Decay Law of Selected Fluorescent Substances

Francesco Giacosa

in collaboration with

A. Kołbus, K. Kyzioł, M. Płódowska,

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Based on 2509.17163 [quant-ph]

UJK Kielce (Poland) **& Goethe U Frankfurt** (Germany)

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UJ Krakow Poland

Outlook

- Decay is non-exponential ar short and long times
- However, diffcoult to measure. Limited evidence so far.
- We report on results of fluorescence decay with two distinct photon detectors
- Spectral function(s) diverging at threshold.
- Two different power laws. Is that a quantum effect?

Crash-course on the decay law

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Survival prob. amplitude A(t) is the Fourier transform of the spectra function

$$\mathcal{A}(t) = \int_{E_{th}}^{\infty} dE\, \rho(E)\, e^{-iEt/\hbar} \quad \Rightarrow \quad P(t) = |\mathcal{A}(t)|^2$$

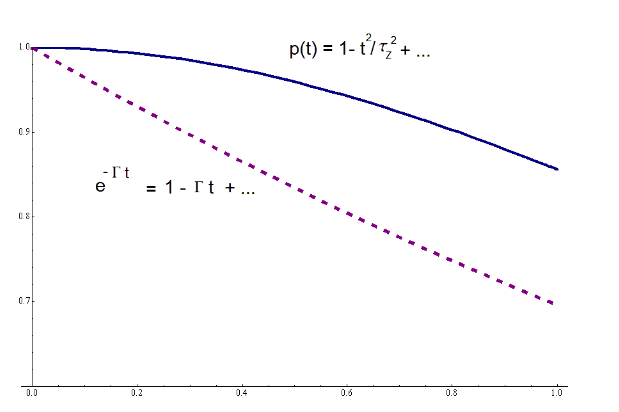
For:

$$E_{th} \to -\infty$$

$$\rho(E) = \frac{\Gamma}{2\pi}\left((E-M)^2 + \Gamma^2/4\right)^{-1} \quad \Rightarrow \quad P(t) = e^{-\Gamma t}$$

1. The spectral function falls faster than 1/E^2: P'(0) = 0 (that leads to Zeno and later on ant-Zeno effect).

2. The threshold energy Eth is finite,
Hence late-time power law



Late-time decay

$$\Gamma(E) \sim (E - E_{th})^{\gamma}$$

$$\rho(E) = \mathcal{N} \frac{E^{\gamma}}{(E - E_0)^2 + \Gamma^2/4} \theta(E)$$

$$A(t) \sim t^{-(\gamma+1)} \qquad P(t) \sim t^{-2(\gamma+1)} \qquad \gamma > -1$$

$$I(t) \propto -P'(t) \sim t^{-\beta}. \qquad \beta = 2\gamma + 3$$

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Experimental confirmation of non-exponential decay: short times

Uniwersytet Jana Kochanowskiego w Kielcach

Cold Na atoms in a optical potential

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

Steven R. Wilkinson, Cyrus F. Bharucha, Martin C. Fischer, Kirk W. Madison, Patrick R. Morrow, Qian Niu, Bala Sundaram* & Mark G. Raizen

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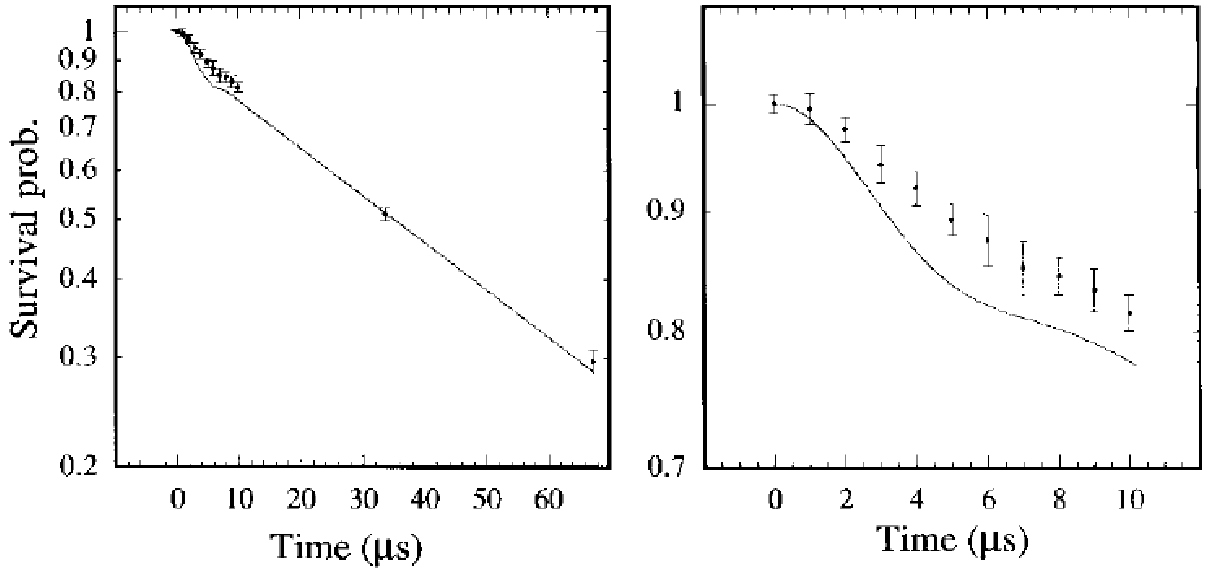
An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.



PRL **96,** 163601 (2006) PHYSICAL REVIEW LETTERS week ending 28 APRIL 2006

Violation of the Exponential-Decay Law at Long Times

C. Rothe, S. I. Hintschich, and A. P. Monkman

Department of Physics, University of Durham, Durham, DH1 3LE, United Kingdom

(Received 4 July 2005; published 26 April 2006)

First-principles quantum mechanical calculations show that the exponential-decay law for any metastable state is only an approximation and predict an asymptotically algebraic contribution to the decay for sufficiently long times. In this Letter, we measure the luminescence decays of many dissolved organic materials after pulsed laser excitation over more than 20 lifetimes and obtain the first experimental proof of the turnover into the nonexponential decay regime. As theoretically expected, the strength of the nonexponential contributions scales with the energetic width of the excited state density distribution whereas the slope indicates the broadening mechanism.

DOI: 10.1103/PhysRevLett.96.163601 PACS numbers: 42.50.Xa, 42.50.Fx

Uniwersytet Jana Kochanowskiego w Kielcach

Within the first-principles treatment of the decay of a metastable state Schrödinger's equation links the probability amplitude, $p(t)$, to the energy distribution density, $\omega(E)$ [1],

$$p(t) = \int e^{-(iEt/\hbar)} \omega(E) dE. \quad (1)$$

For example, taking $\omega(E)$ as a Lorentzian function for all E yields the well-known exponential decay at all times. However, in real physical systems, $\omega(E)$ must always have a lower limit, which is, for example, associated with the rest mass of scattering particles. Similarly, the energy

Experimental confirmation of non-exponential decay: long times

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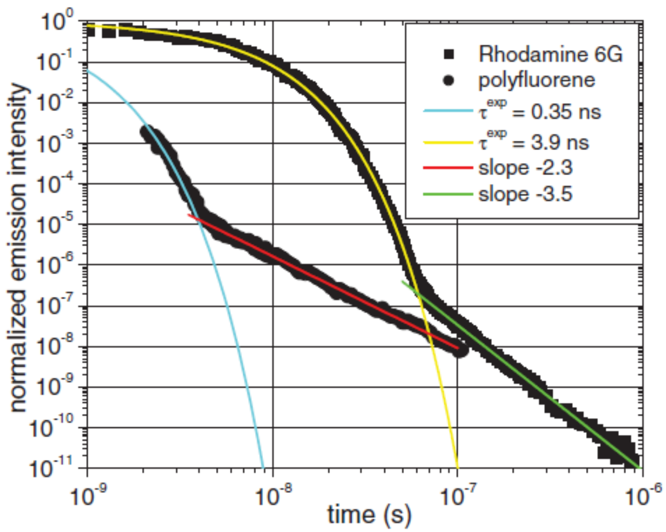

FIG. 2 (color). Corresponding double logarithmic fluorescence decays of the emissions shown in Fig. 1. Exponential and power law regions are indicated by solid lines and the emission intensity at time zero has been normalized.

Confirmation of: L. A. Khalfin. 1957. 1957 (Engl. trans. Zh.Eksp.Teor.Fiz.,33,1371)

$$I(t) \propto -P'(t) \sim t^{-\beta}.$$

TABLE I. Decay characteristics of a range of dissolved materials. Given are the exponential lifetimes (τ), the crossover times ($\tau^{\text{crossover}}$) relative to τ, the exponent of the nonexponential decay contribution, and a relative width computed using Eq. (2).

Organic material	τ (ns)	E (eV)	$\tau^{\text{turnover}}/\tau$	Algebraic exponent	Relative width Γ (10^{-6} eV)
Polyspirobifluorene	2.1	3.0	8.6	−2.08	552
Polyfluorene, $n = 100$	0.35	3.0	11.1	−2.26	45
Polyfluorene, $n = 10$	0.43	3.0	12.7	−2.46	9.1
Polyfluorene, $n = 3$	0.64	3.1	14.7	−2.68	1.3
$Ir(ppy)_3$, frozen solution	3000	2.4	13.0	−2.1	5.4
$Ir(ppy)_3$	933	2.4	13.2	−2.1	4.4
PtOEP, frozen solution	120 000	1.9	15.0	−4.0	0.58
Anthracene	3.6	3.2	14.7	−4.07	1.3
BBO	0.96	3.2	14.7	−2.8	2.4
Coumarin 450	3.6	2.9	16.4	−2.9	0.22
Rhodamine 6G	3.9	2.3	17.5	−3.5	0.058
Coumarin 500	5.0	2.7	17.4	−2.5	0.075

In most cases: $\beta<3$

Limited direct evidence so far.

Additonal indirect evidences:

A. Crespi et al, . **Experimental investigation of quantum decay at short, intermediate, and long times via integrated photonics**. *Phys. Rev. Lett.*, 122:130401, 2019.

N. G. Kelkar, M. Nowakowski, and K. P. Khemchandani. **Hidden evidence of nonexponential nuclear decay**. *Phys. Rev. C*, 70:024601, 2004.

F. Giacosa and G. Pagliara**, Deviation from the exponential decay law in relativistic quantum field theory: the example of strongly decaying particles,** Mod. Phys. Lett. A 26 (2011), 2247-2259

Plenty of exp. evidence that hadrons are not Breit-Wigner, e.g.
S. Schael et al. **Branching ratios and spectral functions of tau decays: Final ALEPH measurements and physics implications**. *Phys. Rept.*, 421:191–284, 2005.

ACTA PHYSICA POLONICA A No. 5 Vol. **146** (2024)

Proceedings of 5th Jagiellonian Symposium on Advances in Particle Physics and Medicine, Kraków, Poland, 2024

Nonexponential Decay Law of the $2P$–$1S$ Transition of the H Atom

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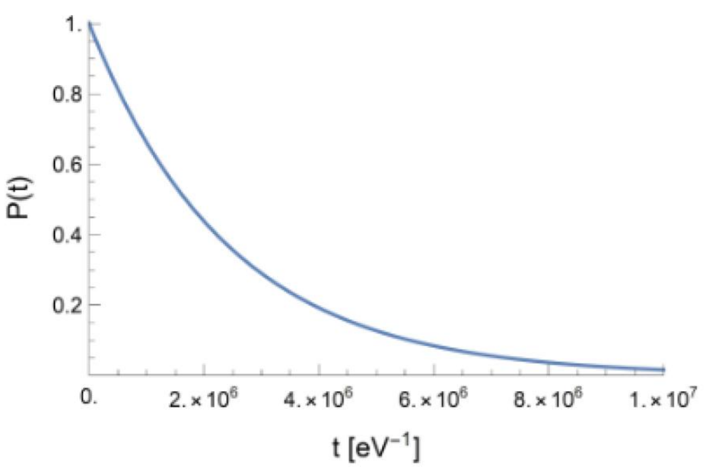


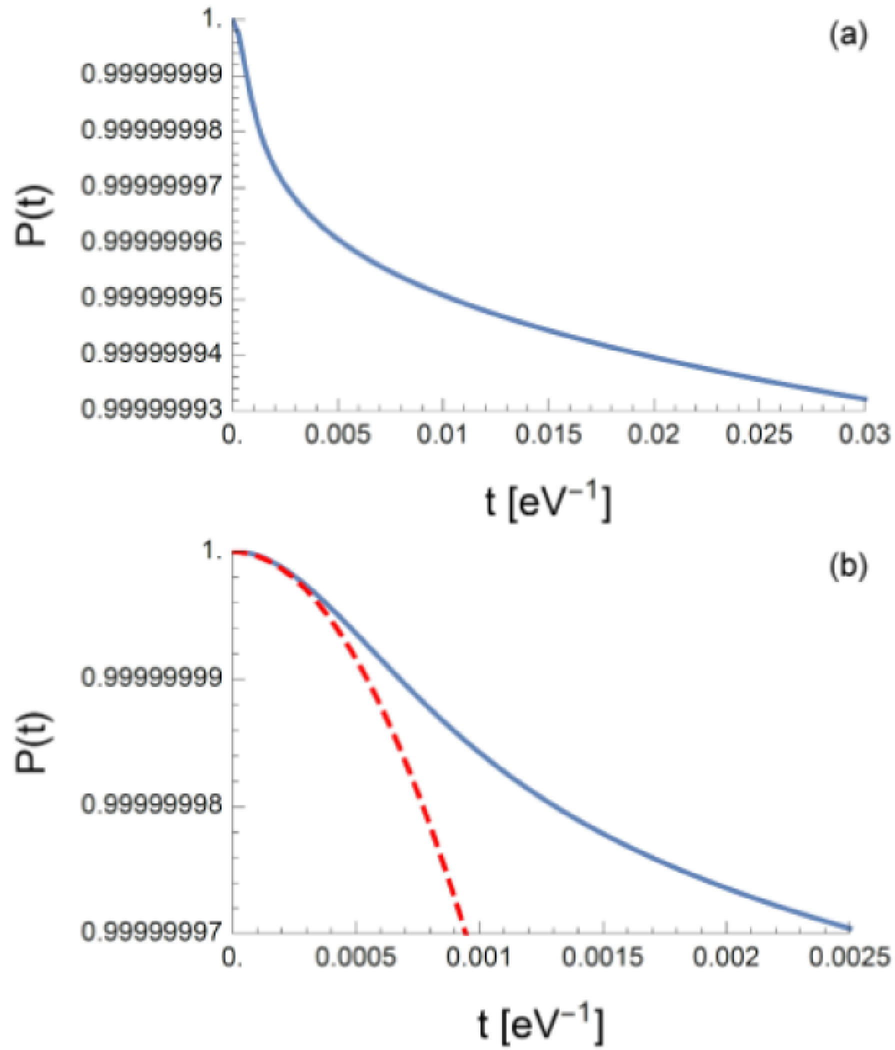
Fig. 2. Survival probability at intermediate times (of the order of $\tau \sim 2.42\times10^6$ eV^{-1}). In this domain, the function is basically indistinguishable from the exponential decay.

