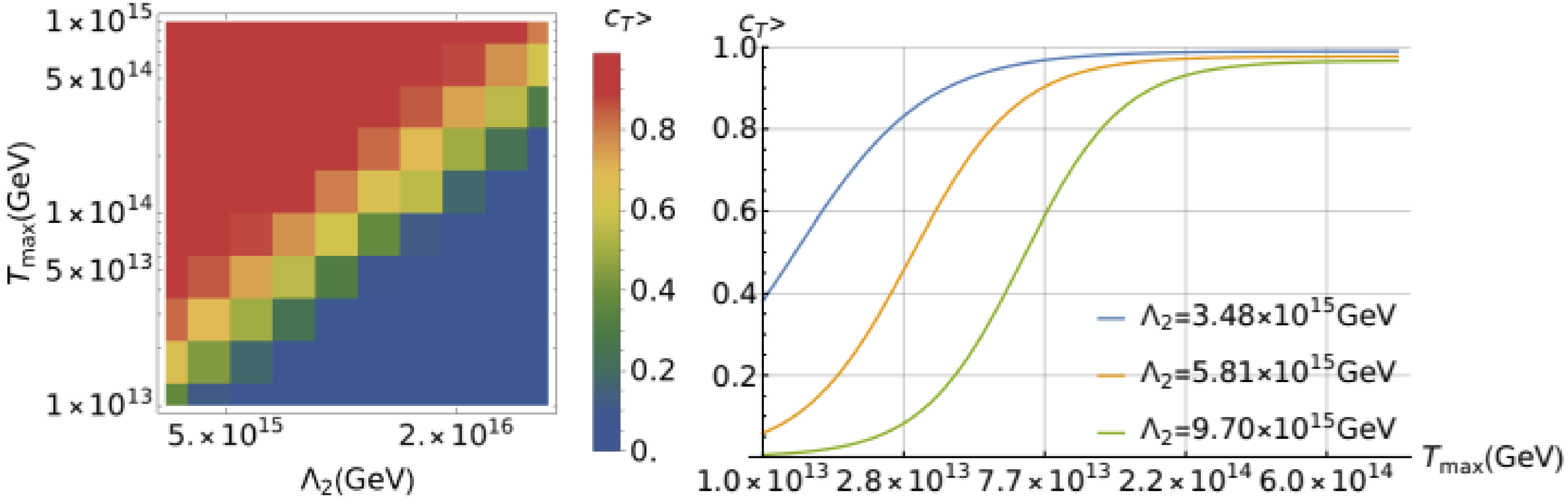


FIG. 5: Lower bound on the gravitational wave speed c_T within Galileon thoery as a function of Λ_2 and $T_{\rm max}$. In these plots $g_{\star s}(T_{\rm today}) = 3.931$, $g_{\star s}(T_{\rm max}) = 1$, $m = 0.1$GeV, and $c_4 = -1$. The bound is set by requiring that the background contribution to the number of effective relativistic species $N_{\rm eff}$ is allowed by BBN. We consider 10 values of Λ_2 versus 10 values of $T_{\rm max}$ in the first plot, which are then fitted for three values of Λ_2 and depicted in the second plot.

DISCUSSION AND CONCLUSIONS

- Violation of the BBN bound is possible in the Horndeski and Galileon cases, but not in the minimal case.
- Optimistic constraint: $1 - c_T < 1.8 \times 10^{-5}$ for $T_{max} = \Lambda_2 = 10^{15}$ GeV
- Graviton production occurs after inflation
- Important step in testing gravitational wave speed and establishing bounds on c_T in Horndeski theories
- Relic graviton background can exceed the BBN bound by a few orders of magnitude
- Future path: generalize this approach to any non-minimally coupled degree of freedom which can generate Cherenkov radiation