Liquid crystal topological defects – particle physics resemblance

Repair classical EM (Faber):

1) $\int_0^\infty E^2 r^2 dr \propto \int_0^\infty r^{-4} r^2 dr = \infty$

Deform/**regularize charge to finite energy**

(**Higgs-like potential** allowing deformation)

2) **Gauss law** charge ∈ ℝ, in nature ∈ ℤ*e*

Interpret **curvature as electric field** to count

(**quantized**) **topological charge** with Gauss law

Skyrmion-like with SO(3) ellipsoid vacuum

3 distinguishable axes like biaxial nematic

Use **real symmetric tensor field** $M = ODO^T$

~**Higgs** e.g. $V = \sum_i (\lambda_i - \Lambda_i)^2$ for D **shape**

Getting **3 leptons**, baryons, nuclei …

with **unified wave-like vacuum dynamics**:

EM >> quantum phase >> GEM

EM	quantum phase	GEM
1st axis rotations	~Klein-Gordon/Dirac? 1st axis twists (~Berry)	0th axis in 4D tiny tilts

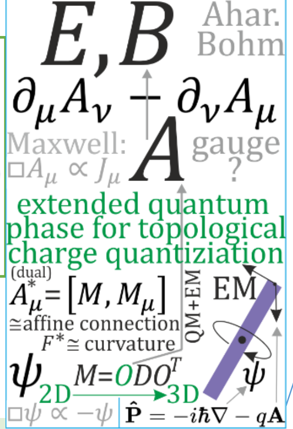
$$\mathcal{L}_{QED} = \bar\psi(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

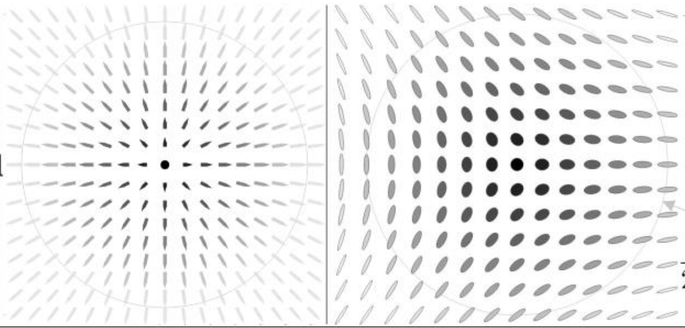
spin/particle + phase

∂_μ?

Lagrangian/Hamiltonian

Jarek Duda article demo video github





2D charge (+1) as director hedgehog

regularized $\vec n \to 0$ by Higgs pot. $V(\vec n) = (\|\vec n\|^2 - 1)^2$

2 axes in 2D

regularized $V(M) = \sum_i (\lambda_i^M - \Lambda_i)^2$ to finite energy

(in 3D: vortex/fluxon)

2D charge/spin = -1/2

$$\frac{1}{2\pi}\oint_L (n_2 n_1' - n_1 n_2')\, dL$$

$\vec n \equiv \vec n(x)$ $\|\vec n\| = 1$

electric/topological (3D winding number) = **3D charge** in **3D**

Gauss law, Jacobian $\mathcal{S} \to S^2$, curvature

$$\oint_{\mathcal{S}} E \cdot dA = \frac{e_0}{4\pi} \oint_{\mathcal{S}(u,v)} du\, dv\ (\partial_u \vec n \times \partial_v \vec n) \cdot \vec n$$

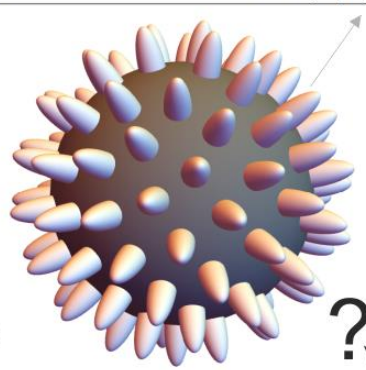
uniaxial nematic

unitary vector in 3D of director field $\vec n(x)$

1 distingused axis

charge + Coulomb

Maxwell equations from director dynamics



biaxial nematic

1 → **3 in 3D**

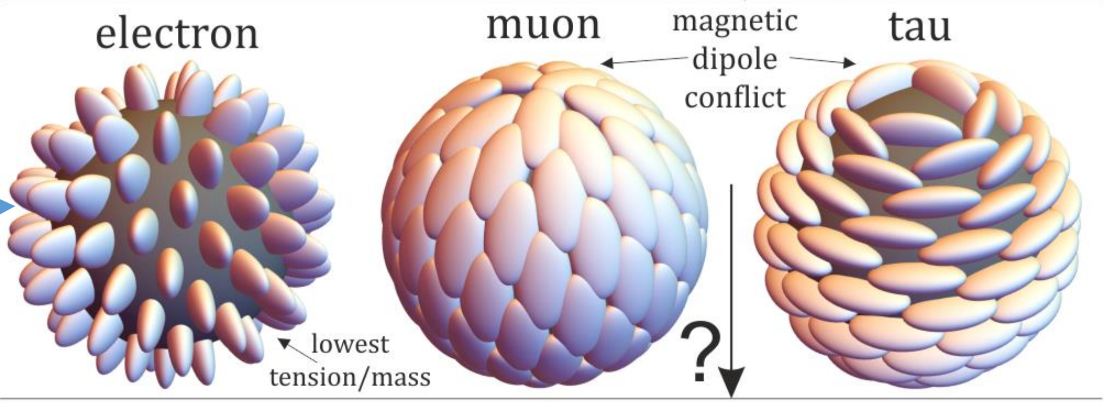
distinguished axes „uniaxial + quantum phase"

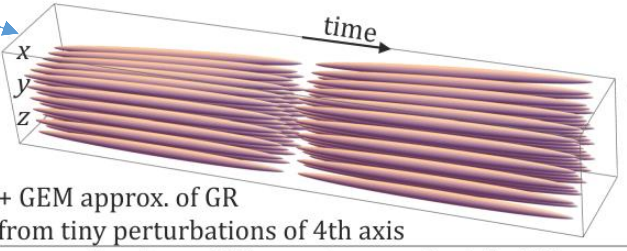
(for „pilot wave")

field of real symmetric M

3 charges with magnetic dipoles

?