Why is the non-exponential decay law so hard to see?
The example of the 2P-12 transition.

See also
P. Facchi and S. Pascazio.
Temporal behavior and quantum zeno time of an excited state of the hydrogen atom.
Phys. Lett. A, 241:139–144, 1998.

2408.06905 [quant-ph]

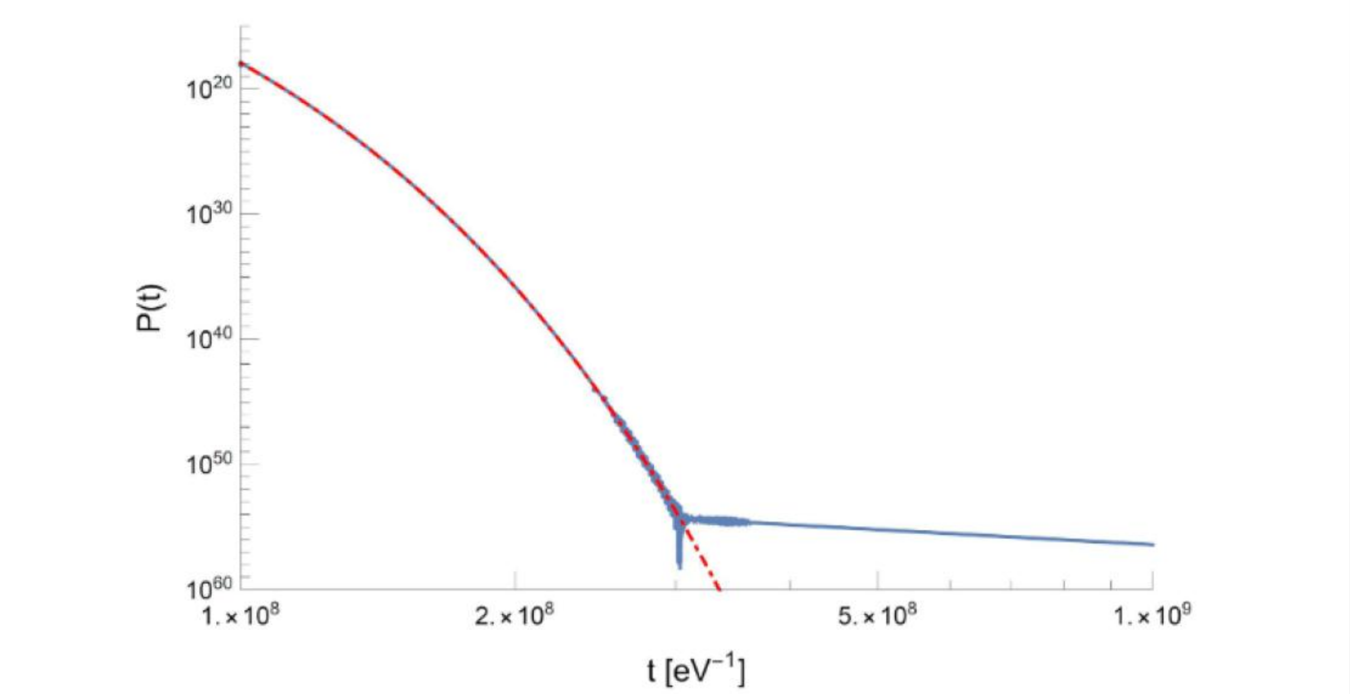
Short-time (Zeno) regime



2408.06905 [quant-ph]

Late-time regime for 2P-1S

β=5



2408.06905 [quant-ph]

Turnover time after more than 100 lifetimes! Not detectable.

- Here: recent experiment on fluorescence
- Different compounds tested: acridine orange, rhodamine B, erythrosine B
- acridine orange, rhodamine B: 1 exp or 2 exp work well.
- erythrosine B: power law favored!

Quantum Late-Time Decay and Channel Dependence

Francesco Giacosa*[1,2], Anna Kolbus[1], Krzysztof Kyziol[1], Magdalena Plodowska[1], Milena Piotrowska[1], Karol Szary[1], and Arthur Vereijken[1]

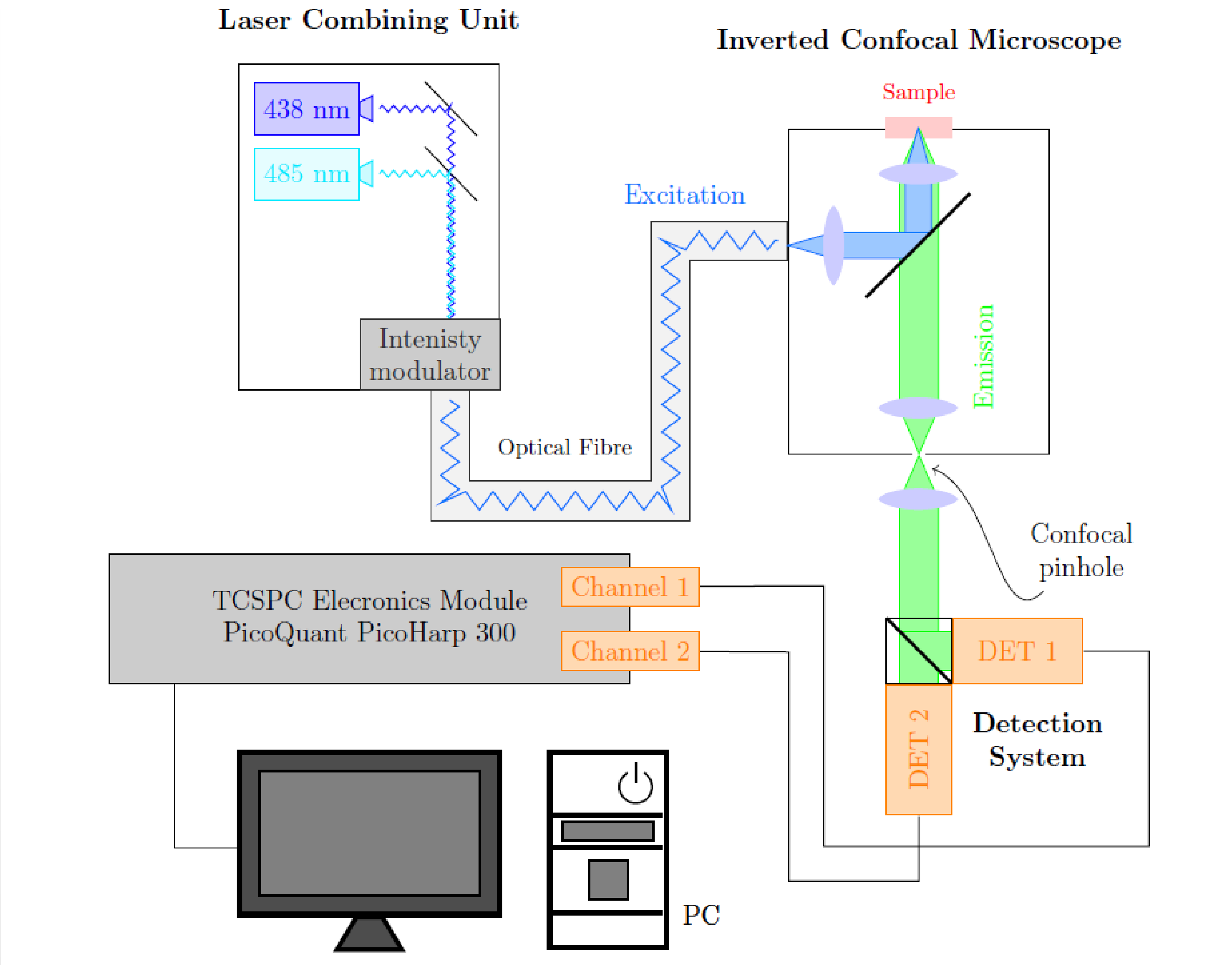
[1]Jan Kochanowski University, Kielce, Poland
[2]Goethe University, Frankfurt am Main, Germany

September 23, 2025

Abstract

Quantum mechanics predicts deviations from exponential decay at short and long times, yet experimental evidence is limited. We report a power-law tail after ~10 lifetimes in erythrosine B fluorescence, confirmed by two detectors probing distinct bands but yielding different power coefficients. The data match a divergent but normalizable spectral density, and theory predicts oscillations as a future test. A novel and general result is that in multichannel QM (and QFT) decay the lifetime is universal, but the late-time deviations exhibit a sizeable time window in which they are channel- (or band-) dependent, a feature consistent with our data.

2509.17163 [quant-ph]



Set-up. (From K. Kyzioł, master thesis, UJK, 2025).

Results for eytrhosine B

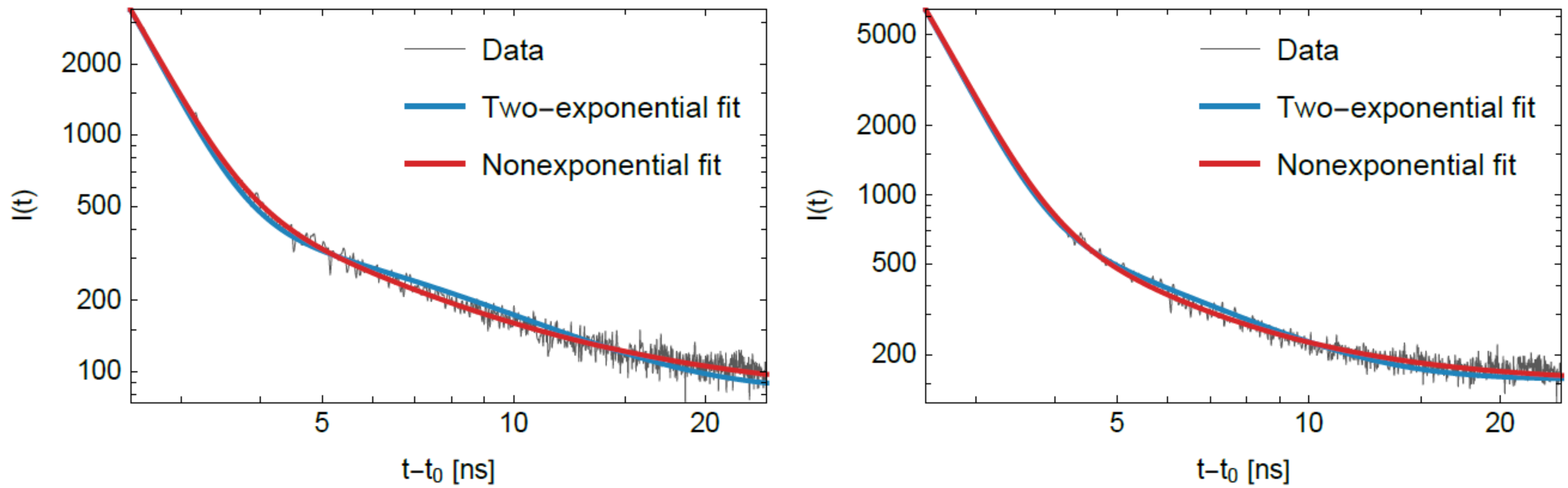
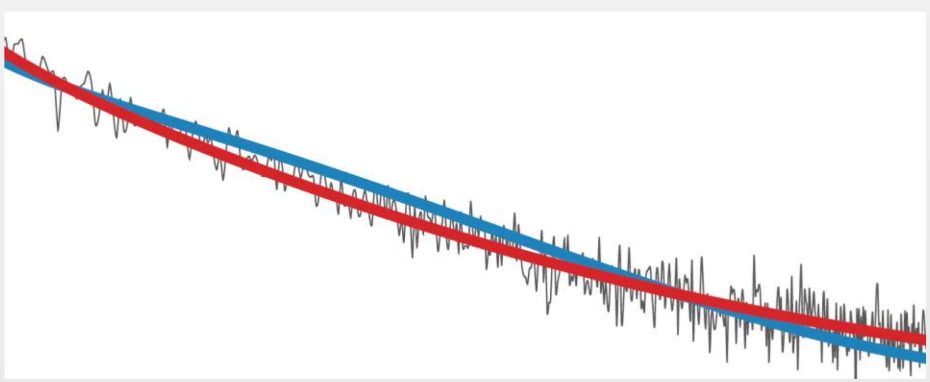


Figure 1: Fluorescence Intensity for both photon detectors (channel 1: left, channel 2: right) - comparison between data and both fitting functions up to ~ 20 ns (data taking up to ~ 95 ns).



2509.17163 [quant-ph]

Fitting functions and chi^2

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2509.17163 [quant-ph]

Table 1: Model functions employed in the analysis; $t \gtrsim 2t_0$ and b is the background. Note, t_0 is not part of the fit parameter, but is taken as the maximum of the intensity.

Model	Fluorescence intensity $I(t)$	Fit parameters
Two-exponential	$I(t) = C_1 \exp\left(-\frac{t-t_0}{\tau_1}\right) + C_2 \exp\left(-\frac{t-t_0}{\tau_2}\right) + b$	$\chi^2(C_1, \tau_1, C_2, \tau_2, b)$
Nonexponential	$I(t) = C \exp\left(-\frac{t-t_0}{\tau}\right) + C_p (t - t_0)^{-\beta} + b$	$\chi^2(C_0, \tau, C_p, \beta, b)$

Table 2: Fitting results for Erythrosine B measurements.

Fitting Range - Channel 1: 0.960 - 94.752 ns; Channel 2: 0.960 - 94.656 ns;						
Two-exponential model						
Channel	χ^2_ν	C_1	τ_1 [ns]	C_2	τ_2 [ns]	b
1	1.219	665 627	0.46105	574.44	5.3898	83.80
2	1.256	1 353 960	0.45583	1 348.40	3.40912	156.53
Nonexponential model						
Channel	χ^2_ν	C	τ [ns]	C_p	β [ns]	b
1	1.049	692 886	0.44802	2 929.81	1.5469	77.03
2	1.080	1 357 800	0.4475	7 687.33	1.9985	150.01

Important remarks

- Both channel show the same life-time (expected)

- …but different power law coefficient.

