Studies of the absorption parameter 3γ/2γ in positronium decays

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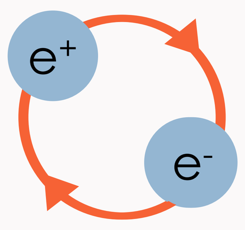
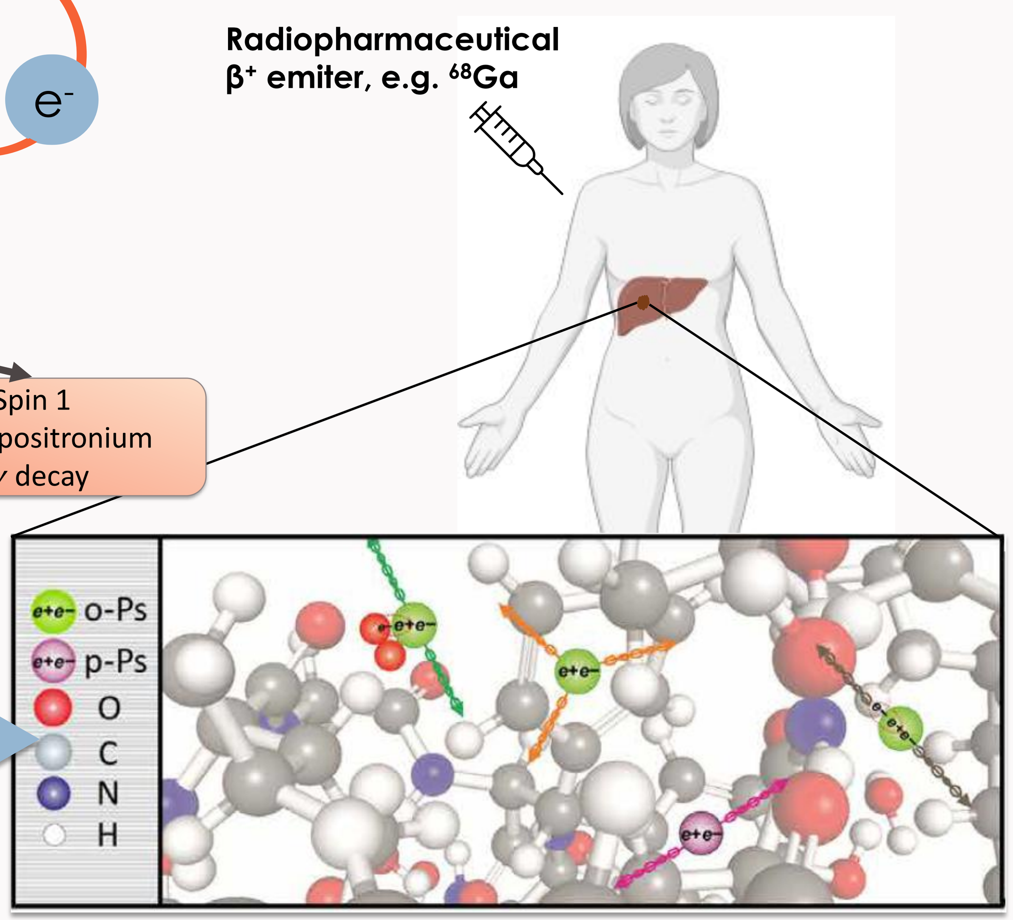
Introduction

During Positron Emission Tomography (PET), as much as 40% of annihilations occur through the formation of positronium inside the patient's body. Its properties, such as lifetime or the fraction of positronium annihilations into three photons (3γ/2γ), are highly influenced by the tissue's submolecular architecture. This has led to the development of a novel PET imaging technique - **positronium imaging** - which provides additional insights into the imaged tissue [1,2].