+ GEM approx. of GR from tiny perturbations of 4th axis

3 → **4 axes** in **4D spacetime**

4th: local time direction

2nd set of **Maxwell equations** for its tiny perturbations: much weaker (longer axis), no mass/energy quantization

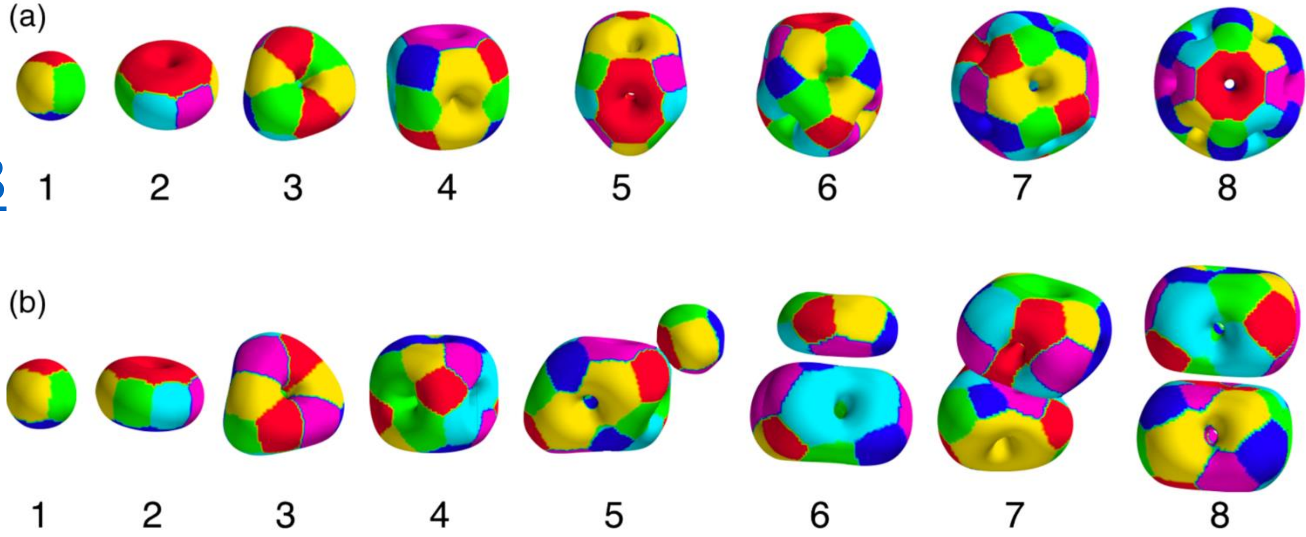
energy density: **EM** | quantized charge | **uniaxial** curvature (Higgs potential) (regularization) | **biaxial/general**, M - matrix curvature (real, symmetric)

$$\mathcal{H} \sim \sum_{\mu,\nu=0}^{3} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \to \sum_{\mu,\nu=0}^{3} \|\partial_\mu \vec n \times \partial_\nu \vec n\|^2 + (\|\vec n\|^2 - 1)^2 \to \sum_{\mu,\nu=0}^{3} \|\partial_\mu A_\nu - \partial_\nu A_\mu\|_F^2 + V(M)$$

$A_\mu = M M_\mu - M_\mu M$

Popular skyrmion models

Solid state, nucleus: PRL 2018



U – tensor field (of matrix)

$\Gamma_i = \partial_i U \, U^{-1}$ local rotation

$E_{kin} = c_1 \sum_i \mathrm{Tr}(\Gamma_i \Gamma_i) + \cdots$

$E_{pot} \propto \mathrm{Tr}(\mathbf{1} - U)$ for unitary – single minimum $U = \mathbf{1}$

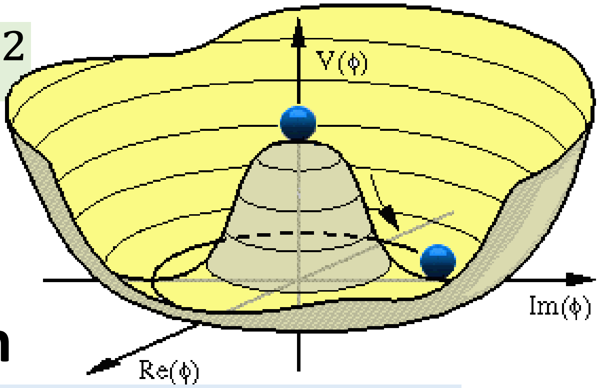
Vacuum (far from particles) filled with $U \approx \mathbf{1}$, only short-range interaction!

No EM, charge (**proton = neutron**), no long-range e.g. Coulomb interaction

(**proton** lighter than **neutron**)

For long-range: use **topologically nontrivial vacuum** (minimum of potential)

E.g. **Higgs potential** “Mexican hat”: $V(\vec{n}) = (||\vec{n}||^2 - 1)^2$



- zero is not minimum (inflation, charge regularization)
- **Dynamics in minimum** corresponds to massless particles (Goldstone bosons), like **electromagnetism**

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu,\ \mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu}$ EM with quantized charge (Faber)

Liquid crystal **long-range interactions** due to **nontrivial vacuum** like for **Higgs** $V(\vec{n}) = (|\vec{n}|^2 - 1)^2$

$F\sim 1/D$:"**Annihilation** dynamics of topological defects induced by microparticles in nematic liquid crystals" Soft Matter

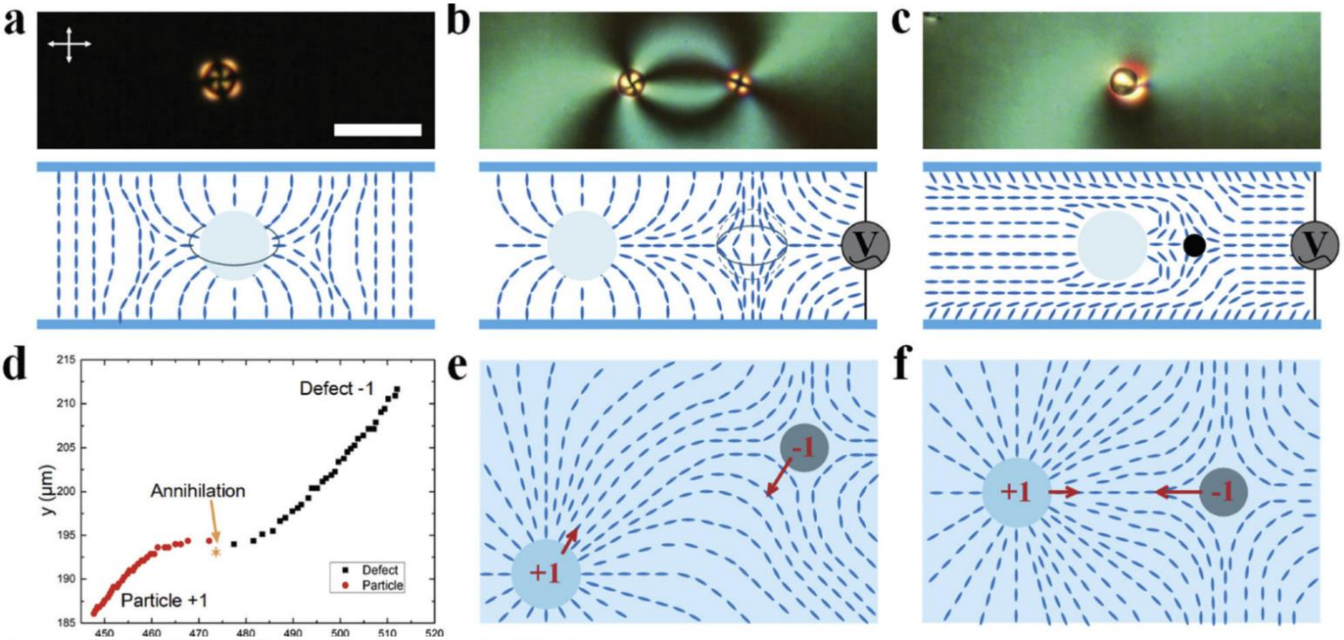
Coulomb: "**Coulomb**-like interaction in nematic emulsions induced by external torques exerted on the colloids" PRE