Channel 1: $\beta = 1.5 < 3$

Channel 2: $\beta = 2.0 \ < 3$

$$I(t) \propto -P'(t) \sim t^{-\beta}.$$

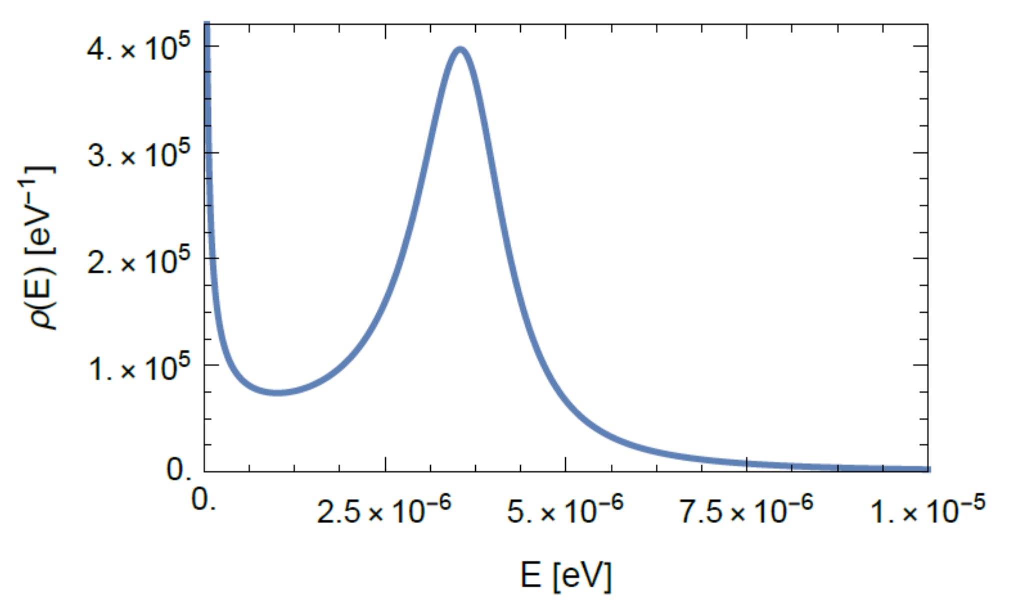
$$\beta = 2\gamma + 3$$

$$\gamma < 0$$

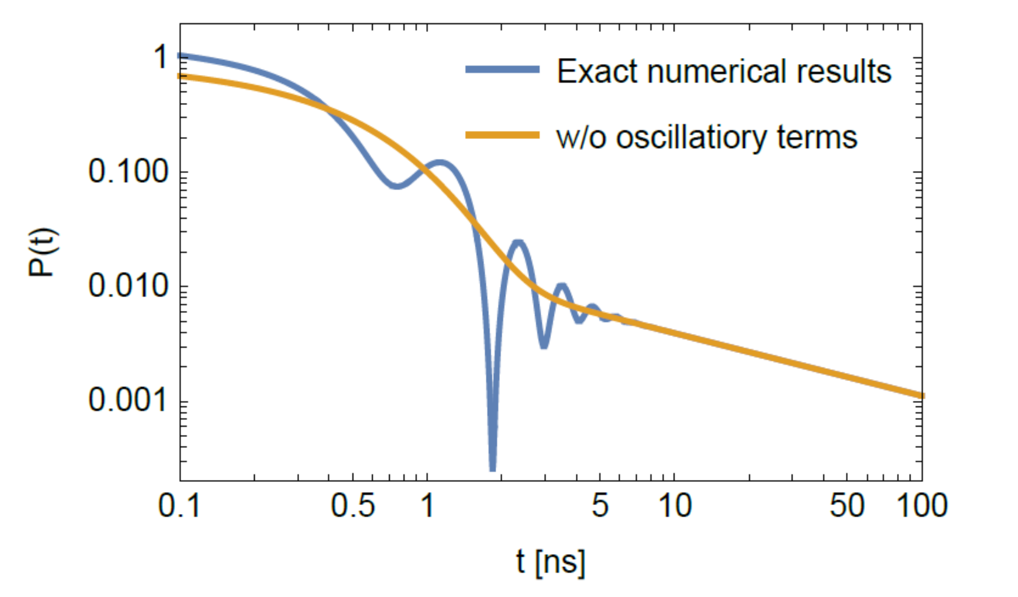
$$\rho(E) = \mathcal{N} \frac{E^{\gamma}}{(E - E_0)^2 + \Gamma^2/4} \theta(E)$$

- Negative γ. What does that mean?

Channel 1: spectral function for reproducing data



This is the required
Shape of the spectral function
for our result
(and for most of Rothe)



Notice the oscillations!
Not visible at present
(coarse graining, binning)
But their existence is
Inherent to the approach

2509.17163 [quant-ph]

Different power coefficients: is that QM?

Consider two decay channel

$$\rho_i(E) = \mathcal{N} \frac{c_i \left(E - E_{th,i}\right)^{\gamma_i}}{(E-M)^2 + \Gamma^2/4}$$

$$w_i(t) = \int_{E_{th,i}}^{\infty} dE \frac{\Gamma_i(E)}{2\pi} \left| \int_0^t \mathcal{A}(t') e^{iEt'/\hbar} dt' \right|^2 , \; \Gamma_i(E) = c_i \left(E - E_{th,i}\right)^{\gamma_i} .$$

F.G. Multichannel decay law. *Phys. Lett. B*, 831:137200, 2022. 2108.07838 [quant-ph]
F. G. Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels. *Found. Phys.*, 42:1262–1299, 2012.

At late times (at the onset of the power law):

$$w_i(t) = w_i(\infty) - a_i t^{-(\gamma_1 + \gamma_2 + 2)} - b_i t^{-2(\gamma_i + 1)} - \ldots$$

Different chanel dependent late-time behavior!

2509.17163 [quant-ph]

For the two intensities for e.g. $\gamma_1 < \gamma_2$.

$$I_1(t) \propto w_1'(t) \propto t^{-2\gamma_1+3}$$

$$I_2(t) \propto w_2'(t) \propto t^{-(\gamma_1+\gamma_2+3)}$$

Extension to bands: different band behavior when the power-law starts to dominante:

$$I_{band}(t) \propto \sum_{i=1}^{N_{band}} \left(a_i t^{-(\gamma_{\min}+\gamma_i+3)} + b_i t^{-(2\gamma_i+3)} \right) \propto t^{-(\gamma_{\min}+\gamma_{B,\min}+3)}$$

2509.17163 [quant-ph]

Discussion/Conclusions

- Power-law describes the fluorescence decay of erythrosine B. Confirmation of Rothe et al (2006).

- Just as (majority of) Rothe's compounds, we obtain $\beta < 3$. That means, the spectral function diverges at threshold!

- We used two different photon detectors. Two different coefficients β were measured. That is *consistent* with QM.

- Late-time power-law: memory effect.
  This memory effect is channel (or band) dependent.

Thank you

- Back-up below

Experiment: setup and results. The experiment was performed on a Nikon Eclipse Ti-E inverted confocal microscope equipped with two picosecond laser diodes as an excitation source (PicoQuant LDH-D-C-440 with wavelength 438 nm and PicoQuant LDH-D-C-485 with wavelength 483 nm – pulse widths < 120 ps, spectral widths ranging from 2 to 8 nm; only the former was used). Fluorescence from the sample was coupled via optical fibres into a dual-channel detection unit (PicoQuant PMA Hybrid 40 photomultipliers, timing resolution <120 ps), separated by a dichroic mirror and bandpass filters (520/35 nm and 600/50 nm). Time-correlated single-photon counting was carried out with a PicoHarp 300 module. This dual-detector scheme allowed us to record fluorescence decay simultaneously in two distinct spectral windows. The four dyes (fluorescein, acridine orange, rhodamine B, erythrosine B) were dissolved in methanol with a solution concentration of 10^{-5} mol/dm^3. While fluorescein, acridine orange, and rhodamine B can be described by a single or two-exponential function (plus background), erythrosine B exhibited a clear non-exponential behaviour at late times. Hence, below we concentrate on erythrosine B. In total, seven fluorescence decay curves were obtained (three measurements (10 min each) in October 2024, two in November 2024, and two in February 2025, 15 min each). These data sets were combined together and are reported in Fig. 1. Also the IRF (Instrument Response Function) was measured. Subsequently, the convolution of the signal was performed, but the influence of the IRF turned out to be negligible for our late-time study.

Basic definitions

Let $|S\rangle$ be an unstable state prepared at $t = 0$.

Survival probabilty ammplitude at $t > 0$:

$$a(t) = \langle S | e^{-iHt} | S \rangle \qquad (\hbar = 1)$$

Survival probability: $p(t) = |a(t)|^2$

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

$$a(t)=\langle S|e^{-iHt}|S\rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

$$a^*(t)=\langle S|e^{-iHt}|S\rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

p(t) decreases quadratically (<u>not linearly</u>);

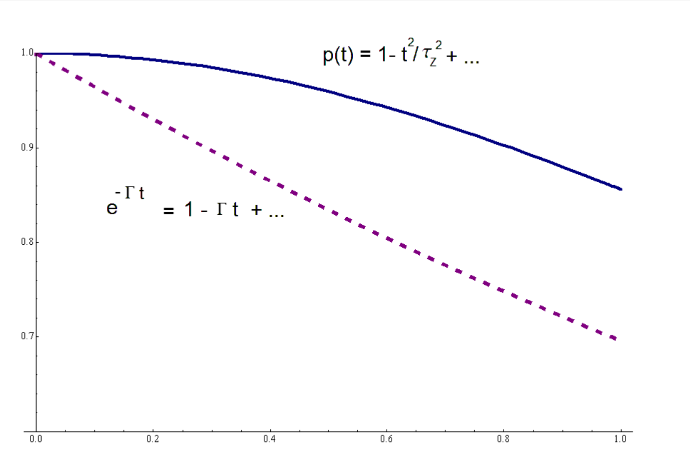
no exp. decay for short times.

τ_Z is the `Zeno time´.

It follows:

$$p(t)=|a(t)|^2 = a^*(t)a(t) = 1 - t^2\left(\langle S|H^2|S\rangle - \langle S|H|S\rangle^2\right) + \ldots = 1 - \frac{t^2}{\tau_Z^2} + \ldots$$

where $\tau_Z = \dfrac{1}{\sqrt{\langle S|H^2|S\rangle - \langle S|H|S\rangle^2}}$.



Note: the quadratic behavior holds
for any quantum transition, not only for decays.

Time evolution and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E)dE = 1$

$$p(t) = \left| \int_{E_{th,1}}^{\infty} dE d_S(E) e^{-\frac{i}{\hbar}Et} \right|^2$$

In stable limit: $d_S(E) = \delta(E-M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Payley and Wiener (1934) theorem: P(t) is not exponential at large time if a left-threshold is present

Time evolution and energy distribution (2)

Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4} \rightarrow a(t) = e^{-iM_0 t - \Gamma t/2} \rightarrow p(t) = e^{-\Gamma t}.$$

The Breit-Wigner energy distribution cannot be exact.

Two physical conditions for a realistic $d_S(E)$ are:

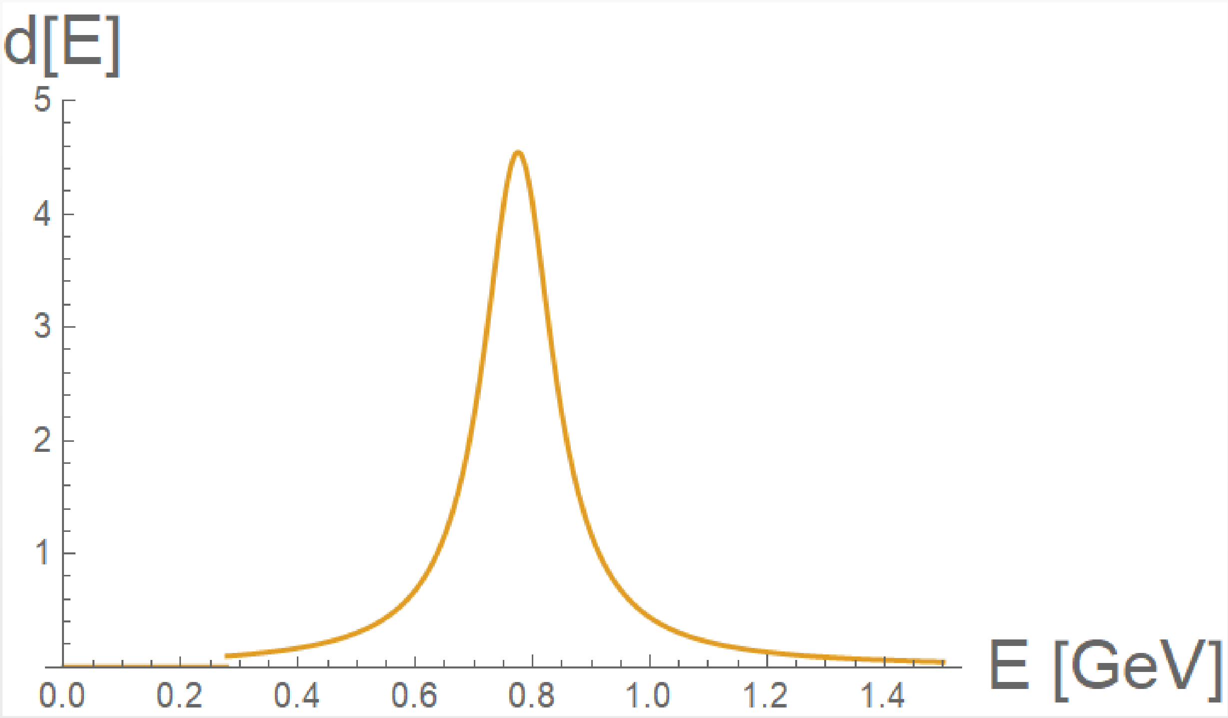
1) Minimal energy: $d_S(E) = 0 \text{ for } E < E_{\min}$

2) Mean energy finite: $\langle \mathrm{E} \rangle = \int_{-\infty}^{+\infty} d_S(E) E dE = \int_{E_{\min}}^{+\infty} d_S(E) E dE < \infty$

How to introudce a threshold?
BW with threshold (naive threatment)

$$d_S(E) = N \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

N is needed because the normalization is lost!