Positronium imaging



- Positron
  - Direct annihilation with an electron (60%) 2γ decay
  - Formation of positronium (40%)
    - Spin 0 Para-positronium 2γ decay
    - Spin 1 Ortho-positronium 3γ decay



Positronium in the tissue:
- o-Ps → 2γ spin-exchange
- o-Ps → 3γ self-annihilation
- p-Ps → 2γ self-annihilation
- o-Ps → 2γ pick-off

J-PET collaboration:
- Promising results reported by Moskal et al. in Refs. [1, 2] regarding **Positronium Lifetime Imaging**
- Development of a complementary positronium imaging method based on the **3γ/2γ annihilation ratio** → needed corrections for physical processes (e.g. **attenuation**)

Volume of free spaces ↑ Oxygenation ↓ → o-Ps lifetime ↑, 3γ/2γ ratio ↑

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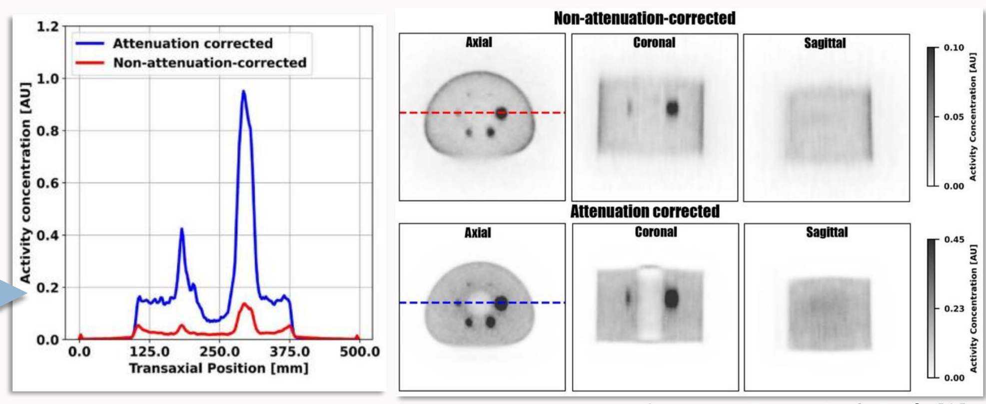
Positronium properties are considered **promising biomarkers** of tissue pathology and oxidation.

Absorption correction

Absorption of gamma quanta in tissues → Underestimation of radiopharmaceutical concentration in deeper tissues

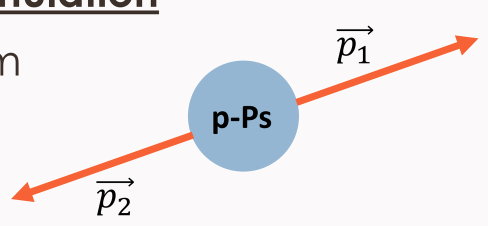
- J-PET: CT-based attenuation correction for two-gamma imaging [3]
- Standard attenuation maps – **not sufficient** for three-gamma imaging
- Absorption map → a basis for developing 3γ attenuation correction



Source: Das et al. Ref. [3]

Monte Carlo Simulation

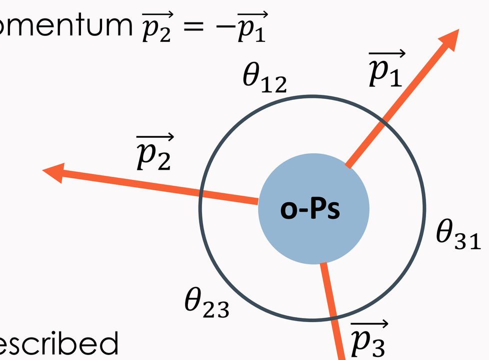
Para-positronium



Isotropic decay into 2 back-to-back photons with energies of 511 keV

Decay simulation:
1. Randomizing the direction of $\vec{p_1}$
2. Assigning the energy of 511 keV
3. Determining the momentum $\vec{p_2} = -\vec{p_1}$

