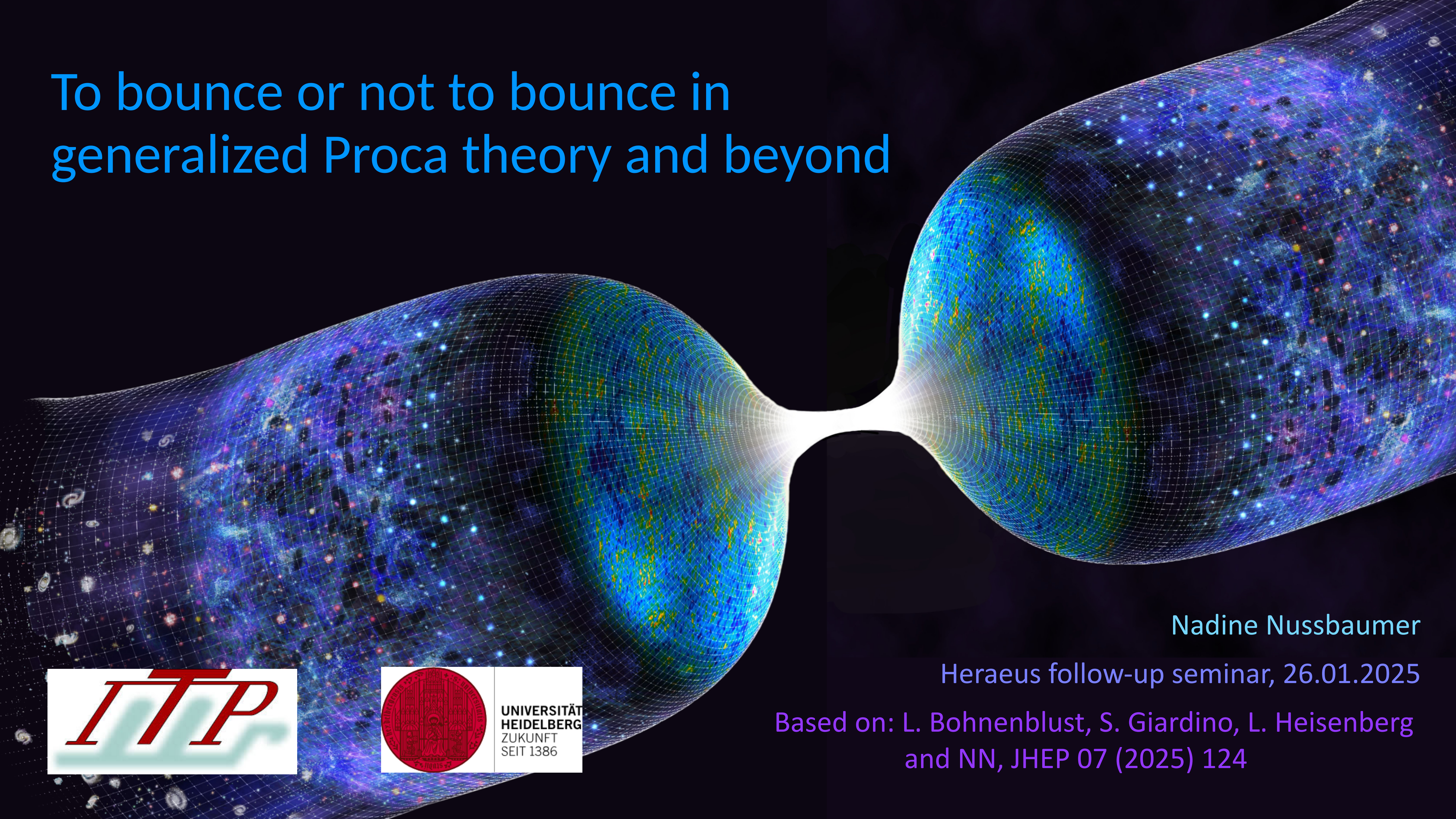
To bounce or not to bounce in generalized Proca theory and beyond