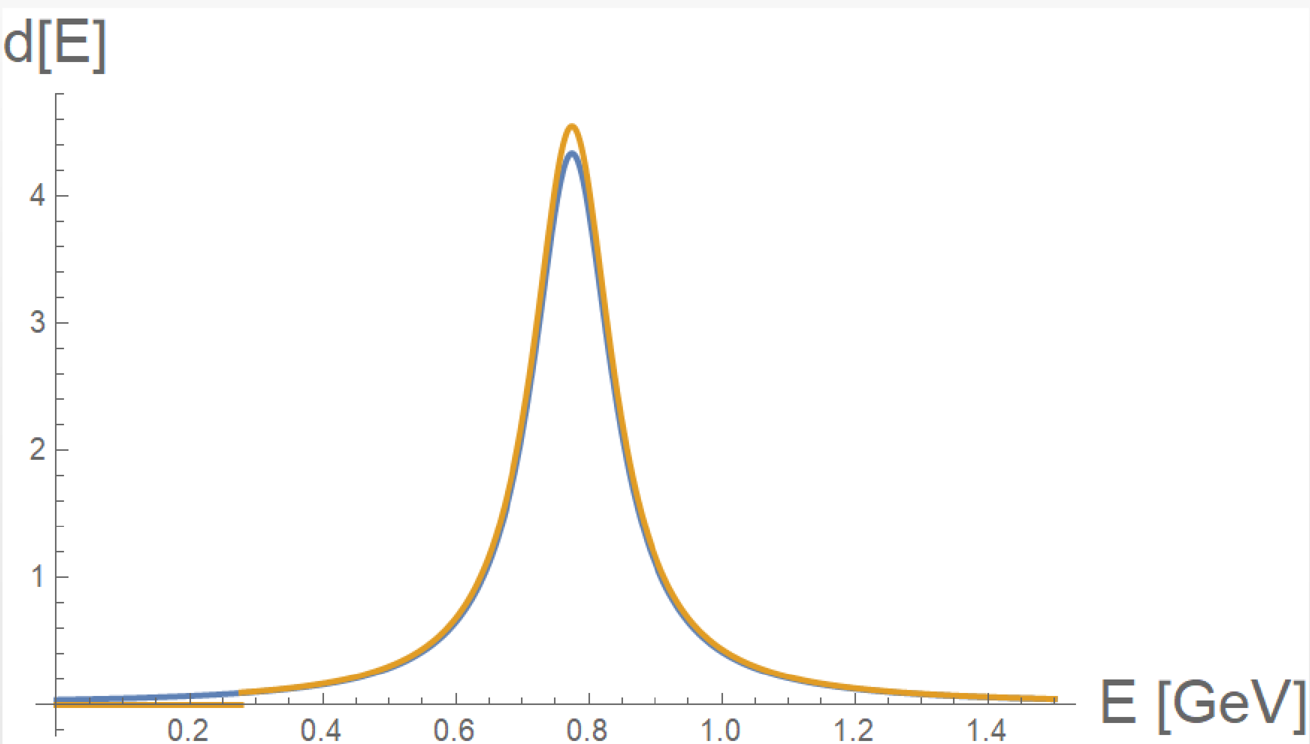


First example: BW with threshold (naive treatment)

$$d_S(E) = N \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

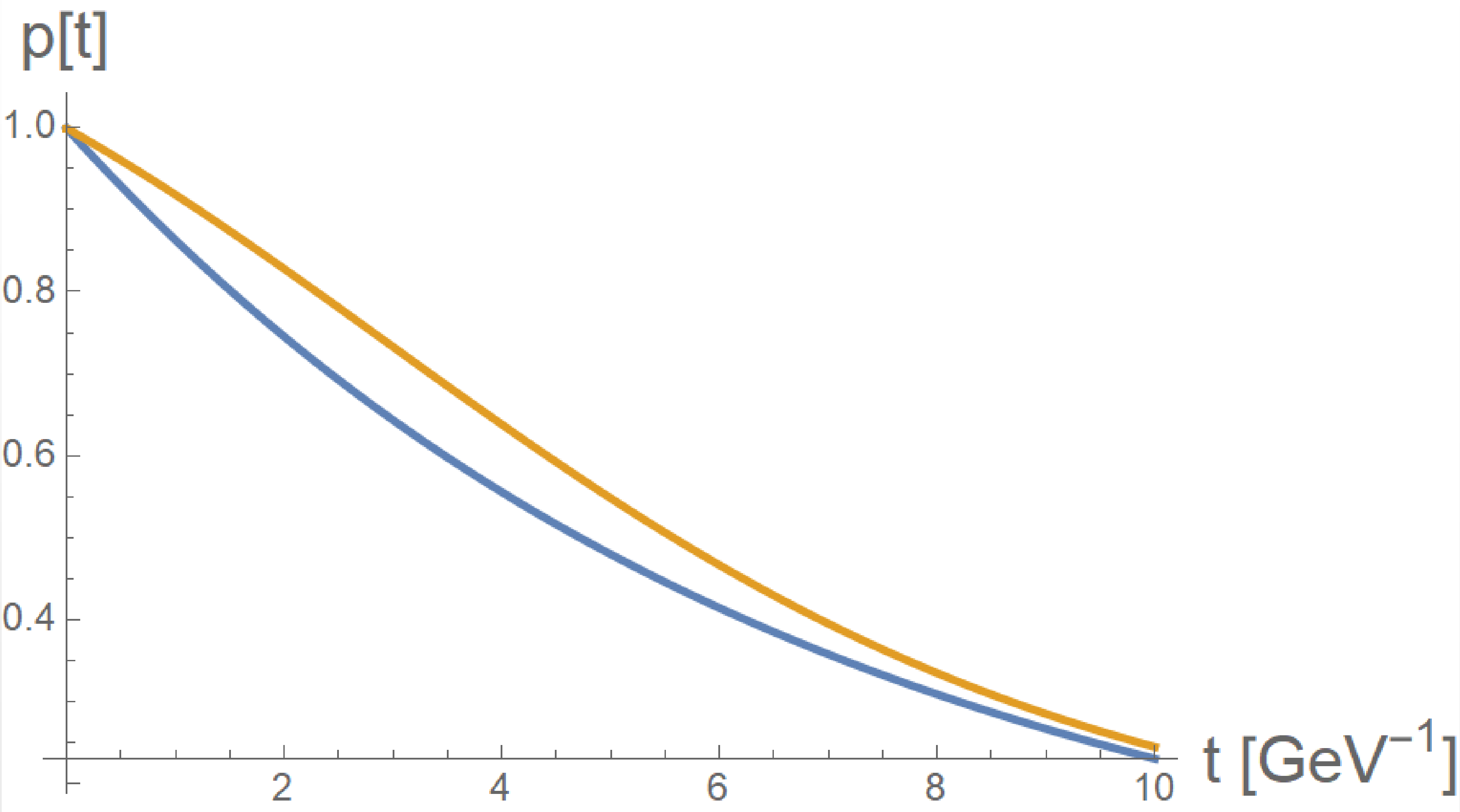
N is needed because the **normalization is lost**!

For the ρ meson, we get $N = 1.05$



$\langle E \rangle = \ln\Lambda = \infty$

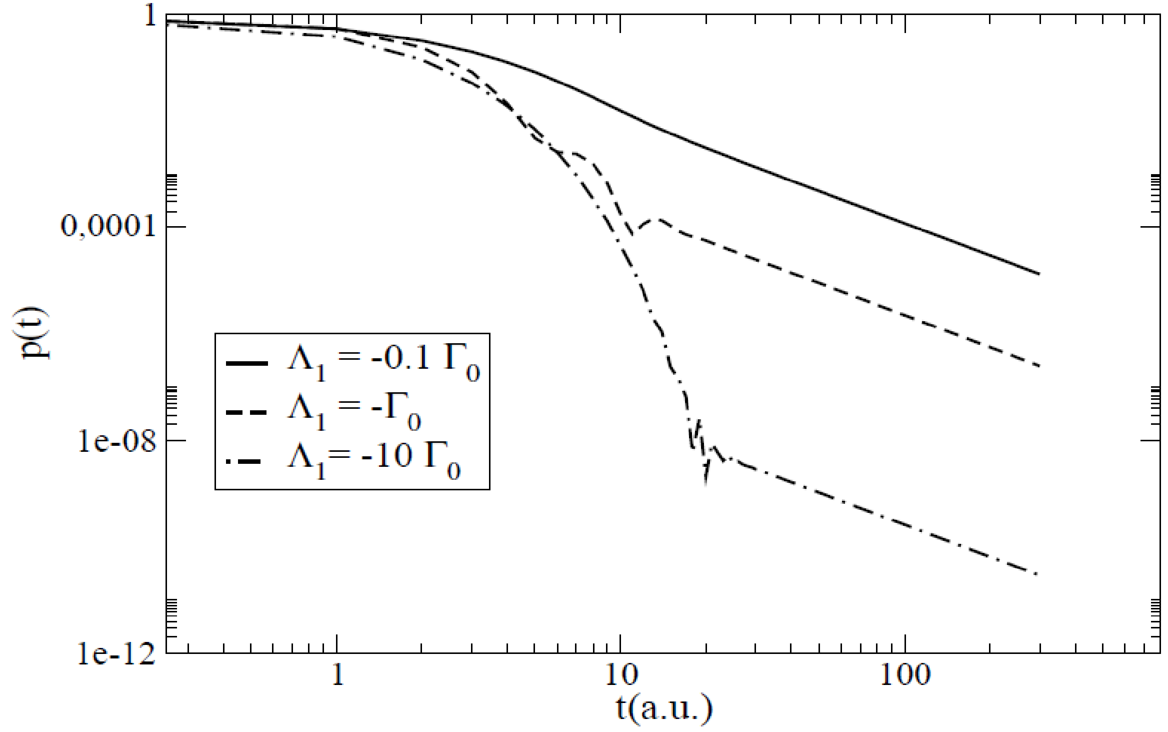
Time evolution



Blue: plain BW, yellow: BW with threshold (naive)

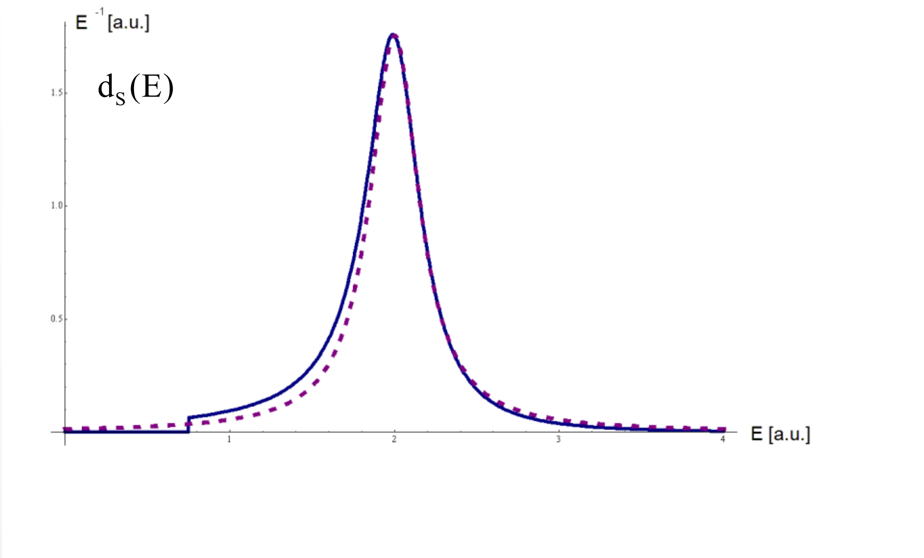
<E>=lnΛ=∞

Single left-threshold at long times



F.G. and G. Pagliara, [arXiv:1204.1896 [nucl-th]].

Second example: threshold plus form factor

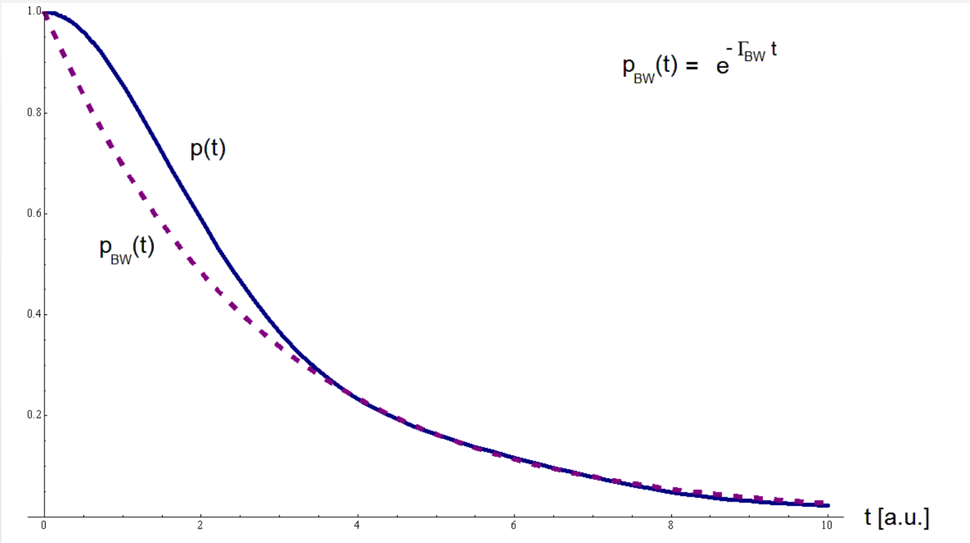


$$M_0 = 2;\ E_{min} = 0.75;\ \ \Gamma = 0.4;\ \Lambda = 3$$

$$d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2-E_0^2)/\Lambda^2}\,\theta(E-E_{min})}{(E-M_0)^2+\Gamma^2/4}$$

$$d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E-M_0)^2+\Gamma_{BW}{}^2/4}$$

Γ_{BW}, such that $d_{BW}(M_0) = d_S(M_0)$



$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt}\, dE\,; \quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW} t}$$

Comments

- The ‘brute force’ threshold and high-energy behavior can be good as a first approximation, but it is just an ‘ad hoc’ modification of Breit-Wigner.
- How to properly describe the theory of decay?
- Which is a suitable energy distribution for e.m. decays?

General non-relativistic approach

Propagator

$$G_S(E) = \frac{1}{E - M + \Pi(E) + i\varepsilon}$$

Self-energy (or loop)
E_{th} is the threshold energy

$$\Pi(E) = -\int_{E_{th}}^{\infty} \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(E')}{E - E' + i\varepsilon} dE'$$

Energy dependent 'decay width'

$$\Gamma(E) = 2\,\mathrm{Im}\,\Pi(E)$$

Energy distribution (or spectral function)

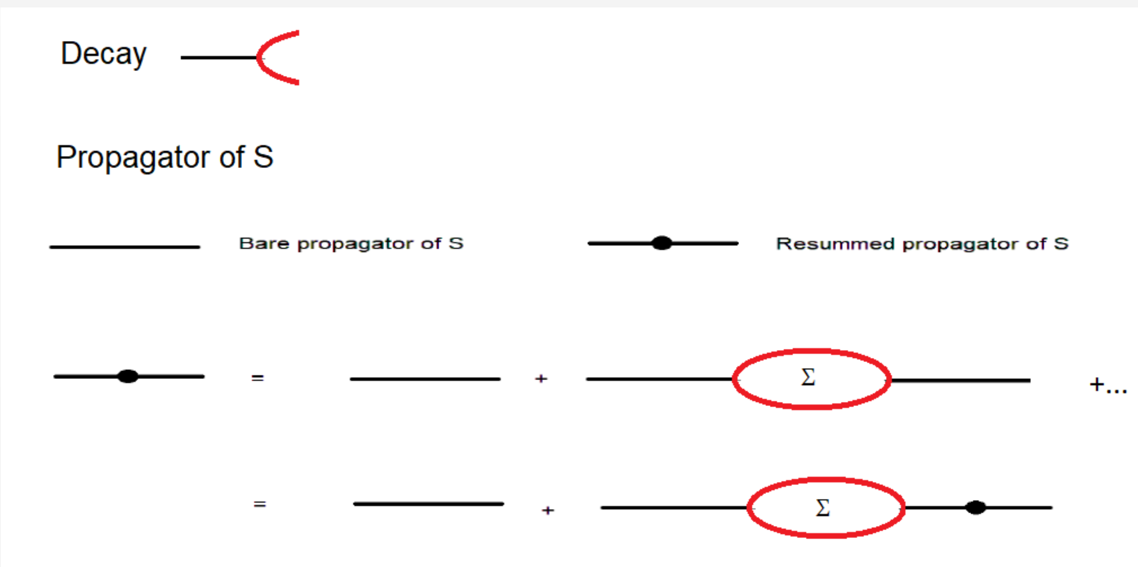
$$d_S(E) = -\frac{1}{\pi}\mathrm{Im}[G_S(E)] = \frac{1}{\pi}\frac{\mathrm{Im}\,\Pi(E)}{(E - M + \mathrm{Re}\,\Pi(E))^2 + (\mathrm{Im}\,\Pi(E))^2}$$

Link between propagator and distribution

The propagator can be expressed as (H being the full Hamiltonian)

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M + \Pi(E) + i\varepsilon} = \int_{E_{th}}^{+\infty} \mathrm{dE}' \frac{d_S(E')}{E - E' + i\varepsilon}$$

out of which $d_S(E) = -\frac{1}{\pi} \mathrm{Im}[G_S(E)]$



Normalization ok!!