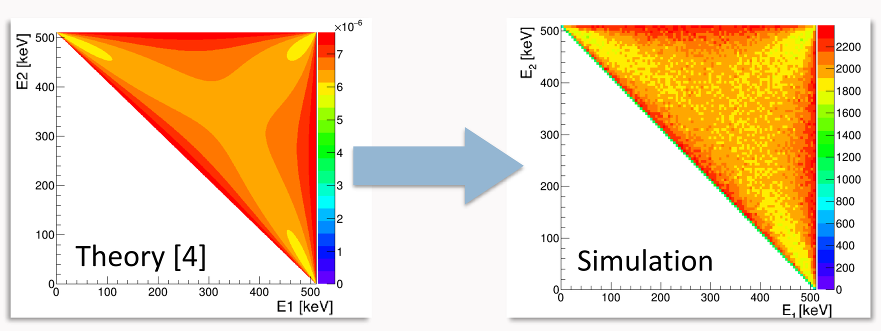
Ortho-positronium



Decay into 3 photons described with the matrix element $M_{oPs\to3\gamma}$:

$$M_{oPs\to3\gamma} = \left(\frac{m_e - E_1}{E_2E_3}\right)^2 + \left(\frac{m_e - E_2}{E_1E_3}\right)^2 + \left(\frac{m_e - E_3}{E_1E_2}\right)^2,$$

where m_e - electron mass, E_1, E_2, E_3 - gamma quanta energies



Absorption of gamma quanta in matter

When passing through a material, a photon can interact with its electrons, nuclei or atoms. The dominant processes when photons loose all or part of their energy are:
1. Photoelectric effect (photoeffect)
2. Compton scattering
3. Pair production (for gamma quanta with an energy exceeding $2m_e$)

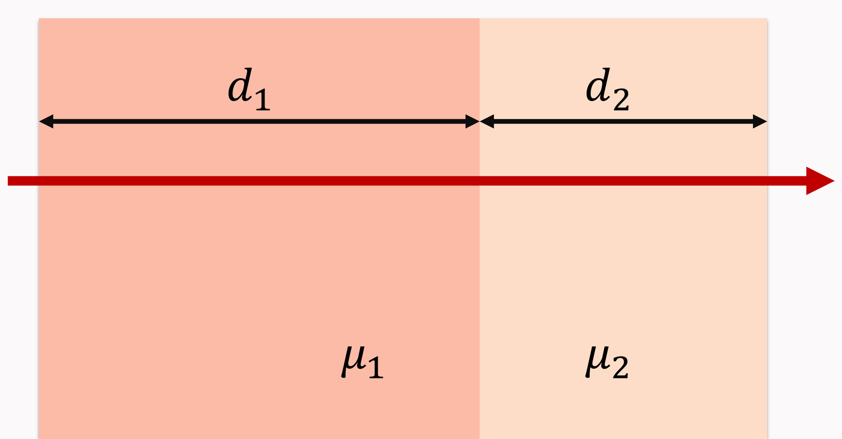
Probability that a photon with energy E_i is **not** absorbed:

$$P_i = \prod_m e^{-\mu_m(E_i)d_m}$$

distance; linear attenuation coefficient

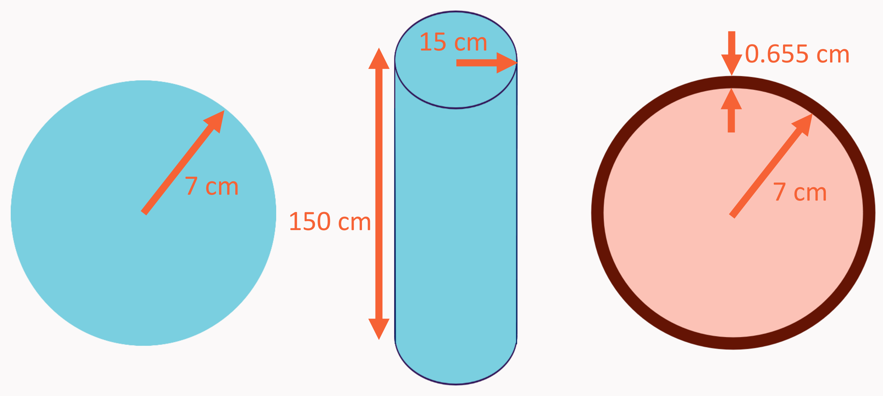
Total probability that **none** of the photons are absorbed:

$$P_{tot} = \prod_i P_i$$



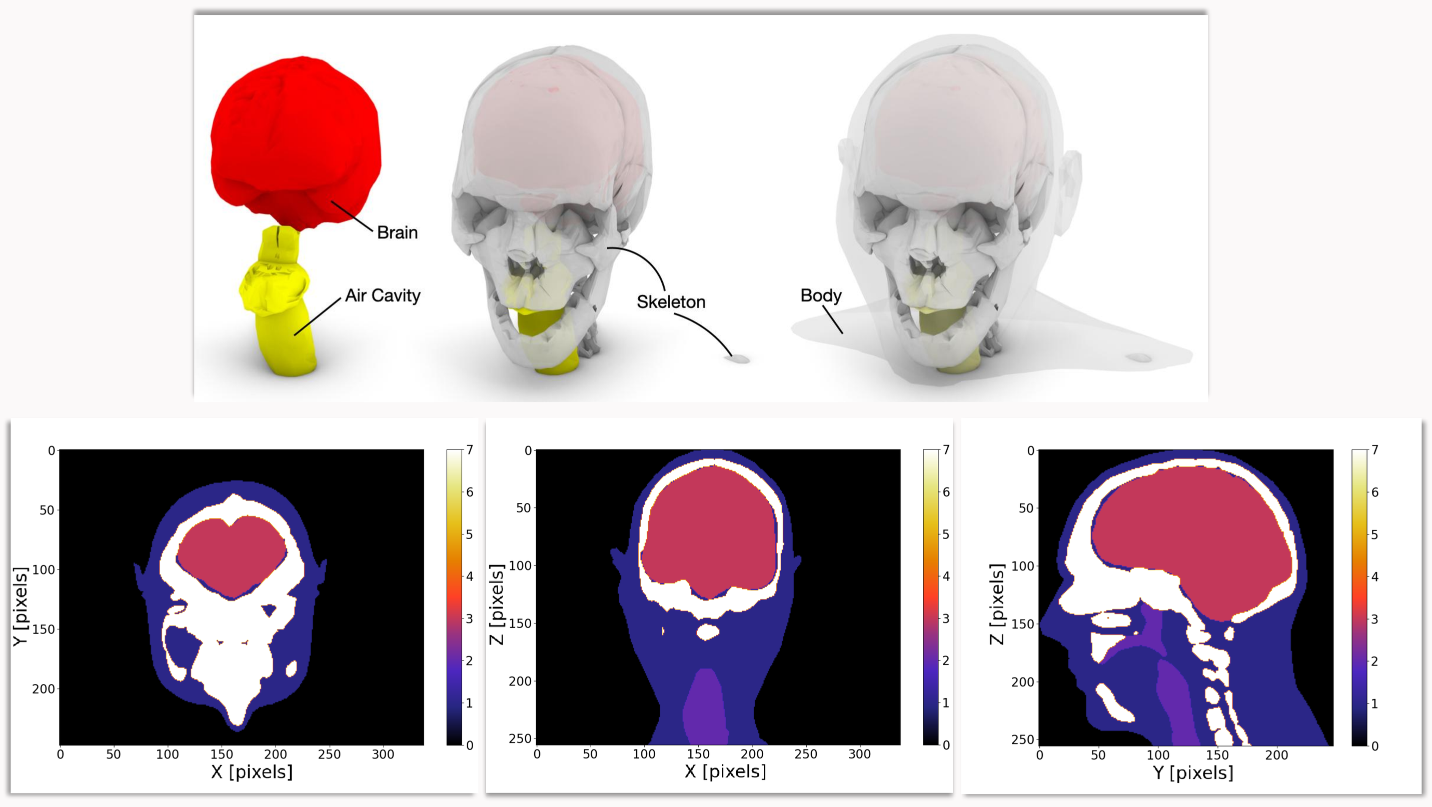
Phantom models – simplified models

Simplified models were implemented in a toy Monte Carlo simulation. The results were validated through comparison with analogous simulations performed using the **GATE** toolkit [5].