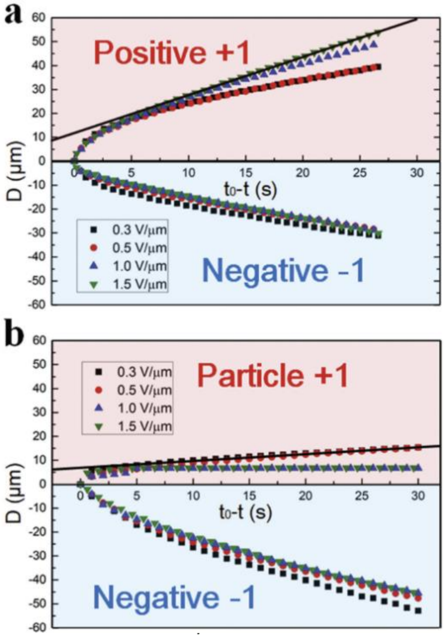
"**Coulomb**-like elastic interaction induced by symmetry breaking in nematic liquid crystal colloids" Scientific Reports

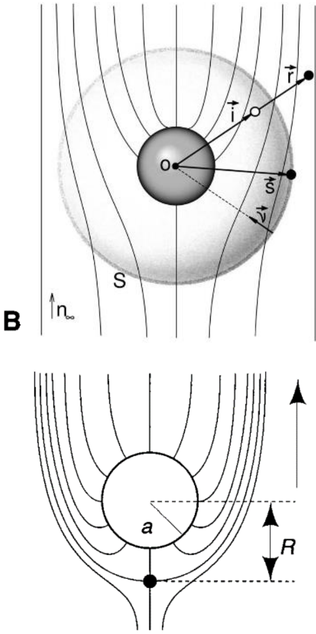
dipole-dipole: "Novel Colloidal Interactions in Anisotropic Fluids" Science

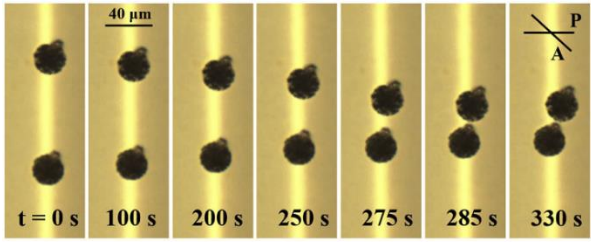
quadrupole-quadrupole: "**Long-range forces** and aggregation of colloid particles in a nematic liquid crystal" PRE

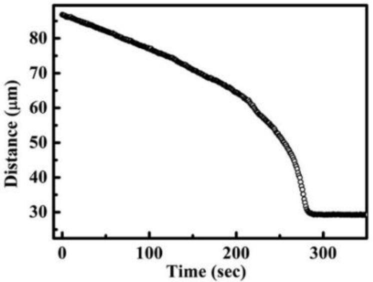
Quantum computers on liquid crystal topological defects? (Science Advances)

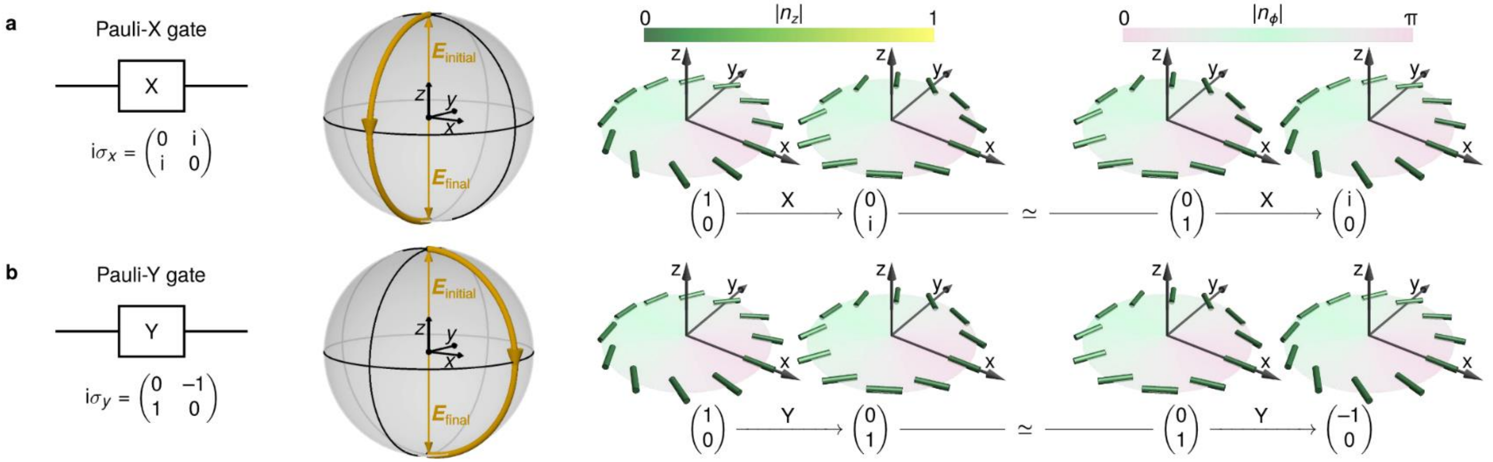




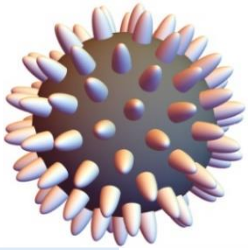








Standard liquid crystal models (review1, review2)

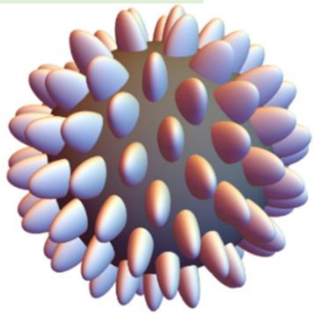


Uniaxial: $\vec{n} \in S^2$ (Higgs potential?), **Oseen-Frank** model (1958) :

$$E[\vec{n}] = \int_\Omega K_1|\nabla \cdot \vec{n}|^2 + K_2|\vec{n} \cdot (\nabla \times \vec{n})|^2 + K_3|\vec{n} \times (\nabla \times \vec{n})|^2 + (|\vec{n}|^2 - 1)^2$$

Biaxial: **Landau-de Gennes** (1974, Nobel Prize in 1991) of preferred shape

$(Q \equiv)$ $M = \sum_{i=1}^3 \lambda_i \vec{n}\,\vec{n}^T$ preferred $(\lambda_1, \lambda_2, \lambda_3)$: $V(M)$ Higgs-like potential:



$$E[M] = \int_\Omega |\nabla M|^2 + \frac{A}{2}\mathrm{Tr}(M^2) - \frac{B}{3}\mathrm{Tr}(M^3) + \frac{C}{4}\left(\mathrm{Tr}(M^2)\right)^2$$

(or $V(M) = \sum_i (\lambda_i - \Lambda_i)^2$ or $\sum_i \left(\mathrm{Tr}\left(M^i\right) - c_i\right)^2$ tough choice!)