$$\int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E) = 1$$

Symmetries in Science XVIII IOP Publishing
Journal of Physics: Conference Series **1612** (2020) 012012 doi:10.1088/1742-6596/1612/1/012012

The Lee model: a tool to study decays

Francesco Giacosa[a,b]
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[arXiv:2001.07781 [hep-ph]]

Time-evolution (general)

$$a_S(t) = Ze^{-iz_{pole}t} + \ldots,$$

The dots describe short- and long-time deviations from the exponential decay

The pole:

$$z_{pole} - M + \Pi_{II}(z_{pole}) = 0\,,$$

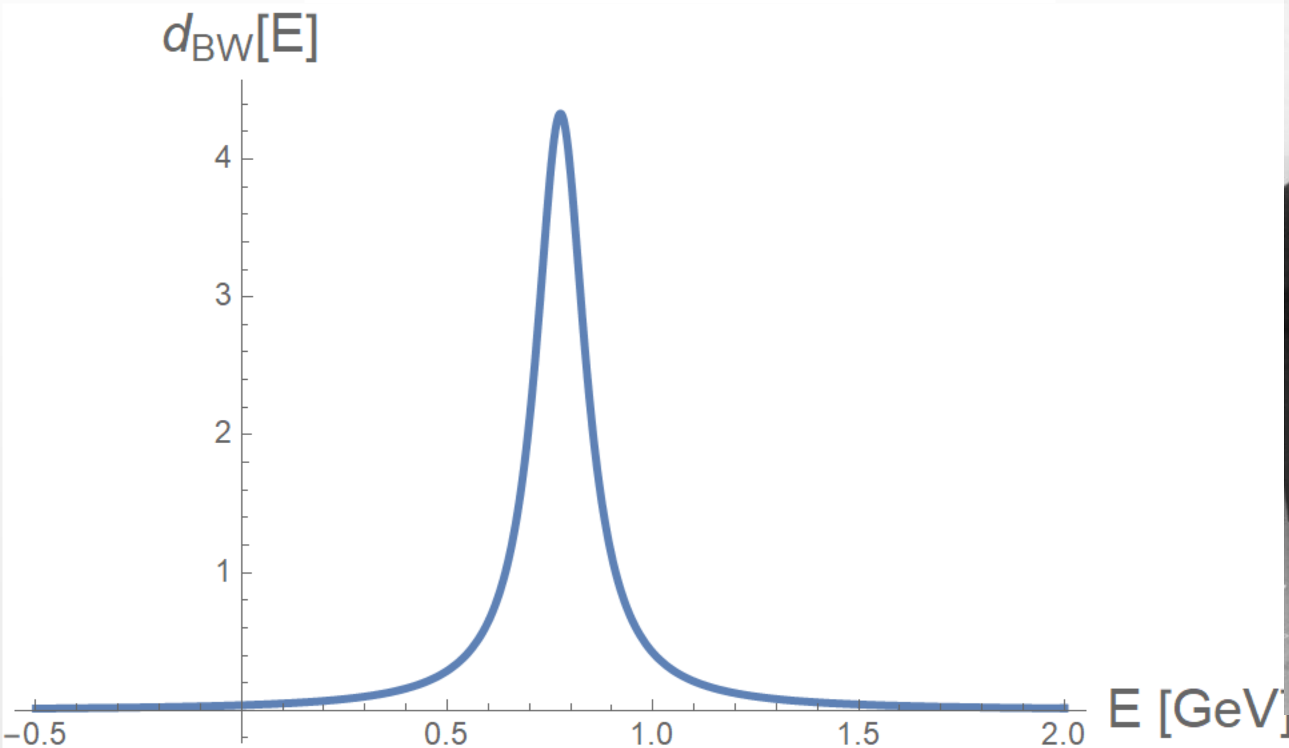
where II refers to the second Riemann sheet. Then:

$$z_{pole} = M_{pole} - i\,\frac{\Gamma_{pole}}{2}\,.$$

Breit-Wigner distribution

Uniwersytet
Jana Kochanowskiego w Kielcach

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$



Rho-meson as example.

BW extends from –inf to +inf. There is no left threshold.

BW: properties

BW-distribution:

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

BW-propagator:

$$G_S^{\mathrm{BW}}(E) = \frac{1}{E - M + i\Gamma/2 + i\varepsilon}$$

Pole:

$$z_{pole}^{\mathrm{BW}} = M - i\Gamma/2$$

Link prop-dist:

$$d_S^{\mathrm{BW}}(E) = -\frac{1}{\pi} \mathrm{Im}[G_S^{\mathrm{BW}}(E)] = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

Normalization:
(important for prob. interpretation)

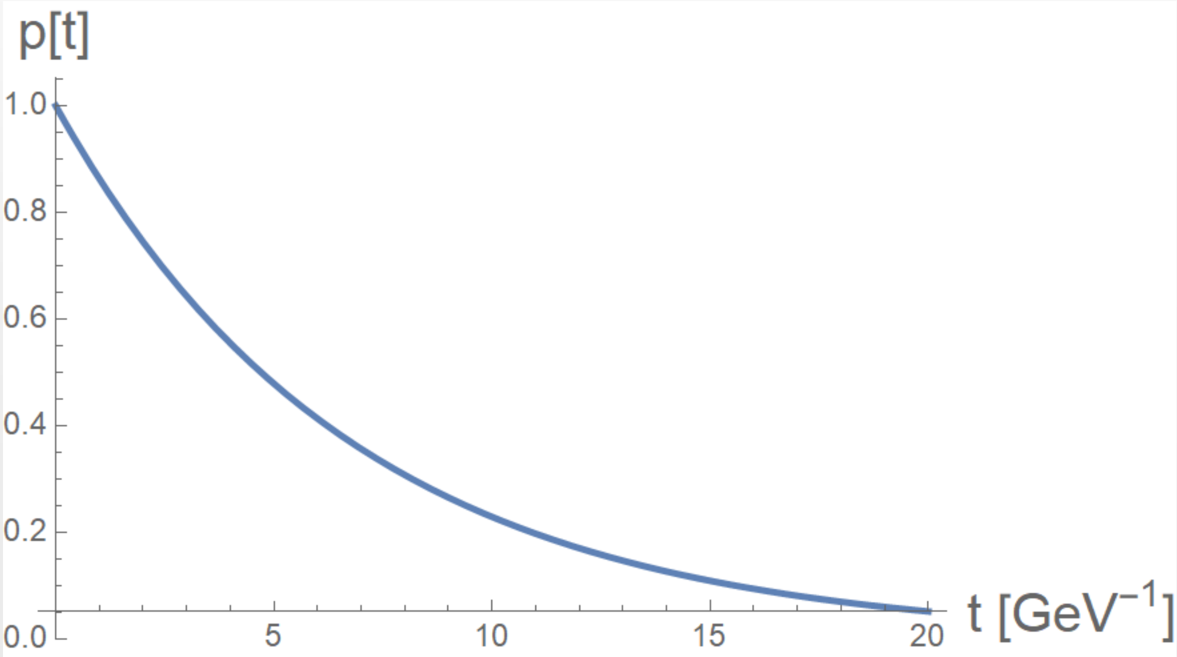
$$\int_{-\infty}^{+\infty} d_S^{\mathrm{BW}}(E) dE = 1$$

BW corresponds to exp. decay

Uniwersytet Jana Kochanowskiego w Kielcach

$$a_S^{\mathrm{BW}}(t) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} \mathrm{dE}\, G_S^{\mathrm{BW}}(E) e^{-iEt} = \int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S^{\mathrm{BW}}(E) = e^{-iMt-\Gamma t/2}$$

$$p^{\mathrm{BW}}(t) = \left|a_S^{\mathrm{BW}}(t)\right|^2 = e^{-\Gamma t}$$



BW with threshold **properly done**

We assume that:

$$\mathrm{Im}\,\Pi(E) = \begin{cases} \frac{\Gamma}{2} \text{ for } E \in (E_{th}, \Lambda) \\ 0 \text{ otherwise} \end{cases}$$

<E>=lnΛ=∞

In the limit $\Lambda \to \infty$ and by using one subtraction we get:

$$\Pi(E) = \frac{\Gamma}{2\pi} \ln\left(\frac{-E_{th} + M}{E_{th} - E}\right)$$

Then:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{\left[E - M + \frac{\Gamma}{2\pi} \ln\left(\frac{M - E_{th}}{E_{th} - E}\right)\right]^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

This is actually the correct Breit-Wigner with threshold!
Correctly normalized to unity, no need of an extra N…but somewhat not handy

Model for e.m. decays: Sill plus form factor

$$\mathrm{Im}\left[\Pi(E)\right] = \frac{\gamma\sqrt{E - E_{th}}}{\left(1 + \frac{(E-E_{th})^2}{\Lambda^2}\right)^2}$$

See also:

27 April 1998

PHYSICS LETTERS A

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Physics Letters A 241 (1998) 139–144

Temporal behavior and quantum Zeno time of an excited state of the hydrogen atom

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Received 4 November 1997; revised manuscript received 28 January 1998; accepted for publication 10 February 1998
Communicated by P.R. Holland

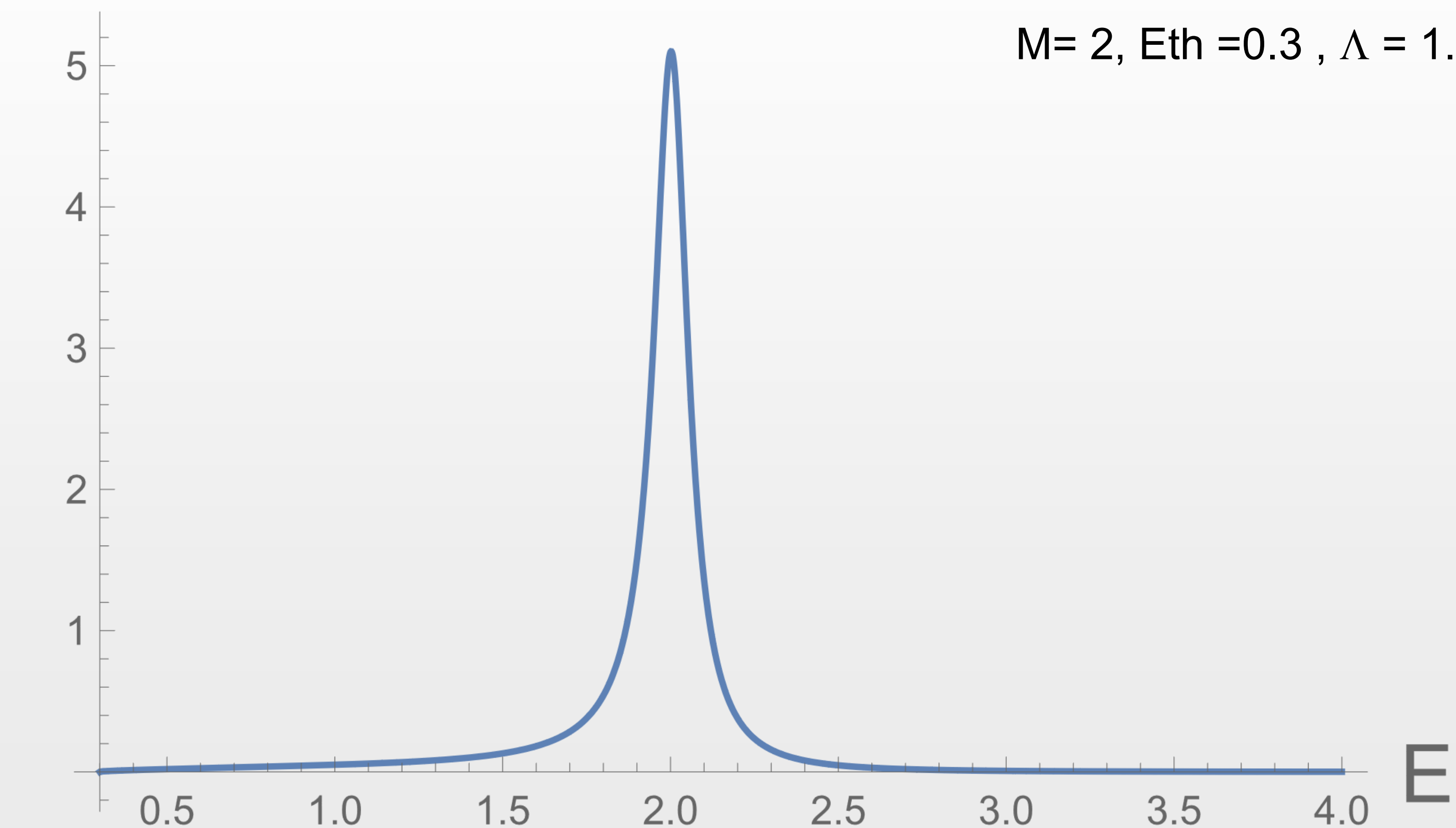
$$\mathrm{Re}[\Pi(E)] = \frac{\sqrt{2}}{8}\,\frac{\gamma\Lambda^{3/2}\left[-(E-E_{th})^3 - \Lambda(E-E_{th})^2 - 5\Lambda^2(E-E_{th}) + 3\Lambda^3\right]}{\left[(E-E_{th})^2 + \Lambda^2\right]^2} + C$$

$$d_S(E) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(E)] = \frac{1}{\pi}\,\frac{\mathrm{Im}\,\Pi(E)}{(E - M + \mathrm{Re}\,\Pi(E))^2 + (\mathrm{Im}\,\Pi(E))^2}$$