Nadine Nussbaumer

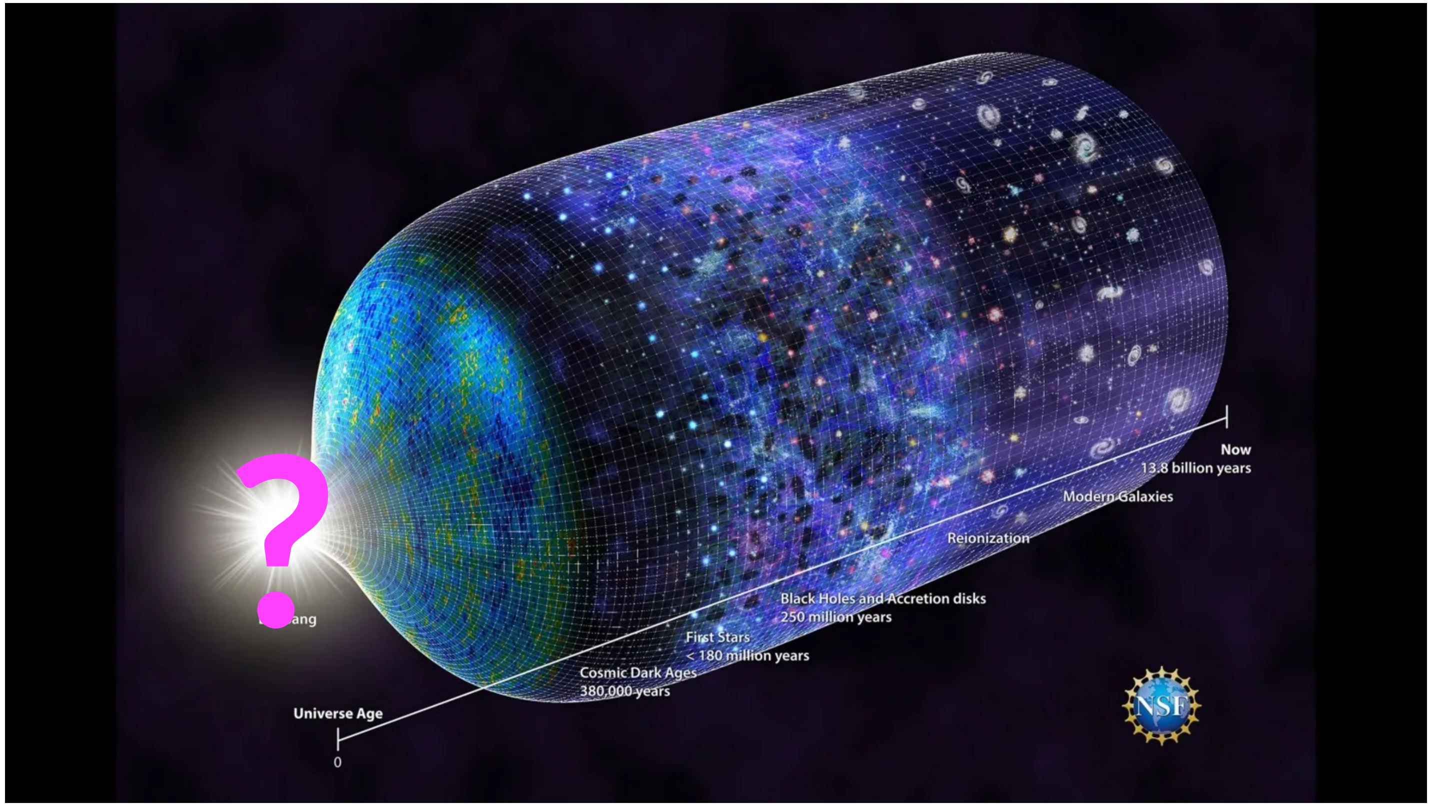
Heraeus follow-up seminar, 26.01.2025

Based on: L. Bohnenblust, S. Giardino, L. Heisenberg and NN, JHEP 07 (2025) 124

ITP

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Why bounces?



National Science Foundation

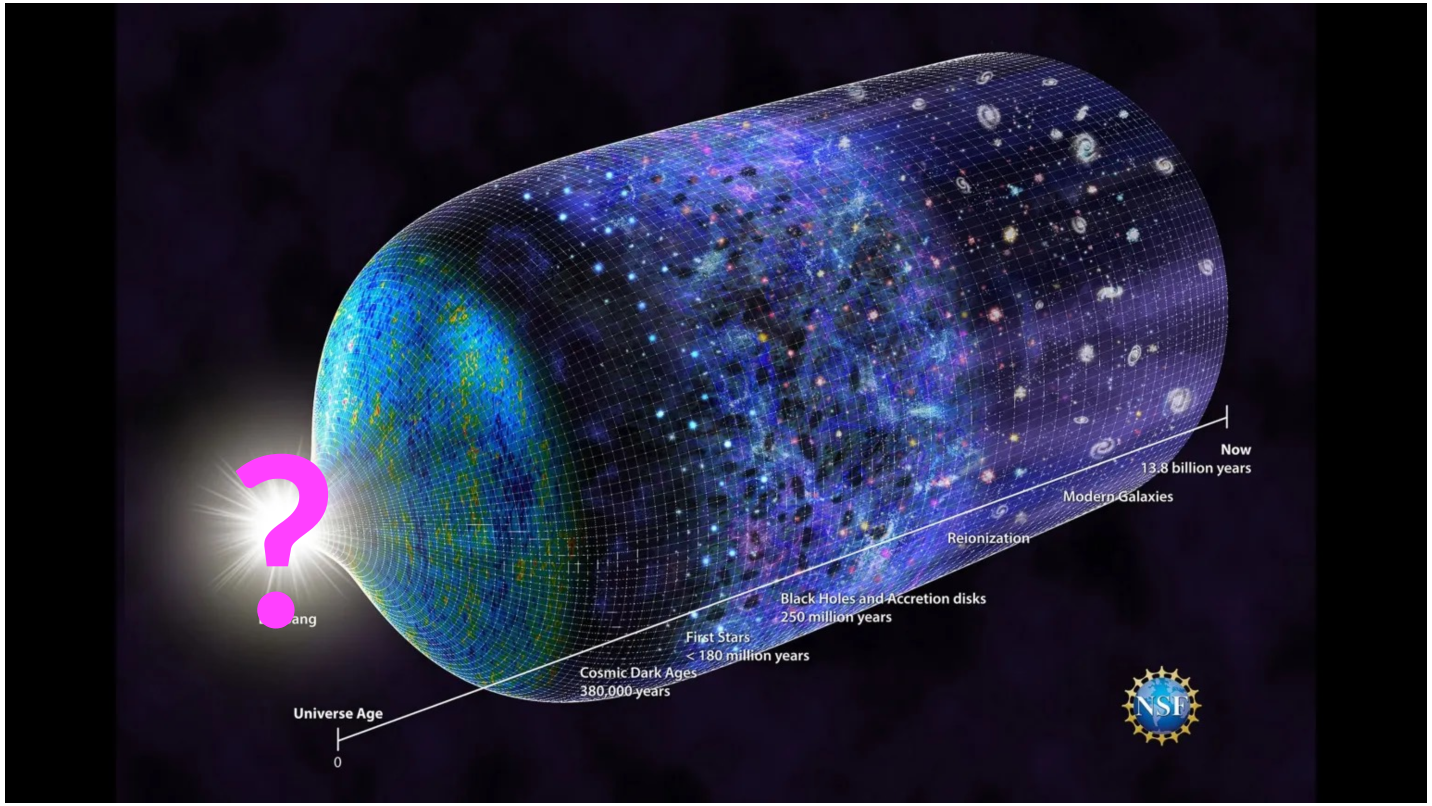
Why bounces?