→**particles?**: Higgs-like potential, e.g. as above, with **Coulomb, EM-like**:

$$\boxed{\boldsymbol{\mathcal{L}_{EM} = F_{\mu\nu}F^{\mu\nu}} \quad \text{for} \quad \boldsymbol{F_{\mu\nu} \propto R_{\mu\nu} = \Gamma_\mu \times \Gamma_\nu} \qquad \boldsymbol{\Gamma_\nu = (\partial_\nu \vec{n}) \times \vec{n}}}$$

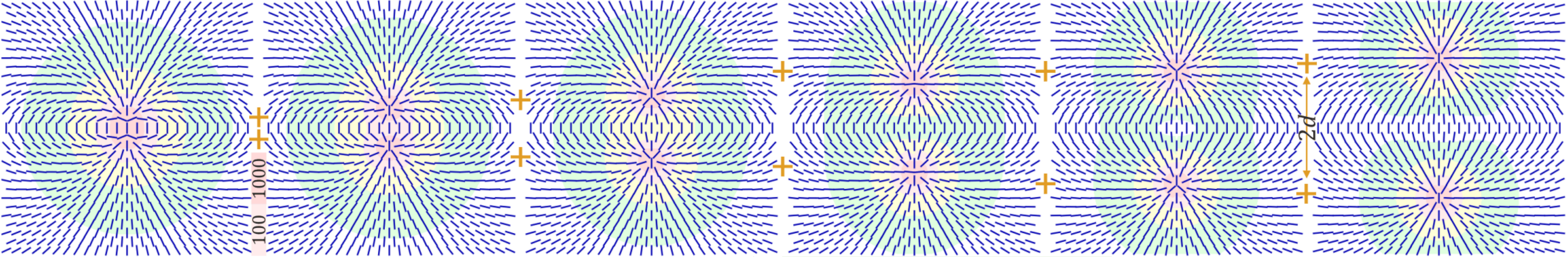
$R_{\mu\nu}$ curvature: Gauss law counts (quantized) topological charge

with **kinetic**: $\mathcal{L} \sim \sum_{i=1}^3 ||\partial_i \vec{n} \times \partial_0 \vec{n}||^2 - \sum_{1 \le i < j \le 3} ||\partial_i \vec{n} \times \partial_j \vec{n}||^2 + V(\vec{n})$

Biaxial: $\boxed{\boldsymbol{R_{\mu\nu}} = \left[\boldsymbol{\partial_\mu M, \partial_\nu M}\right]_{(-1,1,1,1)}}$ $\quad M = ODO^T \quad D \approx (g, 1, \epsilon, 0)$

Lorentz-invariant Lagrangian mechanics leading to **electromagnetism**

plus ~**Klein-Gordon** for twist, gravito-electromagnetism for 0th axis



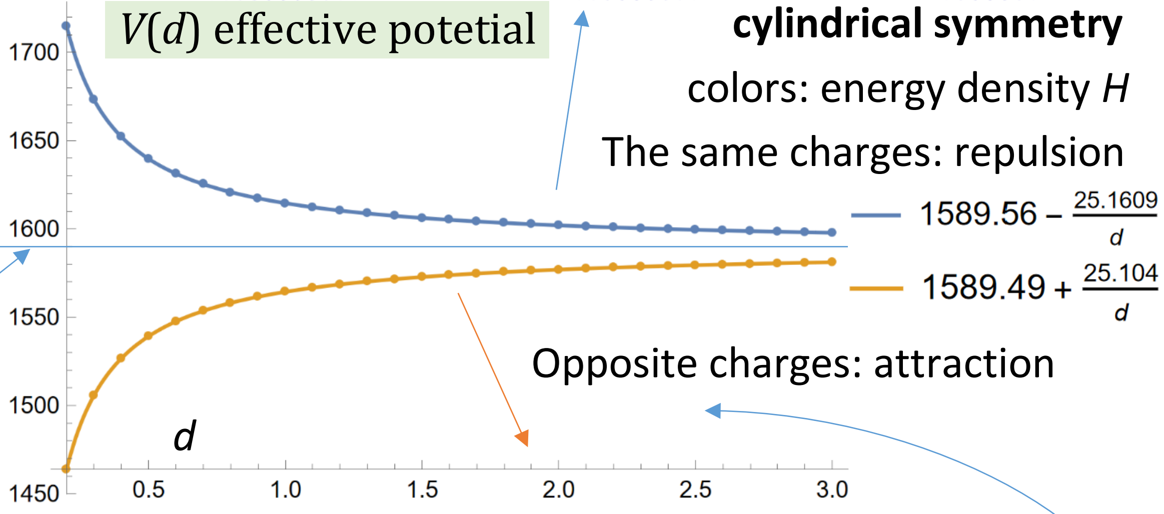
Field energy: ~Coulomb potent.

$$E \approx \frac{m_0}{\sqrt{1-v_1^2}} + \frac{m_0}{\sqrt{1-v_2^2}} + V(d)$$

cutoff ϵ around singularities:

to be **regularized** to rest masses

Lorentz inv.: SRT scaling, magnetism



$V(d)$ effective potetial

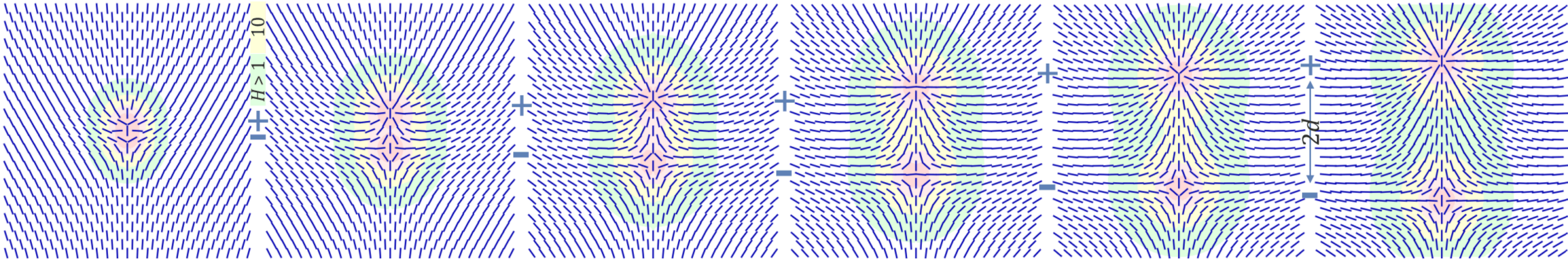
cylindrical symmetry

colors: energy density H

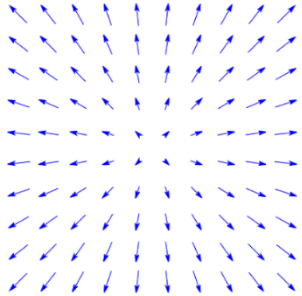
The same charges: repulsion

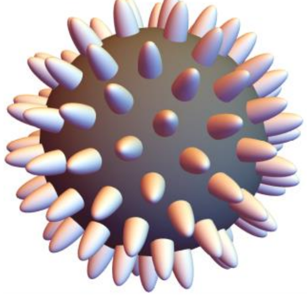
Opposite charges: attraction

```
cos = 1 + (z - d) / Sqrt[(z - d)^2 + r^2] - (z + d) / Sqrt[(z + d)^2 + r^2]; (*Manfried Faber dipole ansatz*)
n = {Sqrt[1 - cos^2] x / r, Sqrt[1 - cos^2] y / r, cos} /. r → Sqrt[x^2 + y^2]; (* cylindrical symmetry *)
M = KroneckerProduct[n, n]; dM = {D[M, x], D[M, y], D[M, z]}; (*vector n → matrix M field*)
H = Simplify[Sum[Total[(dM⟦i⟧.dM⟦j⟧ - dM⟦j⟧.dM⟦i⟧)^2, 2], {i, 2}, {j, i + 1, 3}]]; (*Hamilitonian*)
Es = Table[{d, NIntegrate[4 Pi * x (H /. {y → 0}) * Boole[x^2 + (z - d)^2 > 0.001], (* 0.001 cutoff *)
      {x, 0, Infinity}, {z, 0, Infinity}]}, {d, 0.1, 3, 0.1}]; (*integrate H: total field energy*)
ft = Fit[Es, {1, 1 / d}, d]; Show[Plot[ft, {d, 0.2, 3}], ListPlot[Es]] (*fit Coulomb potentential*)
```



Regularization to finite energy e.g. for “hedgehog”:





Asymptotically (vacuum) $||\vec{n}|| \approx 1$, but $\vec{n}(0) = 0$

thanks to **Higgs**-like potential, e.g. $V(\vec{n}) = (\|\vec{n}\|^2 - 1)^2$

↻: $\vec{\Gamma}_i = (\partial_i \vec{n}) \times \vec{n} \quad = \frac{1}{r} \quad ||\vec{n}||^2 \to \frac{1}{r}$

↻ $\Gamma \propto r^{-1}$ $\quad R \propto r^{-2}$

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu \quad \to \frac{1}{r^2}$ electric field in 3D

$$^*\vec{F}_{\mu\nu} = \frac{-e_0}{4\pi\epsilon_0 c}\vec{R}_{\mu\nu} \sim \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

$\mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}$ → electromagnetism in **vacuum**

Gauss law counting topological charge: $Q_{el}(\mathcal{S}) = \frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} du\,dv\ (\partial_u \vec{n} \times \partial_v \vec{n}) \cdot \vec{n}$

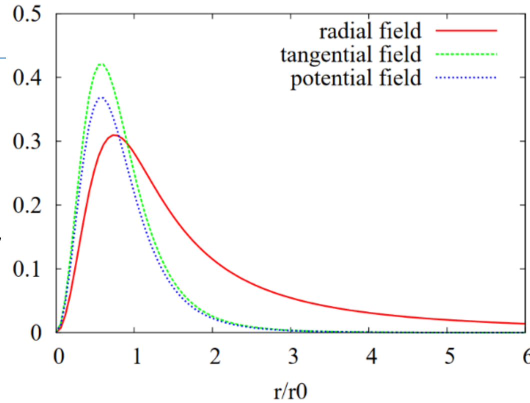
Jacobian of closed surface → S^2: $\det[\vec{n}, \vec{n}_\mu, \vec{n}_\nu] = \vec{n} \cdot (\vec{n}_\mu \times \vec{n}_\nu)$

No parton structure (!) for **electron**,

only field deformation not to exceed 511keVs,

“infinity subtraction from renormalization” – subtracted energy density

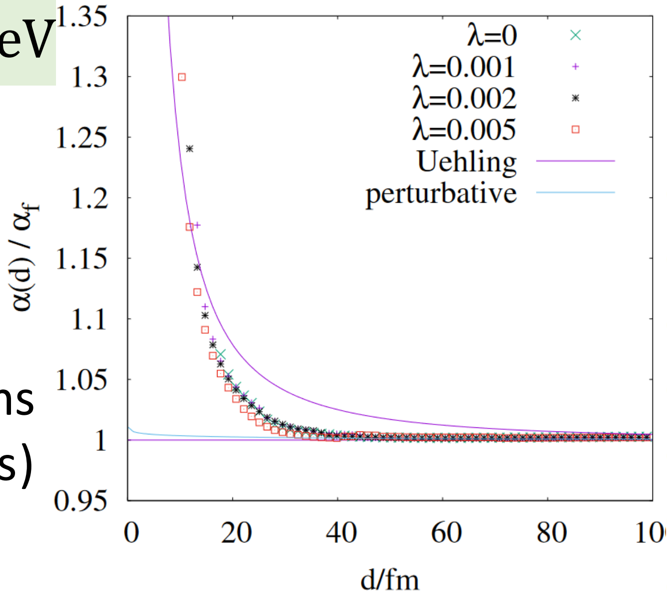
Faber:



$$\int_{\sim 1.4\text{fm}}^{\infty} \frac{1}{2}|E|^2 4\pi r^2 dr = 511\text{keV}$$

Experimental effects of finite size?

Coulomb deformation, **Faber**:

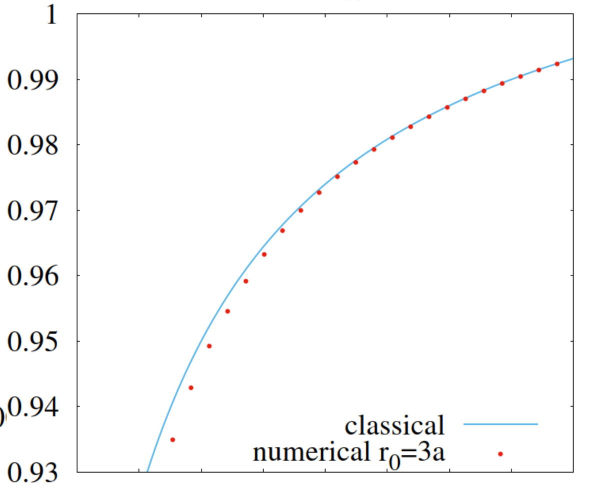


Running coupling:

$\alpha \approx \frac{1}{137} \to \approx \frac{1}{127}$ in 90 GeVs

To hide finite size in Feynman diagrams

(+ renormalization to remove infinities)



Experimental boundaries for size of electron?