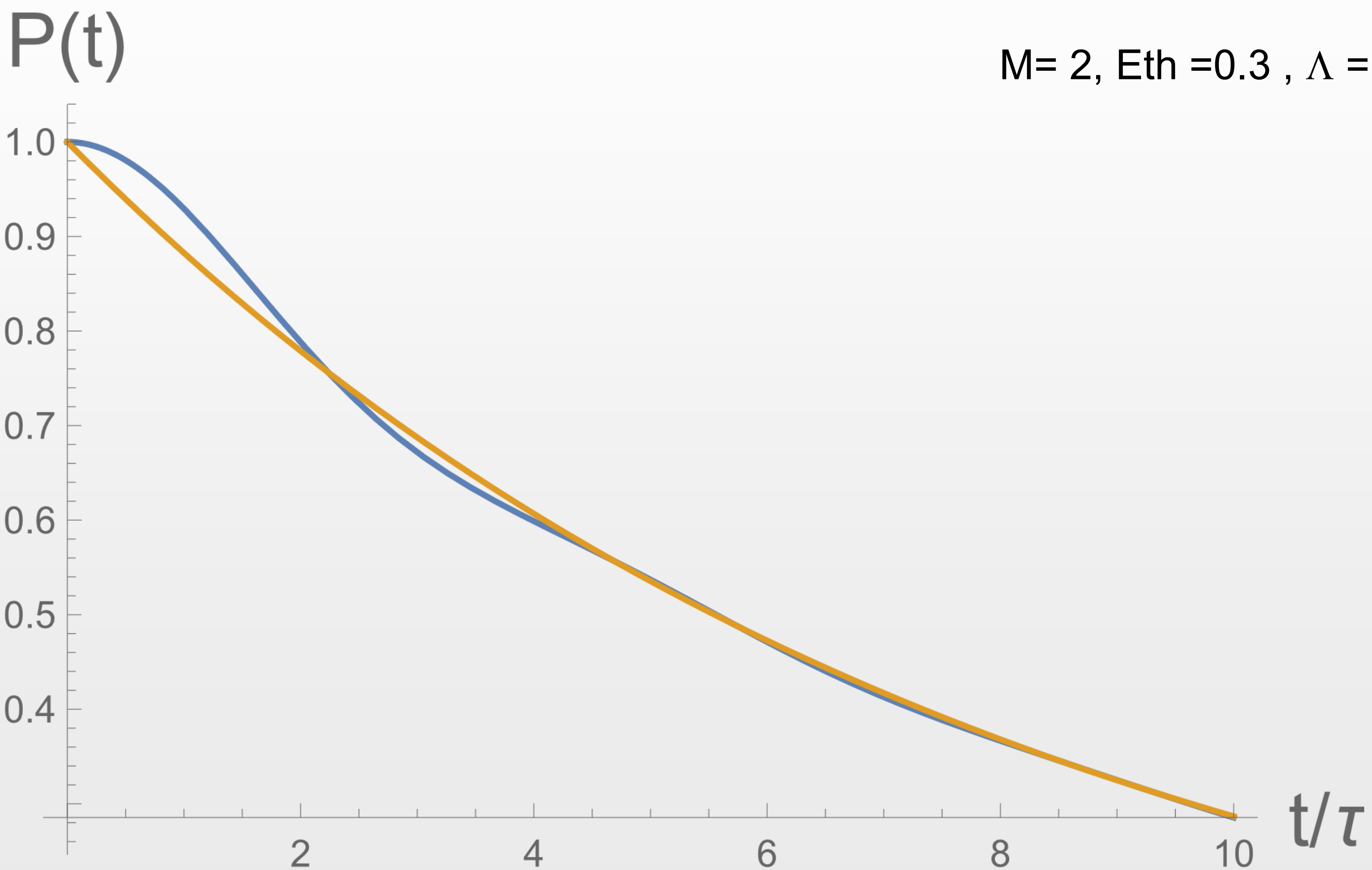
Spectral function

$d_S(E)$

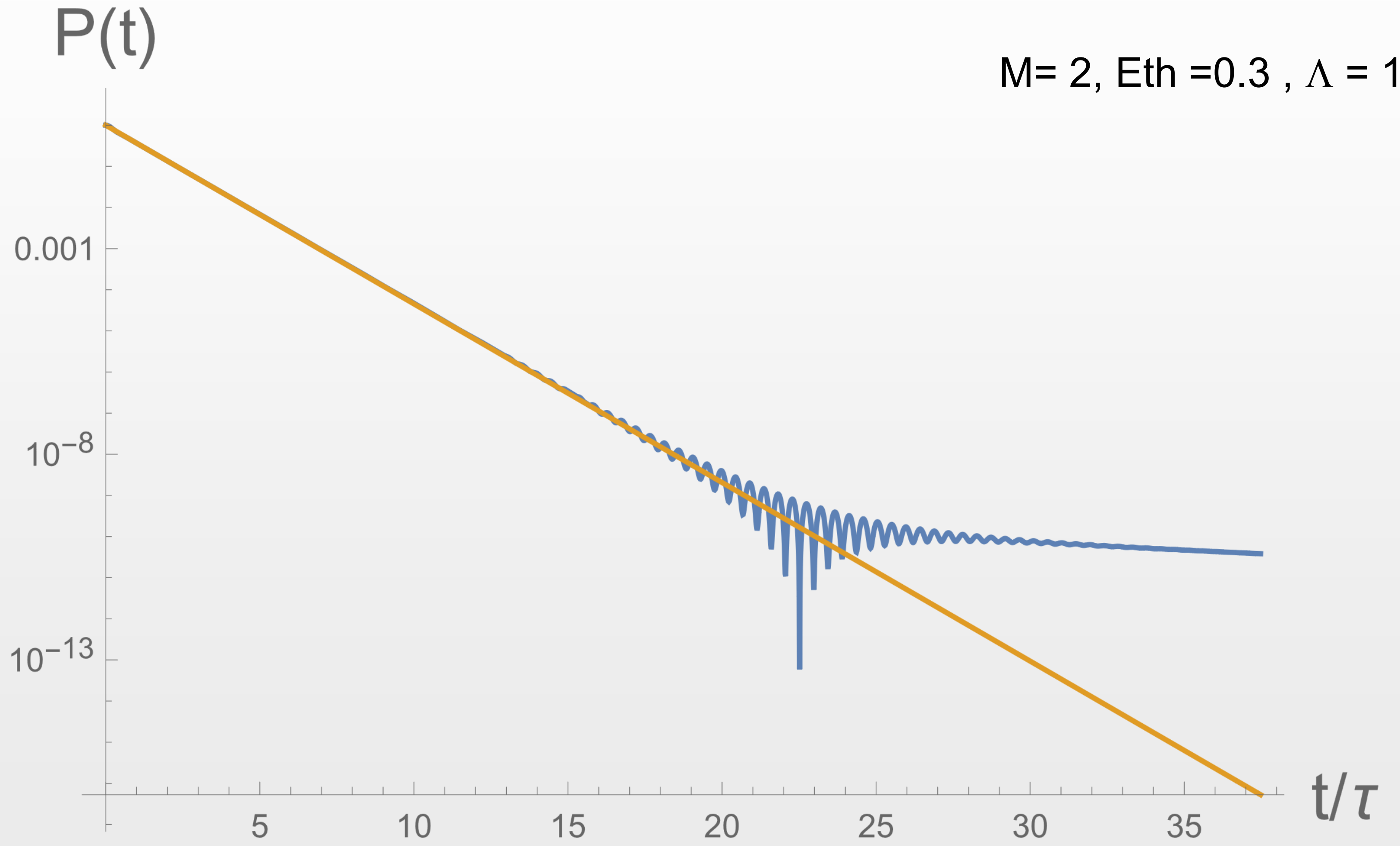
<E> finite and threshold present



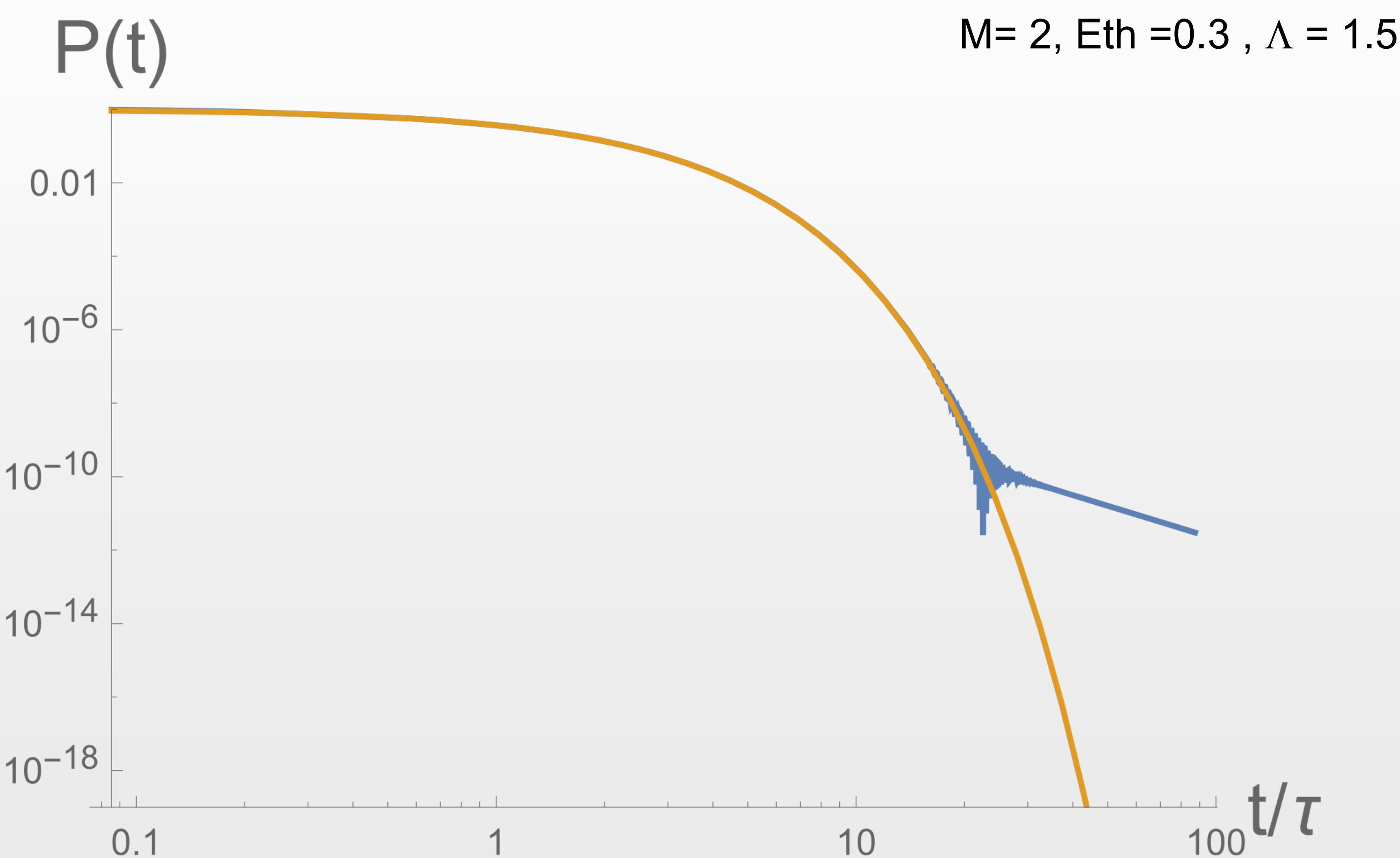
Survival probability



Log-plot



Log-log plot



Hydrogen atm, 2P-1S transition

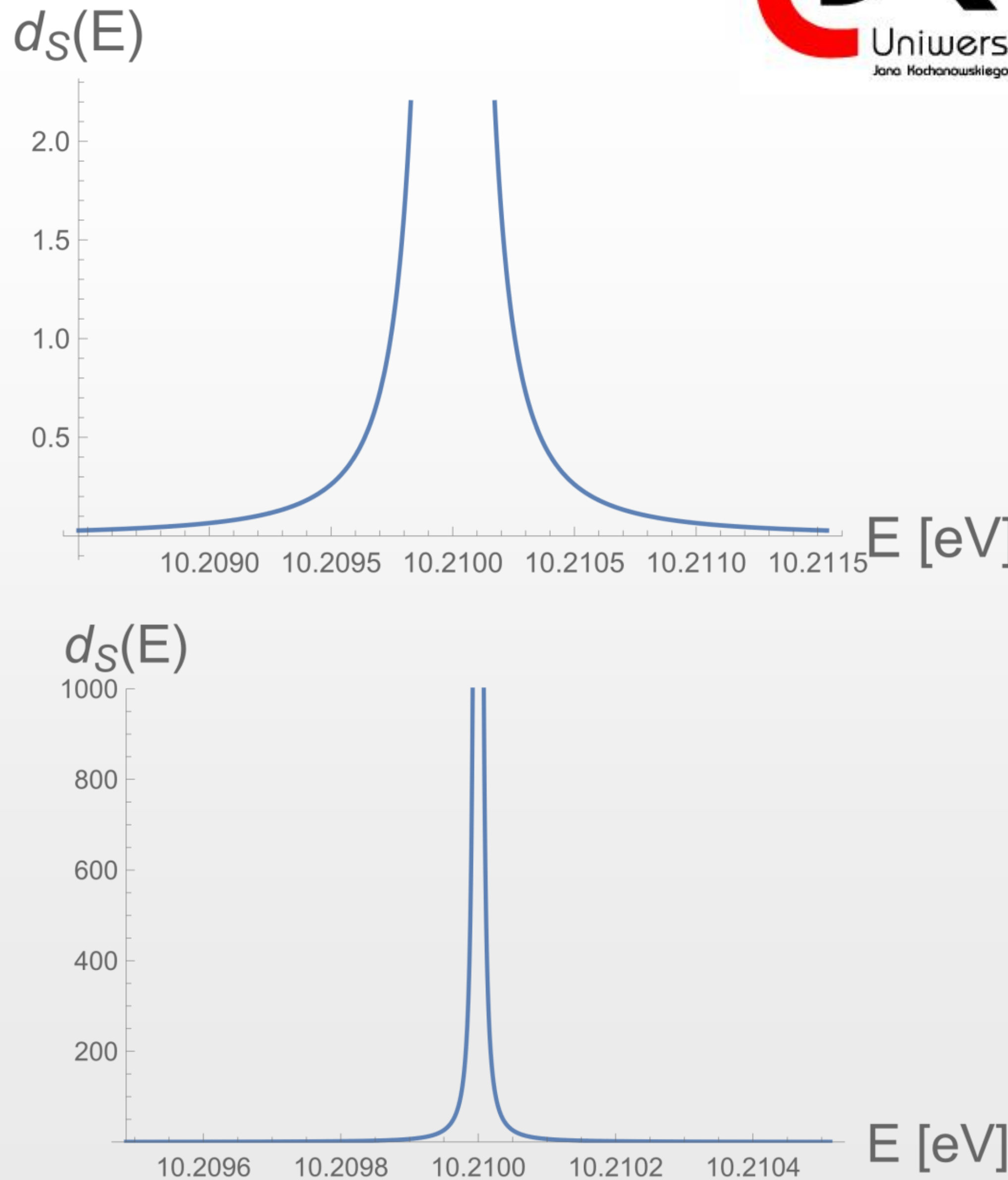
Uniwersytet
Jana Kochanowskiego w Kielcach