- Alternative to inflation
- Inflation has a **singularity** and a **trans-Planckian** problem

Initial singularity $a(t_i) = 0$

If #e-foldings slightly higher than needed -> modes start in trans-Planckian regime

=> problems can be avoided by classical, non-singular bounces



D. Battefeld et al., 2015
R. Brandenberger et al., 2017

National Science Foundation

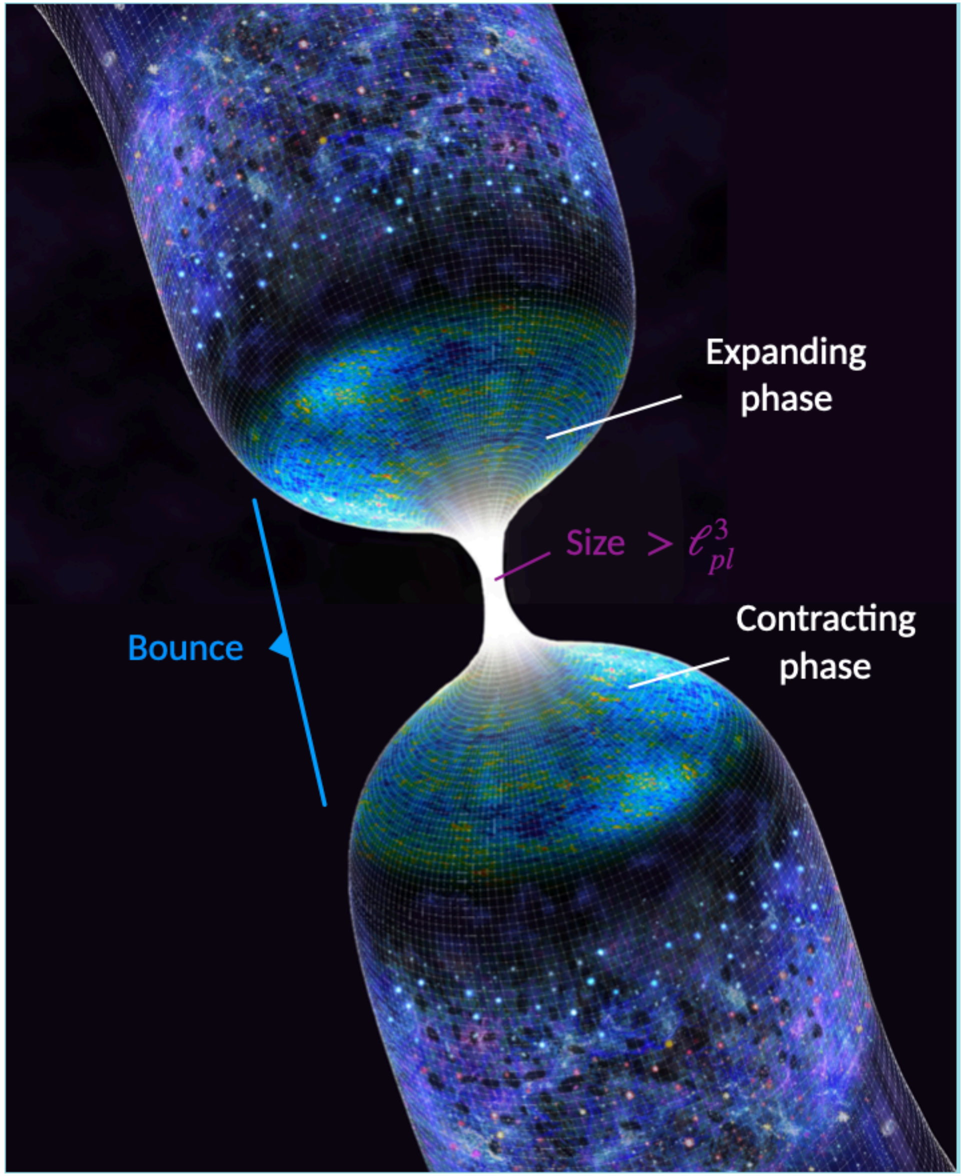
Why bounces?

- Classical, non-singular bounce:

Transition from contracting to expanding phase at finite size with energy density $\rho_B < M_{pl}^2$