Dehmelt 1988, Penning trap (Nobel in 1989): $R < 10^{-22}m$

extrapolating from g-factor: "(…) electron as formed by very tightly binding together **three smaller and much heavier new fermions** [Brodsky, Drell, 1980] (…)"

Neutron (udd): $g \approx -3.8 \qquad < r_{\mathrm{n}}^2 > \approx -0.1\ \mathrm{fm}^2$

Classically: $g = \frac{2m}{q}\frac{\mu}{L} = \frac{2m}{q}\frac{\int AdI}{\omega I} = \frac{m}{q}\frac{\int \rho_q(r)\, r^2 dr}{\int \rho_m(r)\, r^2\, dr}$

Electron-positron cross-section:

Which energy should we use??? (Lorentz contraction!),

~**line** in log-log, $\approx$ 100nb for 1GeV $\qquad \gamma \approx 1000$

Extrapolating to resting: $\gamma = 1 \qquad \sigma \propto \gamma^{-2}$

we get $\approx$ 100mb: $\boldsymbol{r} \approx$ **2 fm**? (energy < 511keV!)

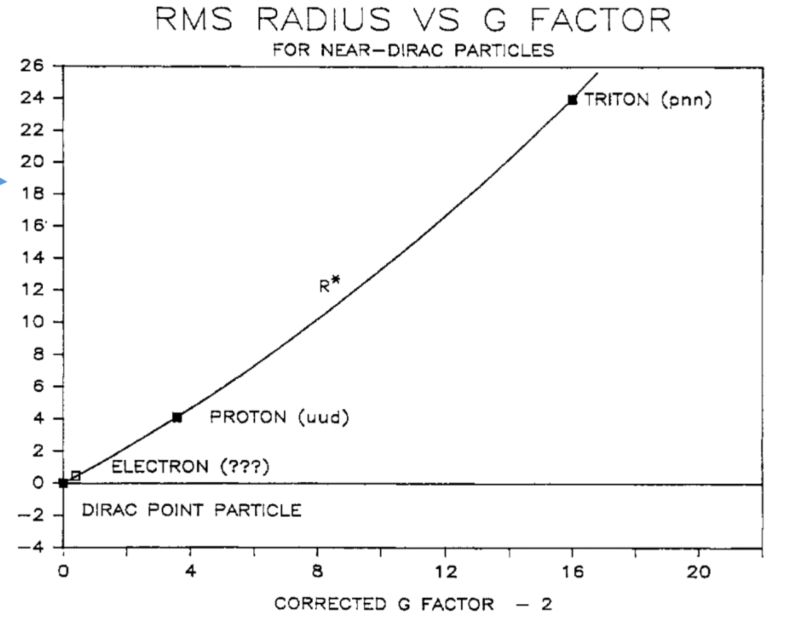
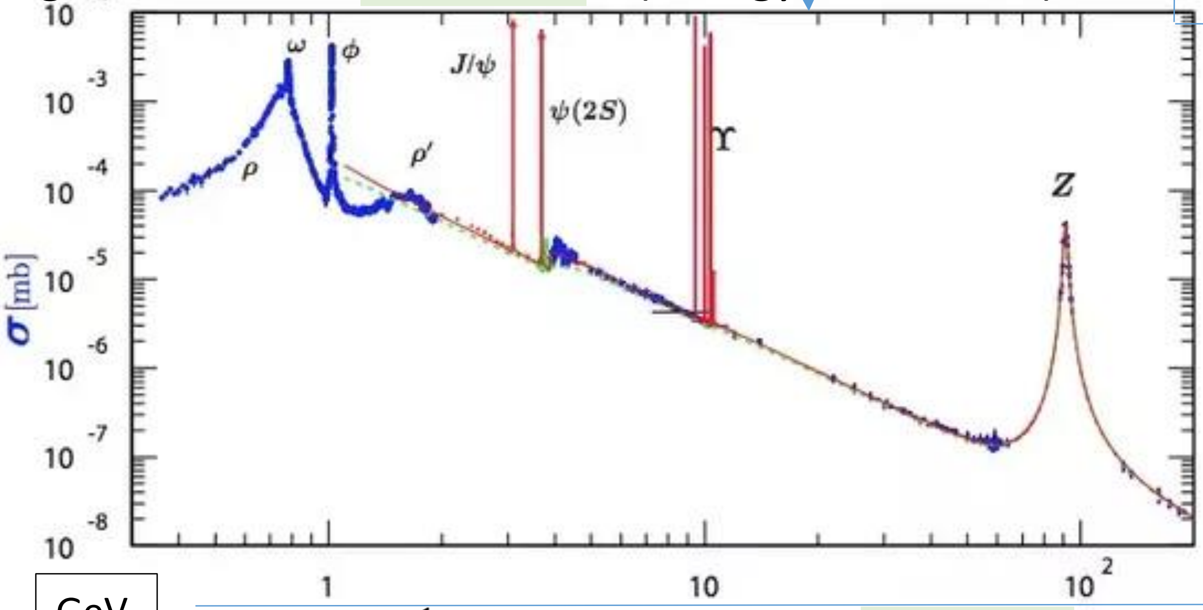


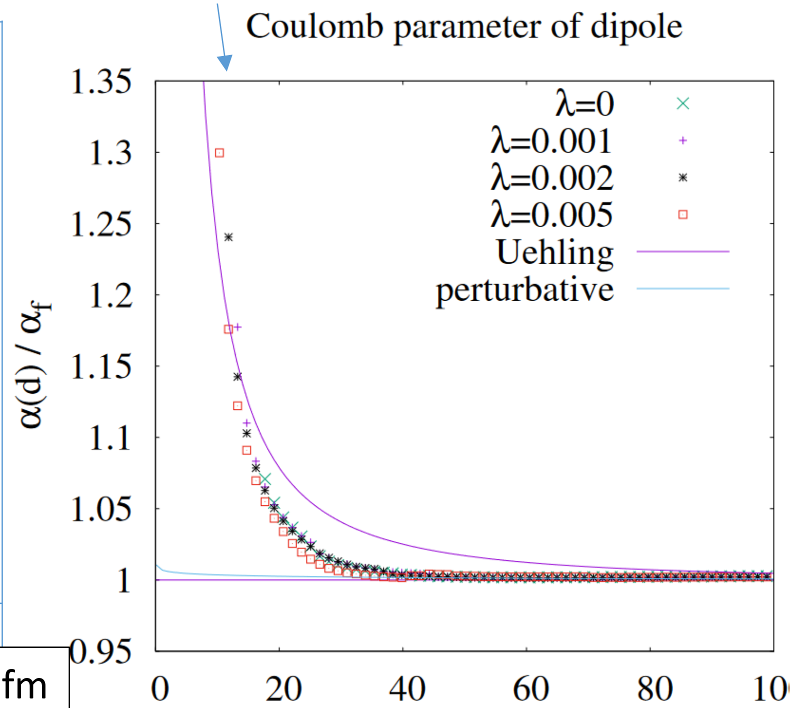
Fig. 8. Normalized RMS radius $R^* = R/\lambda\!\!\!^{-}_C$ vs. corrected *g*-factor minus 2 for near-Dirac particles [3]. A parabola has been fitted to the data points. Recent theories conjecture that the electron, similar to proton and triton, is composed of three smaller fermions. The data point at the origin represents a Dirac point particle of finite arbitrary charge and mass.