τ = 1.9 ns

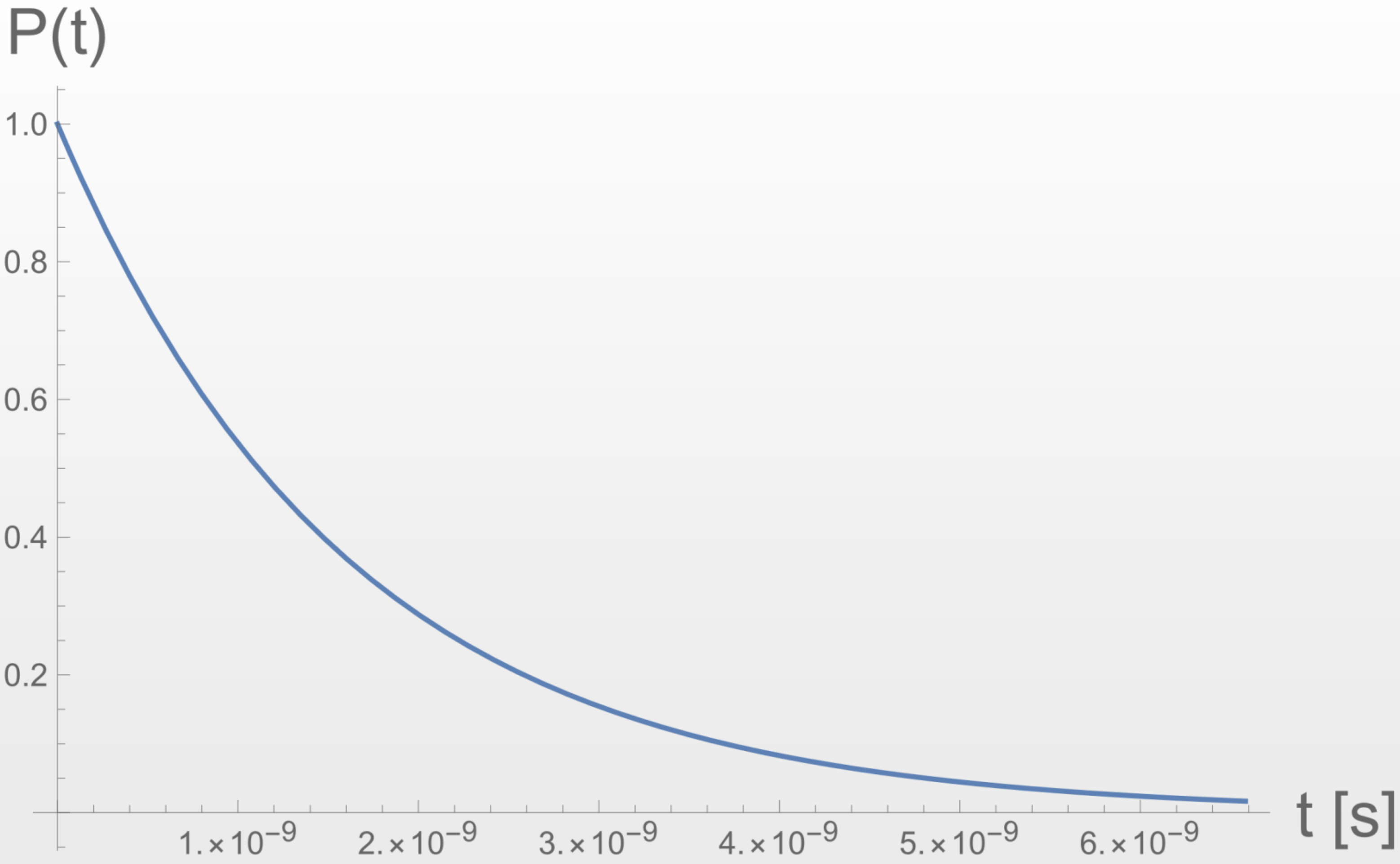
M = 10.21 eV

$\Lambda = 3\alpha m_e/2$

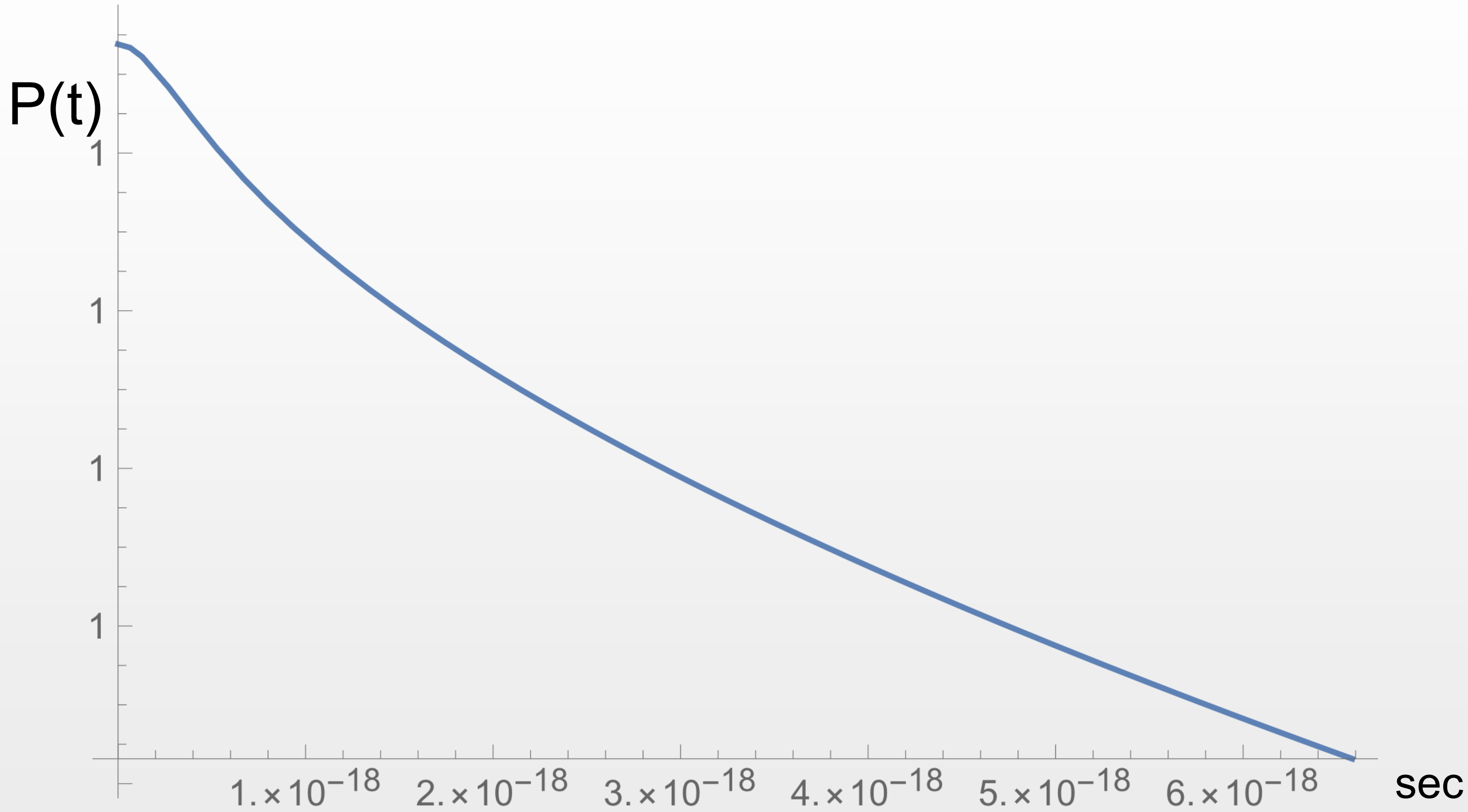
$E_{th} = 0$



Decay law



Short-time decay



Long times: “fighting” with numerics

Multichannel decay law

Physics Letters B 831 (2022) 137200

Contents lists available at ScienceDirect

Physics Letters B

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Multichannel decay law

Francesco Giacosa [a,b,*]

ACTA PHYSICA POLONICA A No. 3 Vol. **142** (2022)

Proceedings of the 4th Jagiellonian Symposium on Advances in Particle Physics and Medicine

Multichannel Decay: Alternative Derivation of the i-th Channel Decay Probability

F. Giacosa [a,b,*]

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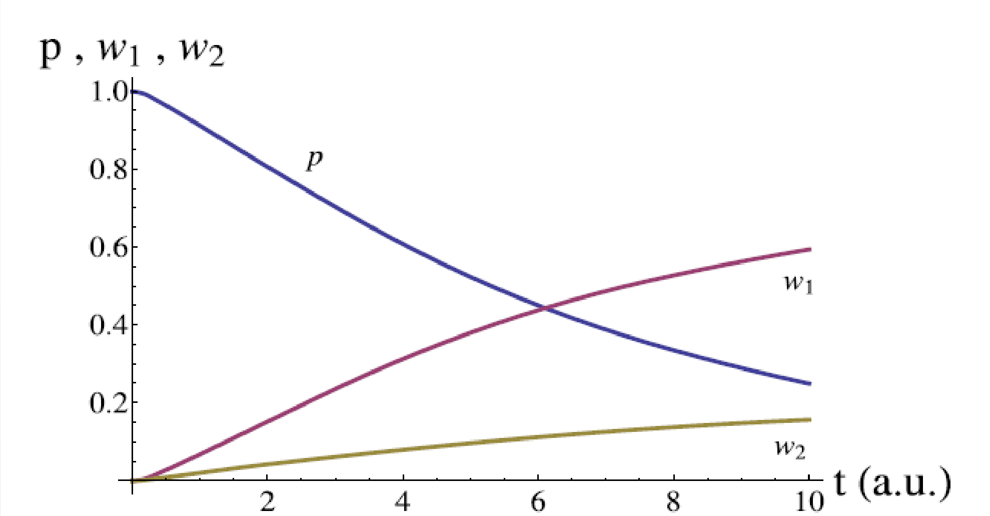


Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p + w_1 + w_2 = 1$ holds. Note, t is expressed in a.u. of $[M^{-1}]$.

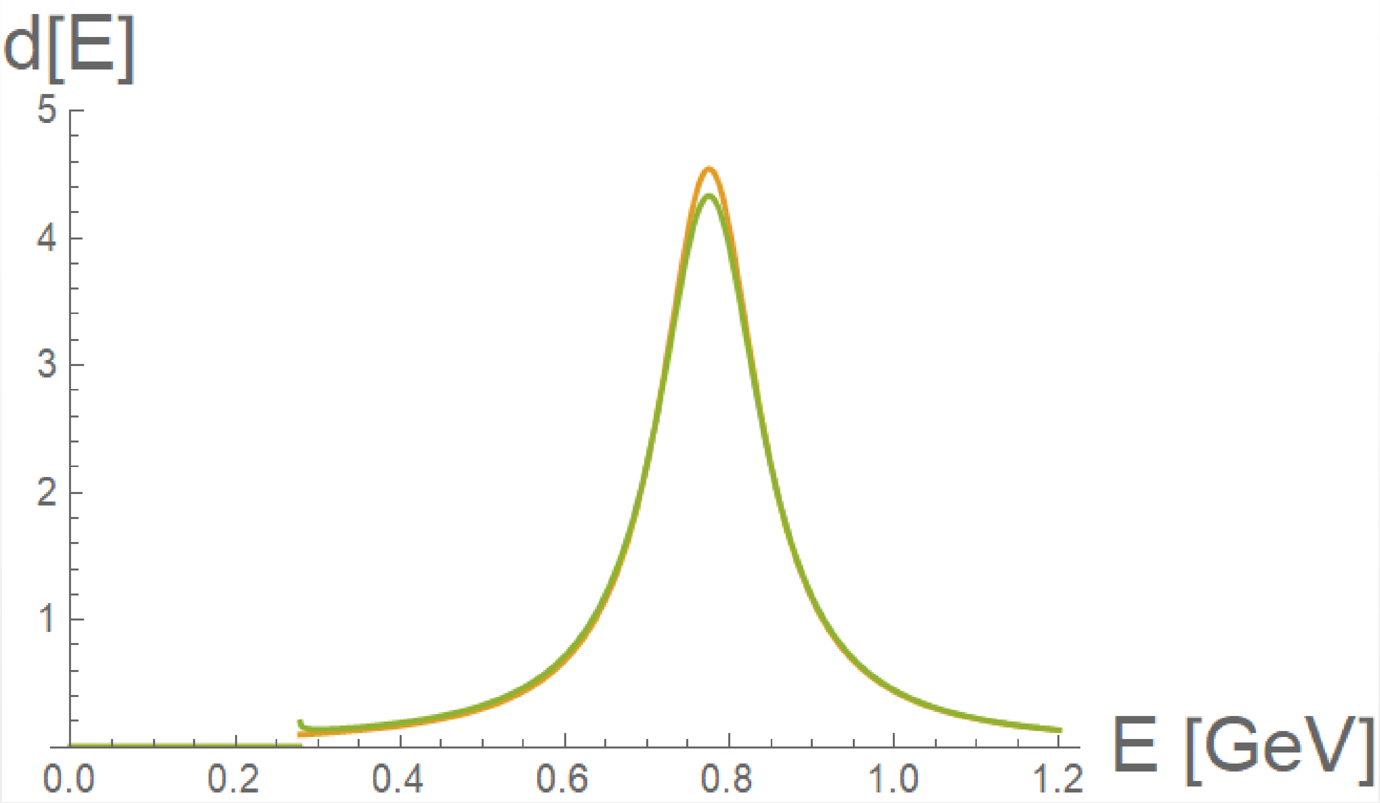
w1(t) is the probability that the decay has occurred in the first channel between (0,t)

$$\sum_{i=1}^{N} w_i = 1 - p(t)$$

$$w_i(t) = \int_{E_{th,i}}^{\infty} \mathrm{dE}\frac{2E^2\Gamma_i(E)}{\pi}\left|\int_{E_{th,1}}^{\infty} \mathrm{dE'} d_S(E')\frac{e^{-iE't}-e^{-iEt}}{E'^2-E^2}\right|^2$$

Thanks!

BW with threshold **properly done**



Comparision with ‘naive’ BW with threshold

Relativistic Sill

Let us consider a resonance with mass M decaying into twoparticles:

$$E_{th} = m_1 + m_2 = \sqrt{s_{th}} \qquad s_{th} = E_{th}^2$$

We **assume** that:

$$\operatorname{Im}\Pi(s) = \sqrt{s - s_{th}}\,\tilde{\Gamma}\,\theta(s - s_{th})$$

$$\Gamma M = \tilde{\Gamma}\sqrt{M^2 - E_{th}^2}$$

Decay width as function of the energy:

$$\Gamma(s) = \frac{\sqrt{s - s_{th}}}{\sqrt{s}}\tilde{\Gamma}$$

Note, it saturates for large s

Relativsitic Sill

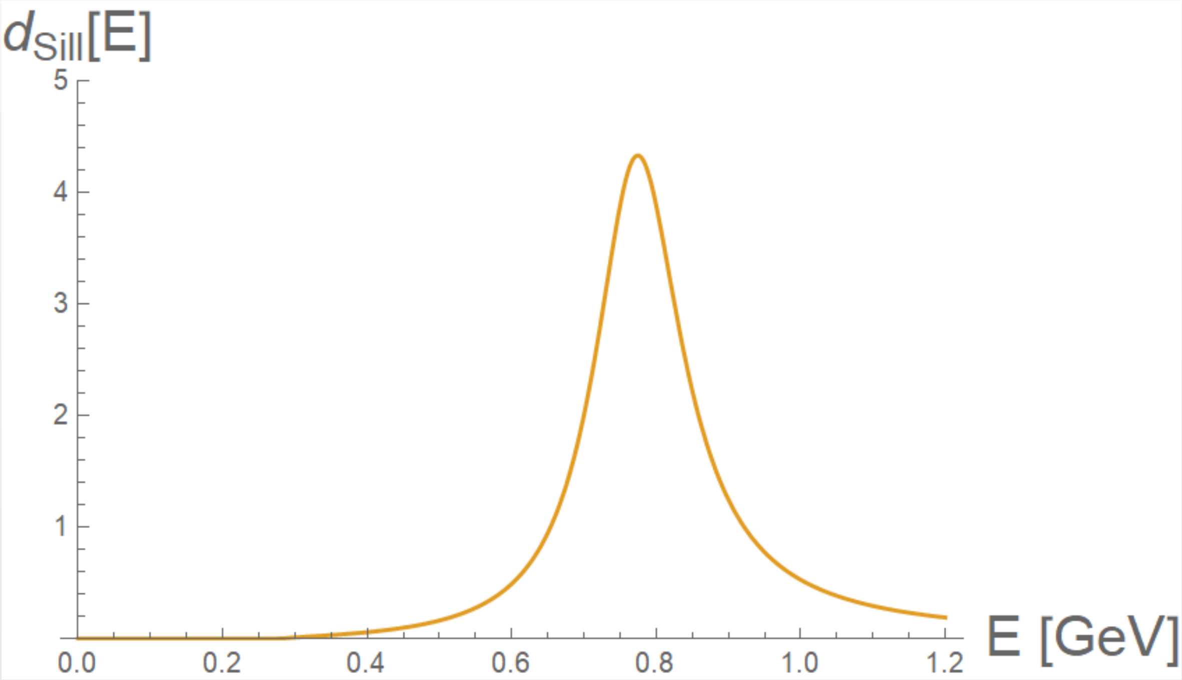
$$\Pi(s) = i\tilde{\Gamma}\sqrt{s - s_{th}}$$