-> $a(t_B) > 0$ and $H(t_B) = \frac{\dot{a}(t_B)}{a(t_B)} = 0$

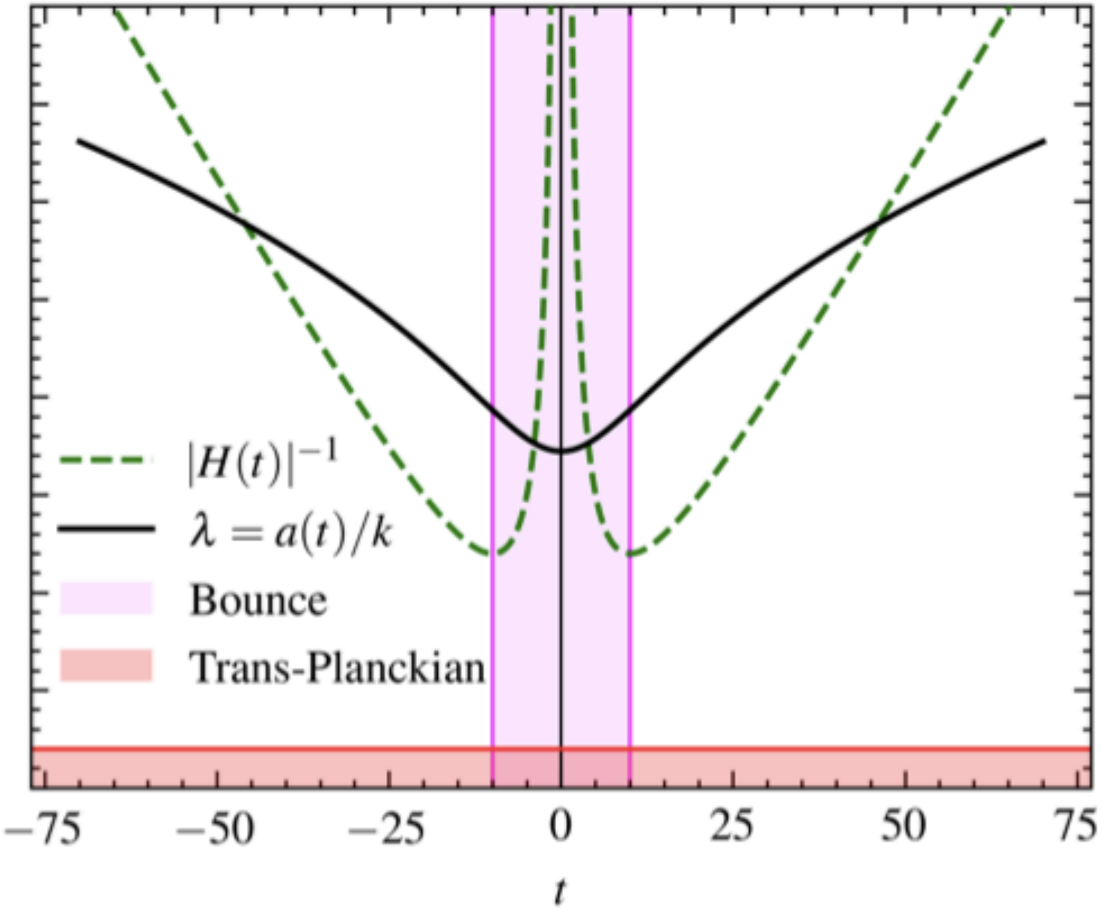
=> **initial singularity** avoided



Why bounces?

- Mechanism to seed primordial fluctuations
- Mode wavelength above Planck length
- Mode becomes superhorizon and reenters horizon later

=> **classical** description below Planck scale



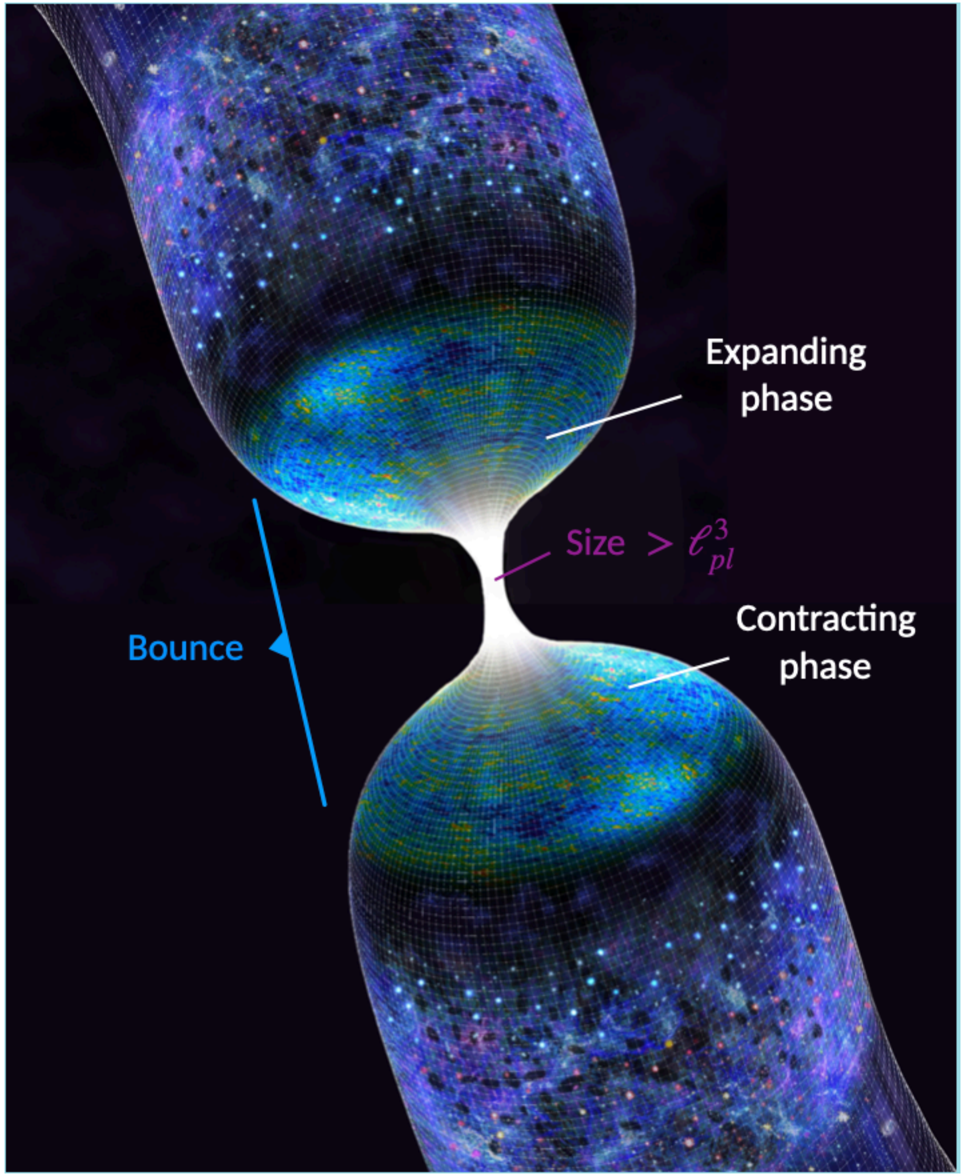
How to bounce

- Classical, non-singular bounce:

Transition from contracting to expanding phase at finite size with energy density $\rho_B < M_{pl}^2$

-> $a(t_B) > 0$ and $H(t_B) = \frac{\dot{a}(t_B)}{a(t_B)} = 0$

-> $\dot{H} > 0$ during the bounce to change from contraction to expansion



How to bounce

- Consider universe with:

(i) Flat FRW metric: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

(ii) Perfect fluid: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$

(iii) Continuity equation: $\dot{\rho} = -3H(\rho + p)$

(iv) **Null energy condition (NEC)**: En.-mom.-tensor obeys $\boxed{T_{\mu\nu}n^\mu n^\nu \geq 0}$ $\forall\, n^\mu$ light-like

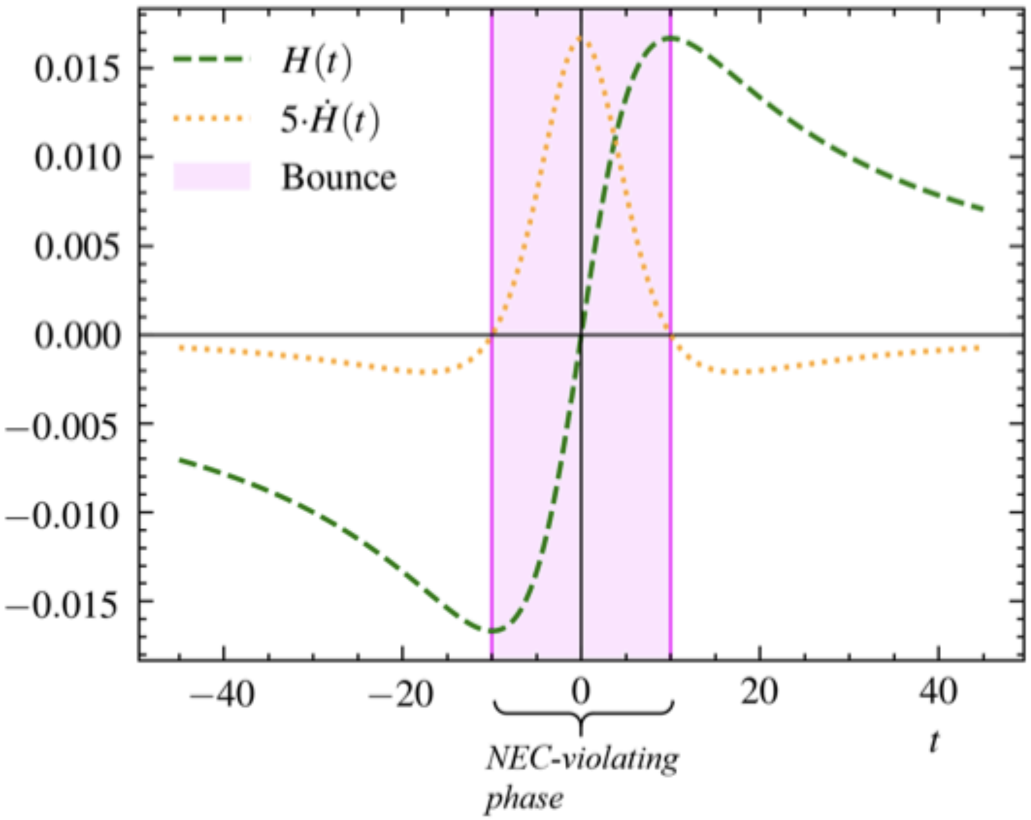
=> $\boxed{(\rho + p) \geq 0}$ for a specifically chosen n^μ

How to bounce

- Consider universe with GR:

-> $$\dot{H} = -\frac{1}{2M_{\text{pl}}}\underbrace{(\rho + p)}_{\geq 0 \text{ (NEC)}} < 0$$

- BUT: bouncing phase requires $\dot{H} > 0$



=> **no NEC-preserving bounce possible** due to BG EOM of GR!

-> **modify gravity** to allow for bouncing phase with $\dot{H} > 0$

How to bounce

- Theoretical consistency: modified gravity theory with new DOF π must be **stable** under small perturbations

$$\mathcal{L}^{(2)}_{\delta\pi} = q_\pi\left(\dot{\delta\pi}^2 - \frac{c_\pi^2}{a^2}(\partial_i\delta\pi)^2\right) - \frac{1}{2}m_\pi^2\delta\pi^2 \qquad \text{with} \qquad \pi = \bar{\pi} + \delta\pi$$

Ghost instability: $q_\pi < 0$ 👻

Stability:

$q_\pi > 0,\ c_\pi^2 \geq 0$

Laplacian/gradient instability: $c_\pi^2 < 0$

Dispersion: $\omega_\pi^2 = c_\pi^2\frac{k^2}{a^2} + m_\pi^2 \simeq c_\pi^2\frac{k^2}{a^2}$ (small scales $k \gg m$)

-> Exp. growing modes: $\delta\pi_k \sim e^{i\omega_\pi t} \sim e^{\pm c_\pi |k|/a}$

How to bounce: Horndeski?

- Pick your favourite modified gravity theory and try to bounce

G.W. Horndeski, 1974

- Scalar-tensor **Horndeski** theory (with $X \equiv -(\nabla\phi)^2/2$) -> maintains second-order EOM:

$$\mathscr{L}_2 = G_2(\phi, X)$$

$$\mathscr{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathscr{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathscr{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

How to bounce: Horndeski?

- Pick your favourite modified gravity theory and try to bounce

G.W. Horndeski, 1974

- Scalar-tensor **Horndeski** theory (with $X \equiv -(\nabla\phi)^2/2$) -> maintains second-order EOM:

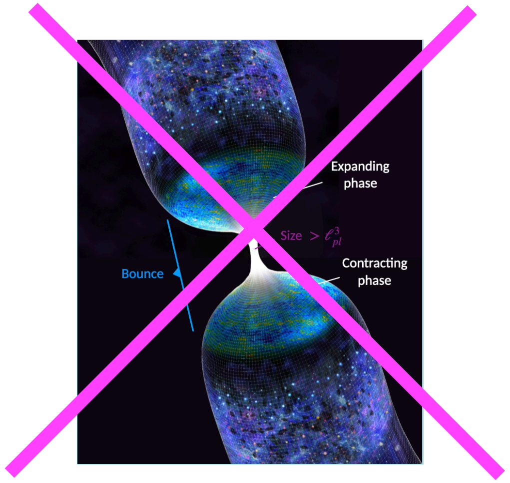
$$\mathcal{L}_2 = G_2(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

NO-GO



- SVT decomposition for perturbations -> no stable region found => **NO-GO theorem**
- Downside: result only valid in certain gauges! M. Libanov et al., 2016, and T. Kobayashi, 2016

How to bounce: generalized Proca?