Running coupling – deformation of α (Coulomb) **proper in Faber's model**



GeV

$$\int_{\sim1.4\mathrm{fm}}^{\infty} \frac{1}{2}|E|^2 4\pi r^2 dr = 511\mathrm{keV}$$



fm

What about quantum phenomena for (topological) solitons? E.g. fluxons:

Experimental demonstration of Aharonov-Casher **interference** in a Josephson junction circuit, PRB 2012 – for **fluxons**, Aharonov-Casher: for magnetic dipole in electric field

Tunneling and resonant tunneling of **fluxons** in a long Josephson junction, PRB 1997

Aharonov-Bohm type forces between magnetic **fluxons**, PRA 1997 **short-range**

Hydrodynamical classical wave-particle duality object "walking droplets":

Single-Particle **Diffraction** and **Interference** at a Macroscopic Scale, **PRL** 2006

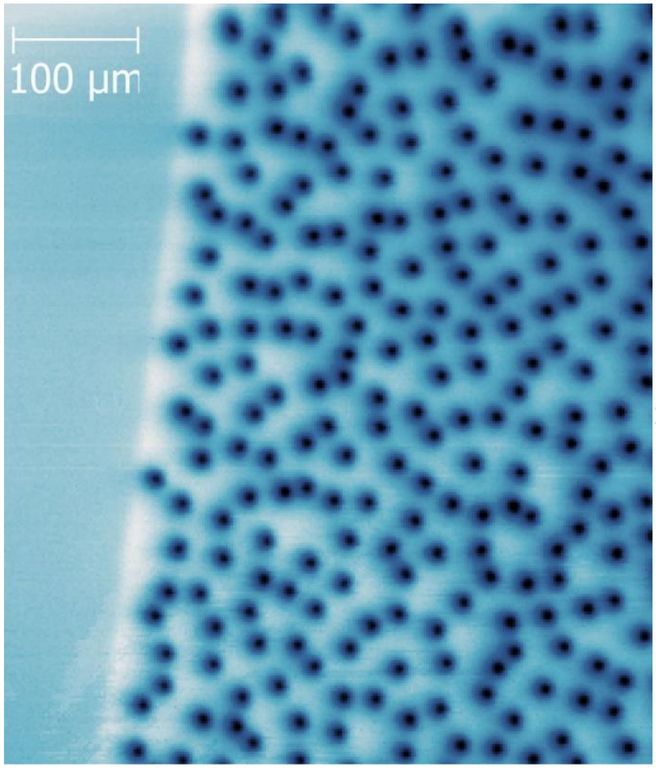
Unpredictable **Tunneling** of a Classical Wave-Particle Association, **PRL** 2009

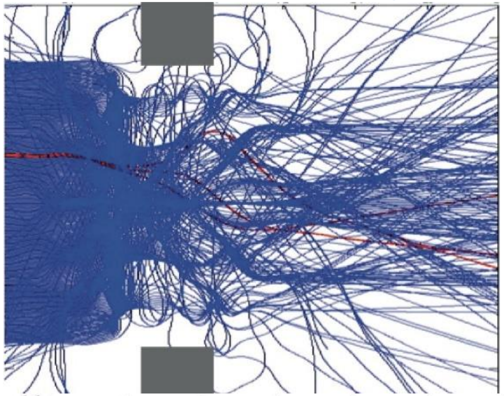
Path-memory induced quantization of classical orbits**,** PNAS 2010

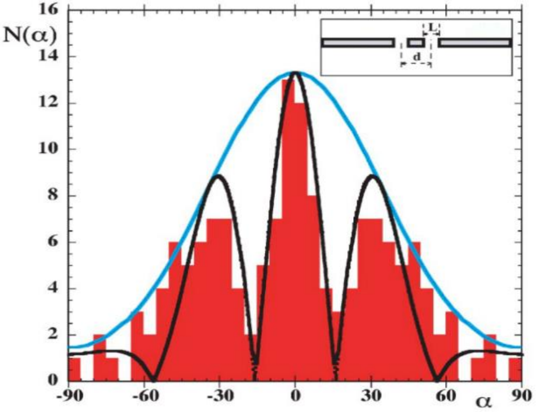
Level Splitting at Macroscopic Scale, PRL 2012 – **Zeeman splitting**

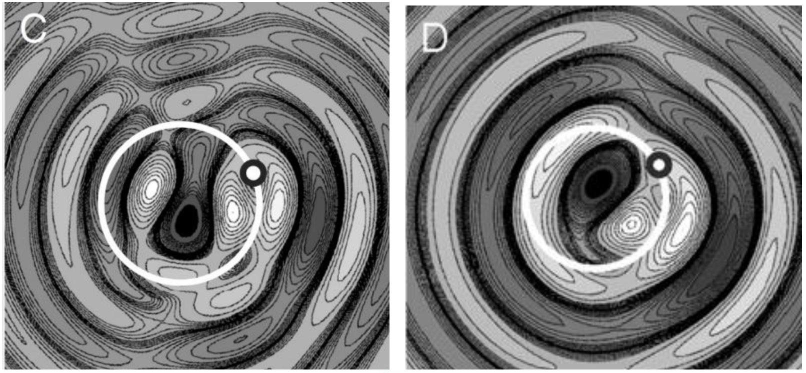
Self-organization into **quantized eigenstates** of a classical wave-driven particle, **Nature** 2014

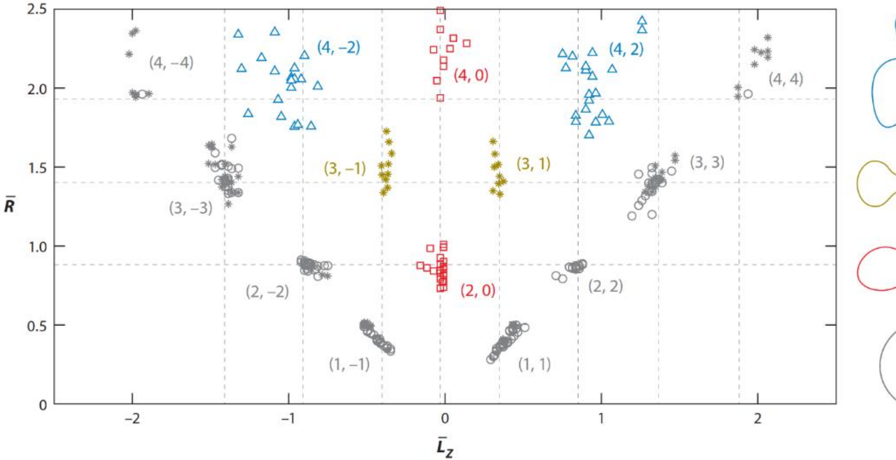
Wavelike statistics from pilot-wave dynamics in a circular corral, PRE 2013