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon}$$

$$\begin{aligned} d_S(s) &= -\frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon} \\ &= \frac{1}{\pi}\frac{\sqrt{s - s_{th}}\,\tilde{\Gamma}}{(s - M^2)^2 + (\sqrt{s - s_{th}}\,\tilde{\Gamma})^2}\theta(s - s_{th}) \end{aligned}$$

Relativistic Sill

$$d_S(E) = d_S^{\text{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + \left(\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}\right)^2} \theta(E - E_{th})$$



Sill for the rho-meson

Comments

$$s_{pole} = M^2 - \frac{\tilde{\Gamma}^2}{2} - i\sqrt{(M^2 - s_{th})\tilde{\Gamma}^2 + \frac{\tilde{\Gamma}^4}{4}} \, .$$

Note, for $\tilde{\Gamma}^2$ sufficiently smaller than $M^2 - s_{th}$, the pole of s can be approximated as

$$s_{pole} \simeq M^2 - i\sqrt{(M^2 - s_{th})}\tilde{\Gamma} = M^2 - i M \Gamma \, ,$$

The normalization

$$\int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S^{\mathrm{Sill}}(E) = 1$$

for any E_{th}, M, and $\tilde{\Gamma}$ is a consequence of the proper treatment of the real part of the loop

Beyond Breit-Wigner

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

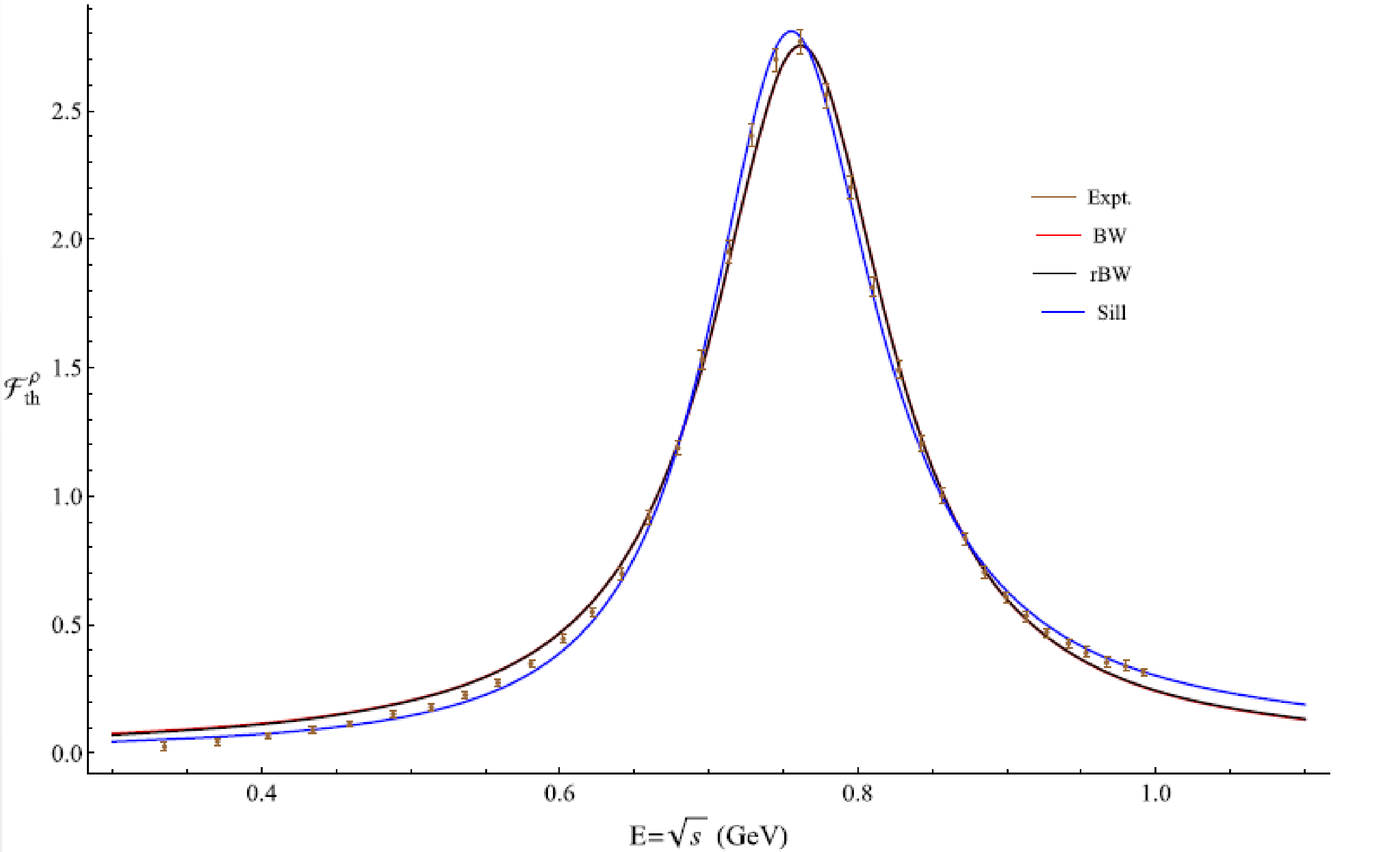
ArXiv: 2106.03749

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2-M^2)^2 + (M\Gamma)^2} \theta(E)$$

$$d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2-M^2)^2 + (\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma})^2} \theta(E - E_{th})$$

Rho meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$

More than a single channel

The extension to the N channels is straightforward:

$$G_S(s) = \frac{1}{s - M^2 + i \sum_{k=1}^{N} \tilde{\Gamma}_k \sqrt{s - s_{k,th}} + i\varepsilon}$$

with

$$\tilde{\Gamma}_k = \Gamma_k \frac{M}{\sqrt{M^2 - E_{k,th}^2}} \text{ and}$$

$$s_{1,th} = E_{1,th}^2 \leq s_{2,th} \leq \ldots \leq_{N,th} = E_{N,th}^2 \,.$$

$$d_s^k(s) = \frac{1}{\pi} \frac{\sqrt{s - s_{\mathrm{th,k}}}\ \tilde{\Gamma}_k}{(s - M^2 - \sum_{i=1}^{Q} \sqrt{s_{\mathrm{th,i}} - s}\ \tilde{\Gamma}_i)^2 + \sum_{i=Q+1}^{N} (\sqrt{s - s_{\mathrm{th,i}}}\ \tilde{\Gamma}_i)^2} \theta(s - s_{\mathrm{th,k}})$$

where, $s_{\mathrm{th,k}}$ is the k^{th} threshold, and the integer Q is such that, for all $i < Q$, $s_{th,i} < s_{th,k}$

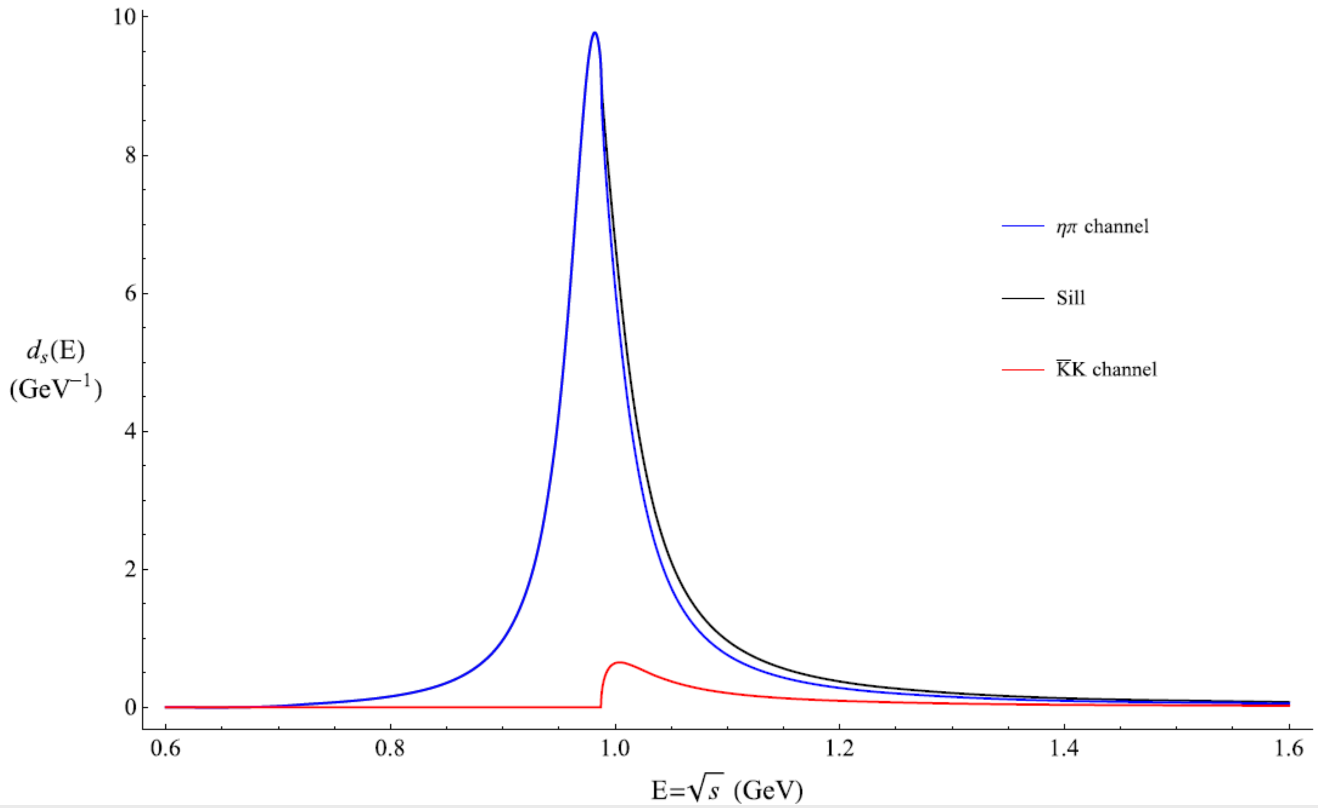
Two-channel case

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}_1\sqrt{s - s_{1,th}} + i\tilde{\Gamma}_2\sqrt{s - s_{2,th}} + i\varepsilon},$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \begin{cases} \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}}}{(s-M^2)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}})^2} & \text{for } s > s_{2,th} \\ \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}}{\left(s-M^2-\tilde{\Gamma}_2\sqrt{s_{2,th}-s}\right)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}})^2} & \text{for } s_{1,th} \le s \le s_{2,th} \\ 0 & \text{for } s < s_{1,th} \end{cases}$$

a0(980) example

Fig. 8 The Sill distribution of the $a_0(980)$ and the $\eta\pi$ and $\bar{K}K$ channels. The non-BW form due to the KK threshold is evident



Other recent Sill application

PHYSICAL REVIEW D **106**, 094009 (2022)

XYZ spectroscopy at electron-hadron facilities. II. Semi-inclusive processes with pion exchange

D. Winney,[1,2,*] A. Pilloni,[3,4,†] V. Mathieu,[5,‡] A. N. Hiller Blin,[6,7] M. Albaladejo,[8] W. A. Smith,[9,10] and A. Szczepaniak[9,10,11]

(Joint Physics Analysis Center)

description of the πp mass distribution in the Δ mass region:

$$d_{\Delta\to\pi p}(M^2) = \frac{1}{\pi}\frac{\rho(M^2)\tilde{\Gamma}_\Delta}{[M^2 - m_\Delta^2]^2 + [\rho(M^2)\tilde{\Gamma}_\Delta]^2}, \quad (39)$$

with $\rho(M^2) = \sqrt{M^2 - M_{\text{min}}^2}$ and $\tilde{\Gamma}_\Delta = \Gamma_\Delta m_\Delta / \rho(m_\Delta^2)$. Interestingly, this function is normalized across the mass

Experimental confirmation of non-exponential decay (1)

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

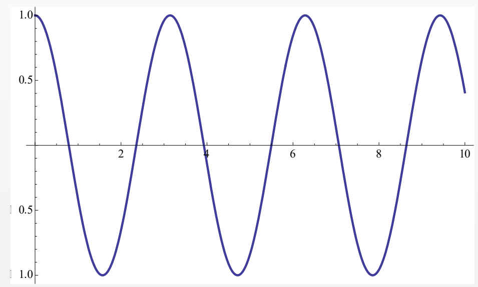
Steven R. Wilkinson, Cyrus F. Bharucha, Martin C. Fischer, Kirk W. Madison, Patrick R. Morrow, Qian Niu, Bala Sundaram* & Mark G. Raizen

Department of Physics, The University of Texas at Austin, Austin, Texas 78712-1081, USA

An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Cold Na atoms in a optical potential

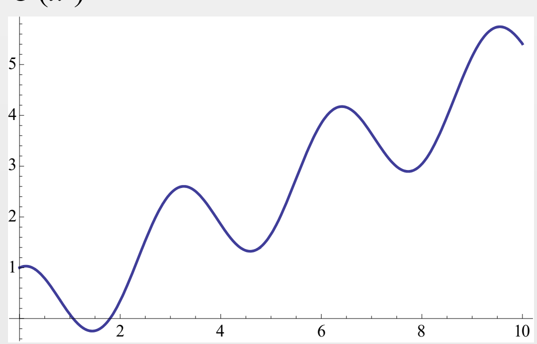
$V(x,0)$



x [a.u.]

$$V(x,t) = V_0 \cos\left(2k_L x - k_L a t^2\right)$$

$U(x')$



x' [a.u.]

$$x' = x - \frac{1}{2} a t^2$$

$$U(x') = V_0 \cos\left(2k_L x'\right) + Max'$$

Normalization and its heuristic justification

One can show that under quite general conditions

$$\int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E) = 1$$

Brief QM recall

$$|S\rangle = \int_{E_{th}}^{\infty} \mathrm{dE}\, a_S(E)\, |E\rangle$$

Eigenstates of Hamilton H

$$H\,|E\rangle = E\,|E\rangle$$

The quantity $d_S(E) = |a_S(E)|^2$ is the 'spectral function'

$$1 = \langle S|S\rangle = \int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E)$$

$$a_S(t) = \langle S\,|e^{-iHt}|\,S\rangle = \int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S(E) e^{-iEt}$$