L. Heisenberg, 2014

- Vector pendant to Horndeski -> **generalized Proca theory**
- Couples massive vector (Proca) field A^μ consistently to gravity, with

Second-order EOM

3 propagating vector DOF

A_0 non-dynamical

$$\mathscr{L}_2 = G_2(X, F_{\mu\nu})$$ (vector field norm: $X \equiv -A^\mu A_\mu/2$)

$$\mathscr{L}_3 = G_3(X)\nabla_\mu A^\mu$$

$$\mathscr{L}_4 = G_4(X)R + G_{4,X}\left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A^\nu \nabla_\nu A^\mu\right]$$

$$\mathscr{L}_5 = G_5(X)G^{\mu\nu}\nabla_\mu A_\nu - \frac{1}{6}G_{5,X}\left[(\nabla_\mu A^\mu)^3 + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3(\nabla_\mu A^\mu)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\right] - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta_\mu \nabla_\alpha A_\beta$$

$$\mathscr{L}_6 = G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu$$

How to bounce: generalized Proca?

- <u>Consider</u>: universe filled with matter modeled as perfect fluid ($\sim$ Schutz-Sorkin action) and gravity modified by generalized Proca
- Stability analysis: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad A_\mu = \bar{A}_\mu + \delta A_\mu$ (+ similarly for matter DOF)
- Background ansatz: $\bar{A}_\mu = (-\bar{A}_0(t), 0, 0, 0), \quad \bar{g}_{\mu\nu}$: flat FRW metric
- EOM for the background metric:

$$G_2 - G_{2,X}\bar{A}_0^2 + 3G_{3,X}H\bar{A}_0^3 + 6G_4H^2 - 6(2G_{4,X} + G_{4,XX}\bar{A}_0^2)H^2\bar{A}_0^2$$
$$-G_{5,XX}H^3\bar{A}_0^5 - 5G_{5,X}H^2\bar{A}_0^3 = \rho_M,$$

$$G_2 + \dot{\bar{A}}_0\bar{A}_0^2G_{3,X} + 2G_4(3H^2 + 2\dot{H}) - 2G_{4,X}\bar{A}_0(3H^2\bar{A}_0 + 2H\dot{\bar{A}}_0 + 2\dot{H}\bar{A}_0)$$
$$-4G_{4,XX}H\dot{\bar{A}}_0\bar{A}_0^3 - G_{5,XX}H^2\dot{\bar{A}}_0\bar{A}_0^4 - G_{5,X}H\bar{A}_0^2(2\dot{H}\bar{A}_0 + 2H^2\bar{A}_0 + 3H\dot{\bar{A}}_0) = -P_M.$$

How to bounce: generalized Proca?

- Consider: universe filled with matter modeled as perfect fluid ($\sim$ Schutz-Sorkin action) and gravity modified by generalized Proca
- Stability analysis: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad A_\mu = \bar{A}_\mu + \delta A_\mu$ (+ similarly for matter DOF)
- Background ansatz: $\bar{A}_\mu = (-\bar{A}_0(t), 0, 0, 0), \quad \bar{g}_{\mu\nu}$: flat FRW metric
- EOM for the background vector field:

$$\mathcal{E}_{\bar{A}_0} = \bar{A}_0[G_{2,X} - 3G_{3,X} H \bar{A}_0 + 6H^2(G_{4,X} + \bar{A}_0^2 G_{4,XX}) + \bar{A}_0 H^3(3G_{5,X} + \bar{A}_0^2 G_{5,XX})] = 0.$$

-> $\bar{A}_0$ does not carry its own dynamics, **fully determined by background evolution**: $\boxed{\bar{A}_0 = \bar{A}_0(H)}$

How to bounce: generalized Proca?

- At the level of perturbations -> do scalar-vector-tensor decomposition:

  - Metric:

$$ds^2 = -(1+2\alpha)dt^2 + 2(V_i + \partial_i\chi)dt dx^i + a(t)^2[(1+2\zeta)\delta_{ij} + 2\partial_i\partial_j E + h_{ij}]dx^i dx^j$$

  - Proca field:

$$A^0 = -\bar{A}_0(t) + \delta A$$

$$A^i = \frac{1}{a^2}\delta^{ij}(\partial_j \chi_V + \partial_j)$$

+ similarly for fluid quantities

Expand total action up to 2nd order in perturbations where S,V,T-modes decouple -> study them separately

How to bounce: generalized Proca?

- **Tensor modes:** Only contribution from the metric! L. Heisenberg et al., 2018

$$S_T^{(2)} = \int dt d^3x \sum_{i=1}^{2} q_T \left[\dot{h}_i^2 - \frac{c_T^2}{a^2}(\partial h_i)^2 \right]$$

- Stability coefficients:

$$q_T \equiv 2G_4 - 2\bar{A}_0^2 G_{4,X} - \bar{A}_0^3 H G_{5,X}$$

$$c_T^2 \equiv \frac{2G_4 - \bar{A}_0^2 \dot{\bar{A}}_0 G_{5,X}}{q_T}$$

How to bounce: generalized Proca?

- **Vector modes:** After integrating out non-dynamical and redefining DOF L. Heisenberg et al., 2018

$$S_V^{(2)} \simeq \int dt d^3x \frac{a^3}{2} q_V \left[\dot{Z}_i^2 - \frac{c_V^2}{a^2} (\partial_j Z_i)^2 \right]$$

(in the small-scale limit)

- Stability coefficients:

$$q_V \equiv G_{2,F} + 2\bar{A}_0^2 G_{2,Y} - 4\bar{A}_0 H g_5 + 2H^2(G_6 + \bar{A}_0^2 G_{6,X})$$

$$c_V^2 \equiv 1 + \frac{\bar{A}_0^2(2G_{4,X} + \bar{A}_0 H G_{5,X}^2}{2q_T q_V} + \frac{2[\dot{H}G_6 - \bar{A}_0^2 G_{2,Y} - (\bar{A}_0 H - \dot{A}_0)(\bar{A}_0 H G_{6,X} - g_5)]}{q_V}$$

How to bounce: generalized Proca?