Phantom models – human body phantom

- Mesh-based **XCAT** phantom (mesh50_XCAT) developed and maintained by B. Auer [6, 7]
- Highly detailed male anatomy for subject in 50th percentile
- Source – voxelized version of the phantom
- Head Only model (computational limitations)
- Simulations performer using the **GATE** toolkit [5]



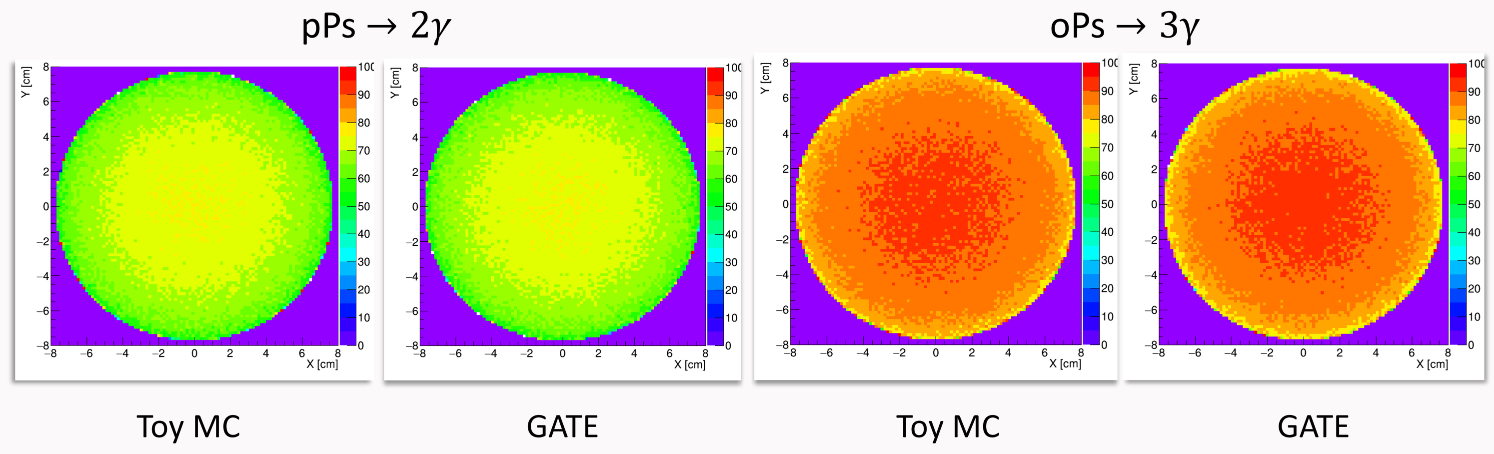
Results

Simplified head model

- Percentage of gamma quanta pairs or triplets, from which **at least one** of the photons **interacted** with the model
- Specific to the annihilation point
- 10^7 events