- **Scalar modes:** $S_S^{(2)} = \int dt d^3x (\mathscr{L}^{\text{flat}} + \mathscr{L}_\zeta + \mathscr{L}_E)$ L. Heisenberg et al., 2018

w_i: collections of G_i

$$\mathcal{L}^{\text{flat}} = a^3 \Bigg[\left(\frac{w_2}{\bar{A}_0}(\partial_i \alpha) - w_1(\partial_i \delta A) \right) \frac{\partial^i \chi}{a^2} - w_3 \frac{(\partial \alpha)^2}{a^2} + w_4 \alpha^2 + w_3 \frac{(\partial_i \delta A)(\partial^i \alpha)}{a^2 \bar{A}_0} + w_3 \left(\partial_i \alpha - \frac{1}{2\bar{A}_0}(\partial_i \delta A) \right) \frac{\partial_i \dot{\psi}}{a^2 \bar{A}_0}$$
$$+ w_8 \frac{\delta A \alpha}{\bar{A}_0} + \left(w_6(\partial_i \alpha) + \frac{w_2 - \bar{A}_0 w_6}{2\bar{A}_0^2}(\partial_i \delta A) \right) \frac{\partial^i \psi}{a^2} - w_3 \frac{(\partial \delta A)^2}{4a^2 \bar{A}_0^2} + w_5 \frac{\delta A^2}{\bar{A}_0^2} - w_3 \frac{(\partial \dot{\psi})^2}{4a^2 \bar{A}_0^2} + w_7 \frac{(\partial \psi)^2}{2a^2} - \alpha \delta \rho_M$$
$$- (\rho_M + P_M) \frac{(\partial_i v)(\partial^i \chi)}{a^2} - v \delta \dot{\rho}_M - 3H(1 + c_M^2) v \delta \rho_M - \frac{1}{2}(\rho_M + P_M) \frac{(\partial v)^2}{a^2} - \frac{c_M^2}{2(\rho_M + P_M)} \delta \rho_M^2 \Bigg], \quad (42)$$

$$\mathcal{L}_\zeta = a^3 \Bigg[3 \left(\frac{w_2}{\bar{A}_0} \delta A - w_1 \alpha - (\rho_M + P_M) v \right) \dot{\zeta} - \frac{2}{a^2} \left[q_T (\partial_i \chi) + \alpha_3 (\partial_i \psi) \right] \partial^i \dot{\zeta} - 3 q_T \dot{\zeta}^2$$
$$+ \left[2(q_T - 2\bar{A}_0 \alpha_3) \partial_i \alpha + 2\alpha_3 (\partial_i \delta A) \right] \frac{\partial^i \zeta}{a^2} + \mathcal{F}_T \frac{(\partial \zeta)^2}{a^2} \Bigg], \quad (43)$$

$$\mathcal{L}_E = a^3 \Bigg[- 2 q_T (\partial_i \ddot{\zeta}) - 2 B_2 (\partial_i \dot{\zeta}) - w_1 (\partial_i \dot{\alpha}) - (\dot{w}_1 + 3H w_1) \partial_i \alpha + \frac{w_2}{\bar{A}_0} (\partial_i \dot{\delta A}) - B_5 (\partial_i \delta A)$$
$$- (\rho_M + P_M) \left[\partial_i \dot{v} - 3H c_M^2 (\partial_i v) \right] \Bigg] \partial^i E. \quad (44)$$

How to bounce: generalized Proca?

- **Scalar modes:** After redefining and integrating out non-dynamical DOF

$$S_S^{(2)} \simeq \int dt d^3x a^3 \left[q_S \left(\dot{\mathscr{R}}_k^2 - c_S^2 \frac{k^2}{a^2} \mathscr{R}_k^2 \right) + q_M \left(\delta\dot{\tilde{\rho}}_k^2 - c_M^2 \frac{k^2}{a^2} \delta\tilde{\rho}_k^2 \right) \right]$$

- Stability coefficients:

$$q_S \equiv \frac{q_T(3w_2^2 + 4q_T w_5)}{(w_1 - 2w_2)^2}, \qquad q_M \equiv \frac{a^2}{2(\rho + p)}$$

+ c_S^2 and c_M^2

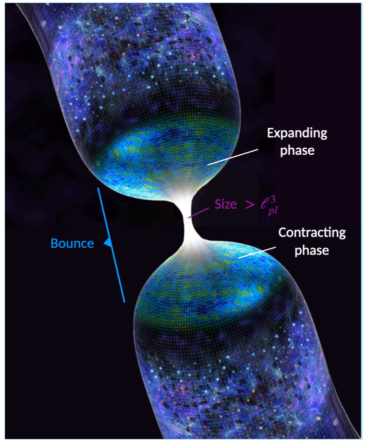
How to bounce: generalized Proca?

- **Stability analysis:** stable bounce requires over full cosmological time evolution $(-\infty < t < +\infty)$

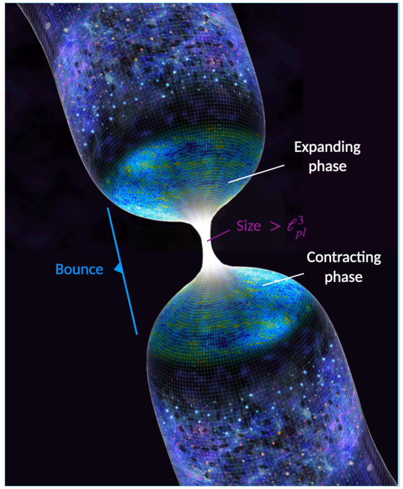
$$q_T > 0, \quad c_T^2 \geq 0, \quad q_V > 0, \quad c_V^2 \geq 0, \quad q_S > 0, \quad c_S^2 \geq 0, \quad q_M > 0, \quad c_M^2 \geq 0$$

-> gives constraints on coupling functions G_i and their derivatives

- Additional constraint for bouncing phase: $\dot{H} > 0$ (during some finite time interval)



How to bounce: generalized Proca?

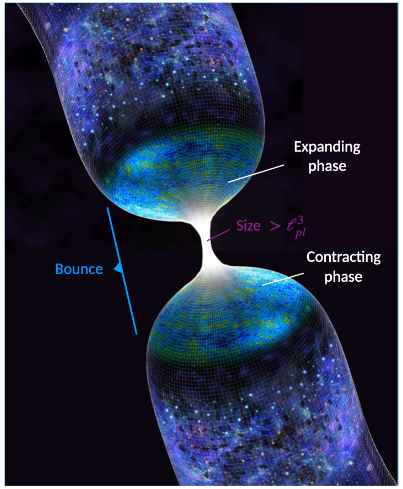


- Stability in the matter sector:

require $q_M = \dfrac{a^2}{2(\rho+p)} > 0$ -> $\rho + p \geq 0$

- Combining background EOM: $\rho + p = -\left(2q_T + \dfrac{3w_2^2}{2w_5}\right)\dot{H}$

How to bounce: generalized Proca?



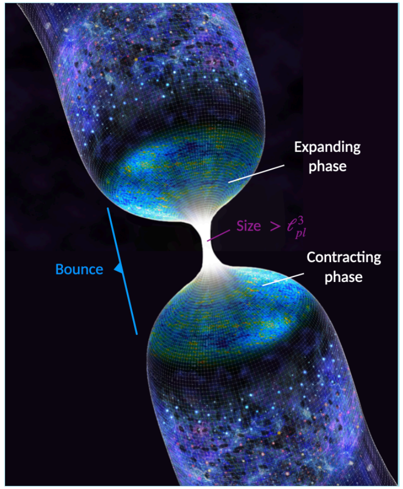
- Stability in the matter sector:

require $q_M = \dfrac{a^2}{2(\rho+p)} > 0$ -> $\rho + p \geq 0$

- Combining background EOM: $\rho + p = -\left(2q_T + \dfrac{3w_2^2}{2w_5}\right)\dot{H}$

(i) $\rho + p > 0:$ At the bounce $\dot{H}(t_B) > 0$ -> $0 < 2q_T(t_B) < -\dfrac{3w_2(t_B)^2}{2w_5(t_B)}$ but $w_2(t_B) = 0$

How to bounce: generalized Proca?



- Stability in the matter sector:

require $\quad q_M = \dfrac{a^2}{2(\rho + p)} > 0 \quad$ -> $\quad \rho + p \geq 0$

- Combining background EOM: $\quad \rho + p = -\left(2q_T + \dfrac{3w_2^2}{2w_5}\right)\dot{H}$

(i) $\rho + p > 0$: At the bounce $\dot{H}(t_B) > 0 \quad$ -> $\quad 0 < 2q_T(t_B) < -\dfrac{3w_2(t_B)^2}{2w_5(t_B)}$ but $w_2(t_B) = 0$

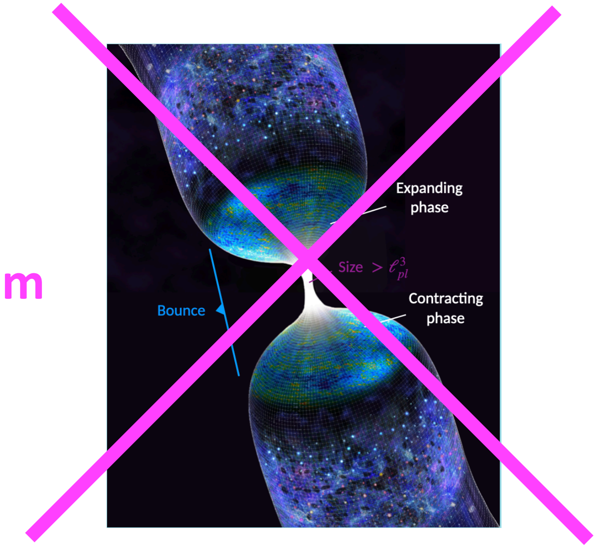
(ii) $\rho + p = 0$: scalar kinetic coefficient vanishes $\quad q_S = 3w_2^2 + 4w_5 q_T = 0$

How to bounce: generalized Proca?

- Stability in the matter sector:

require $q_M = \dfrac{a^2}{2(\rho + p)} > 0$ -> $\rho + p \geq 0$

NO-GO theorem



- Combining background EOM: $\rho + p = -\left(2q_T + \dfrac{3w_2^2}{2w_5}\right)\dot{H}$

(i) $\rho + p > 0$: At the bounce $\dot{H}(t_B) > 0$ -> $0 < 2q_T(t_B) < -\dfrac{3w_2(t_B)^2}{2w_5(t_B)}$ but $w_2(t_B) = 0$

(ii) $\rho + p = 0$: scalar kinetic coefficient vanishes $q_S = 3w_2^2 + 4w_5 q_T = 0$

=> either encounter **instabilities** in the **tensor** or **scalar** sector, **no stable bounce** possible!

Conclusion

- non-singular cosmological bounces provide an alternative to inflation that do not suffer from problems afflicting inflation (e.g. singularity and trans-Planckian problem)
- bounces not possible in pure GR -> modified gravity theories provide a way out
- scalar-tensor theories: Horndeski cannot bounce (gauge-dependent)
- **No-Go theorem** for **bounces mediated by generalized Proca fields**: Either run into instabilities in the matter, tensor or scalar sector already at the linear level of perturbations!
- No-go theorem gauge-independent -> theoretically robust