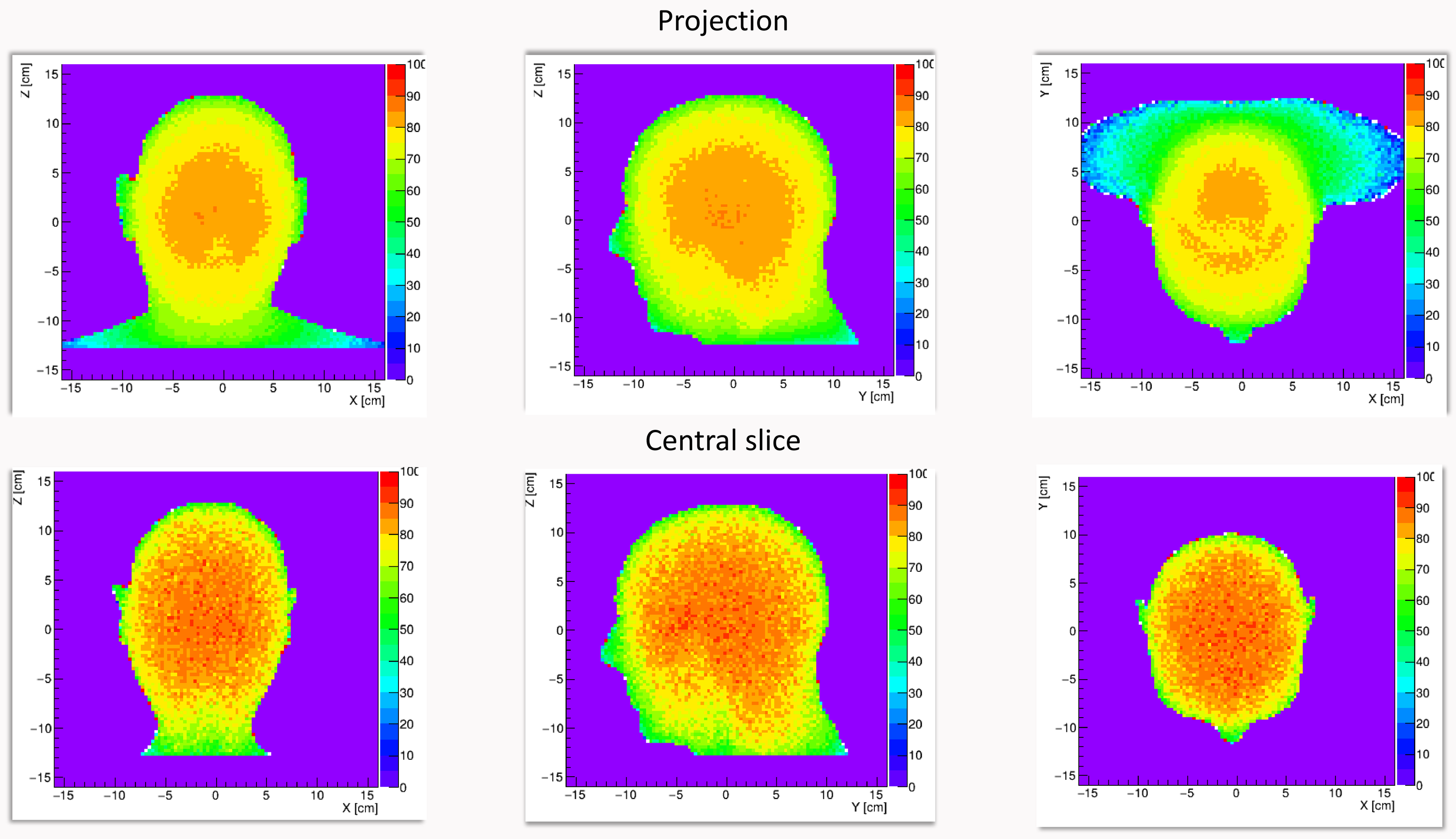
Surviving photon multiplets within the whole model

	Toy MC	GATE
p-Ps	30.156 ± 0.017%	30.052 ± 0.017%
o-Ps	11.879 ± 0.011%	11.919 ± 0.011%
$3\gamma/2\gamma$	0.39392 ± 0.00043	0.39392 ± 0.00043

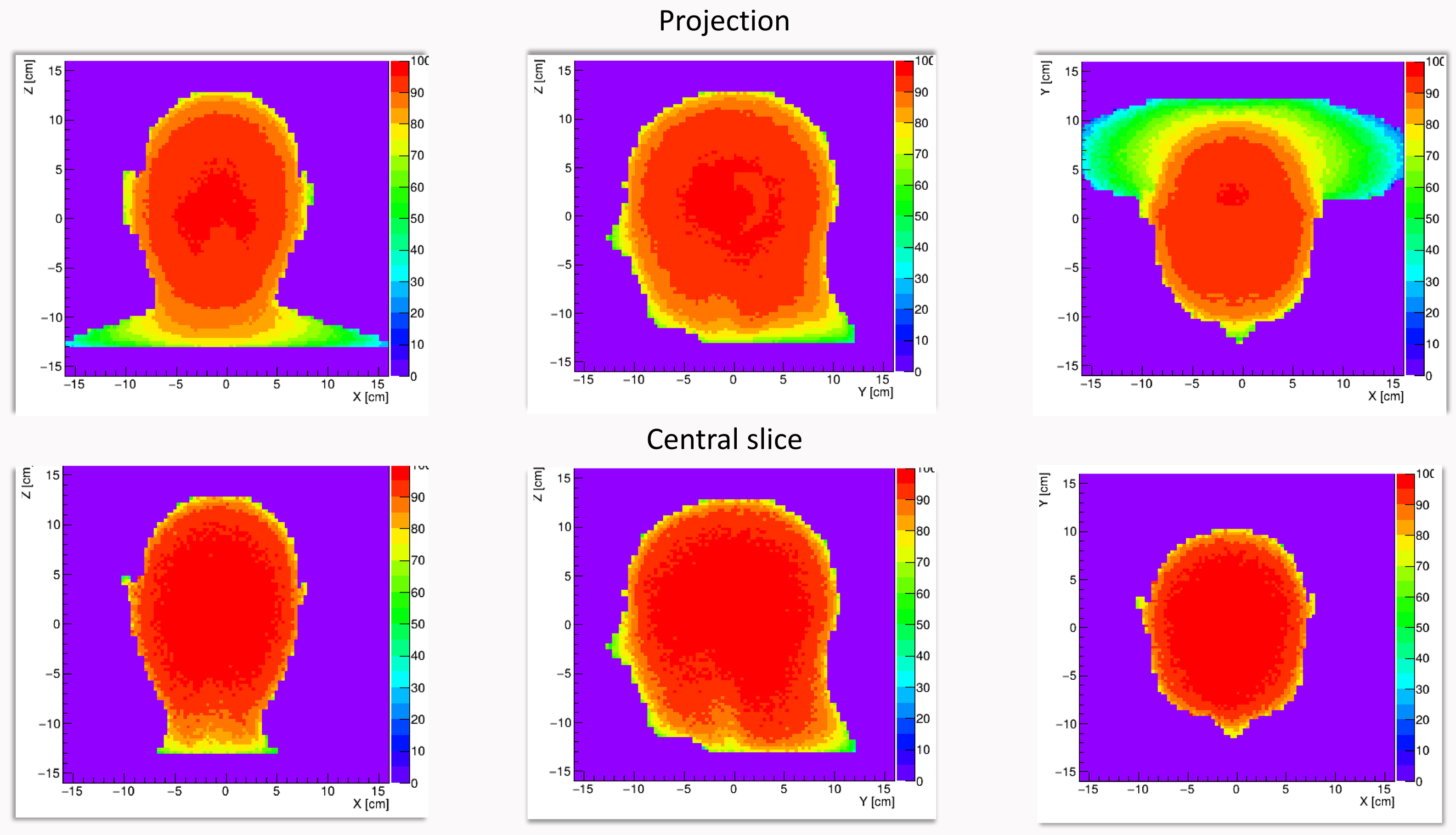


Human body model

Para-positronium, 10^7 events, 24.922 ± 0.016% pairs not absorbed



Ortho-positronium, $4.435 \cdot 10^7$ events, 10.256 ± 0.010% triplets not absorbed



Perspectives

Simulations using whole-body XCAT human phantom → Introducting detector acceptance into the simulation → Simulation using a phantom created based on the patient's CT scan

Acknowledgements

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References

[1] P. Moskal et al. Science Advances 10 (2024)
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[7] B. Auer https://github.com/BenAuer2021/Mesh-based-Human-Phantom-for-Simulation