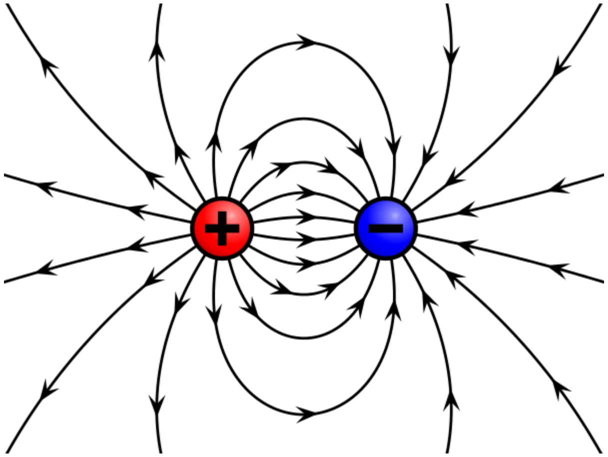


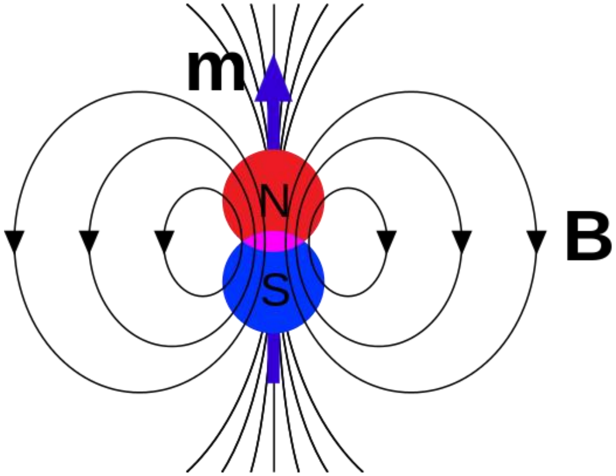


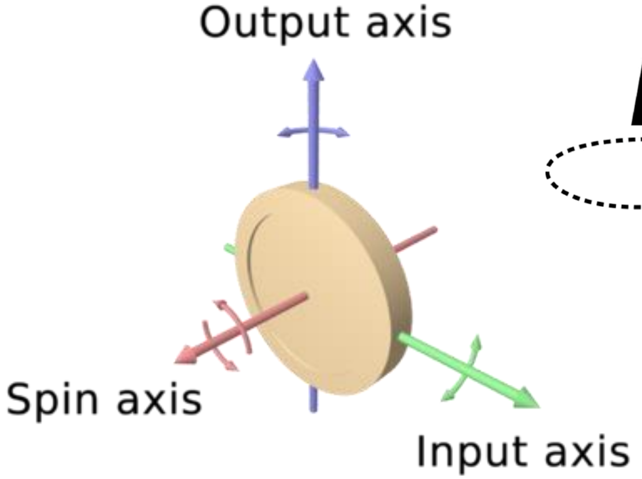


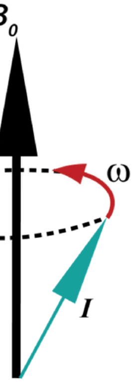


Electron – (at least) a complex configuration of electromagnetic field … Larmor





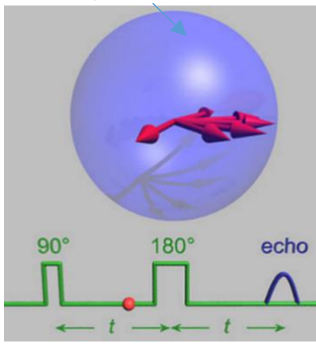




Electric charge $\left(E \propto \frac{1}{r^2}\right)$ + magnetic dipole $\left(B \propto \frac{1}{r^3}, \text{magnets}\right)$ + "gyroscope" $\left(L = \frac{\hbar}{2}\right)$

+ $\approx 10^{21} Hz$ zitterbewegung (observ.)/de Broglie's clock $(E = mc^2 = \hbar\omega)$:

spin echo

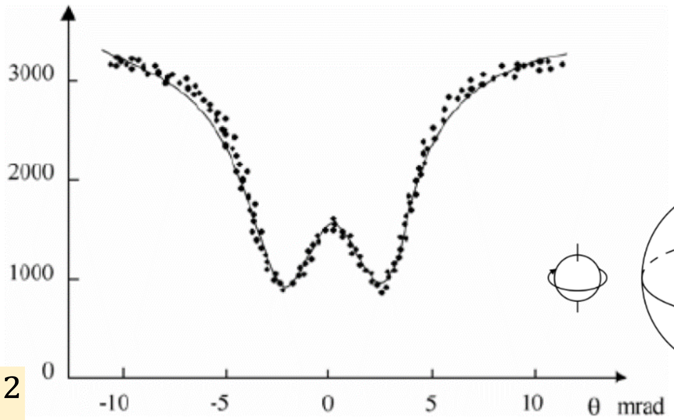


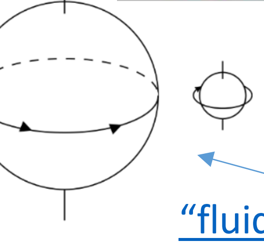
some internal periodic process

Along <110> axis of silicon crystal atomic spacing correspond to 'internal clock' period for $E \approx 81MeV$ electrons – observed resonance:

P. Catillon, N. Cue, M.J. Gaillard, R. Genre, M. Gouanère, R.G. Kirsch, J.-C. Poizat, J. Remillieux, L. Roussel, M. Spighel, *A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling,* Found Phys (2008) 38: 659–664

stationary **Schrödinger**: $\psi = \psi_0 e^{iEt/\hbar}$ for $E = mc^2$



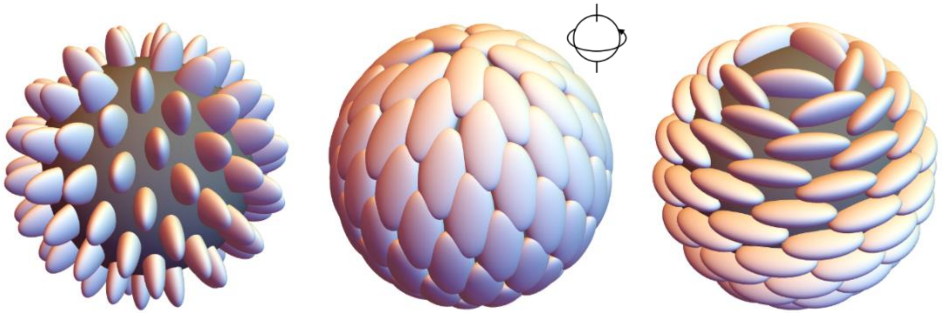


"fluid drag"

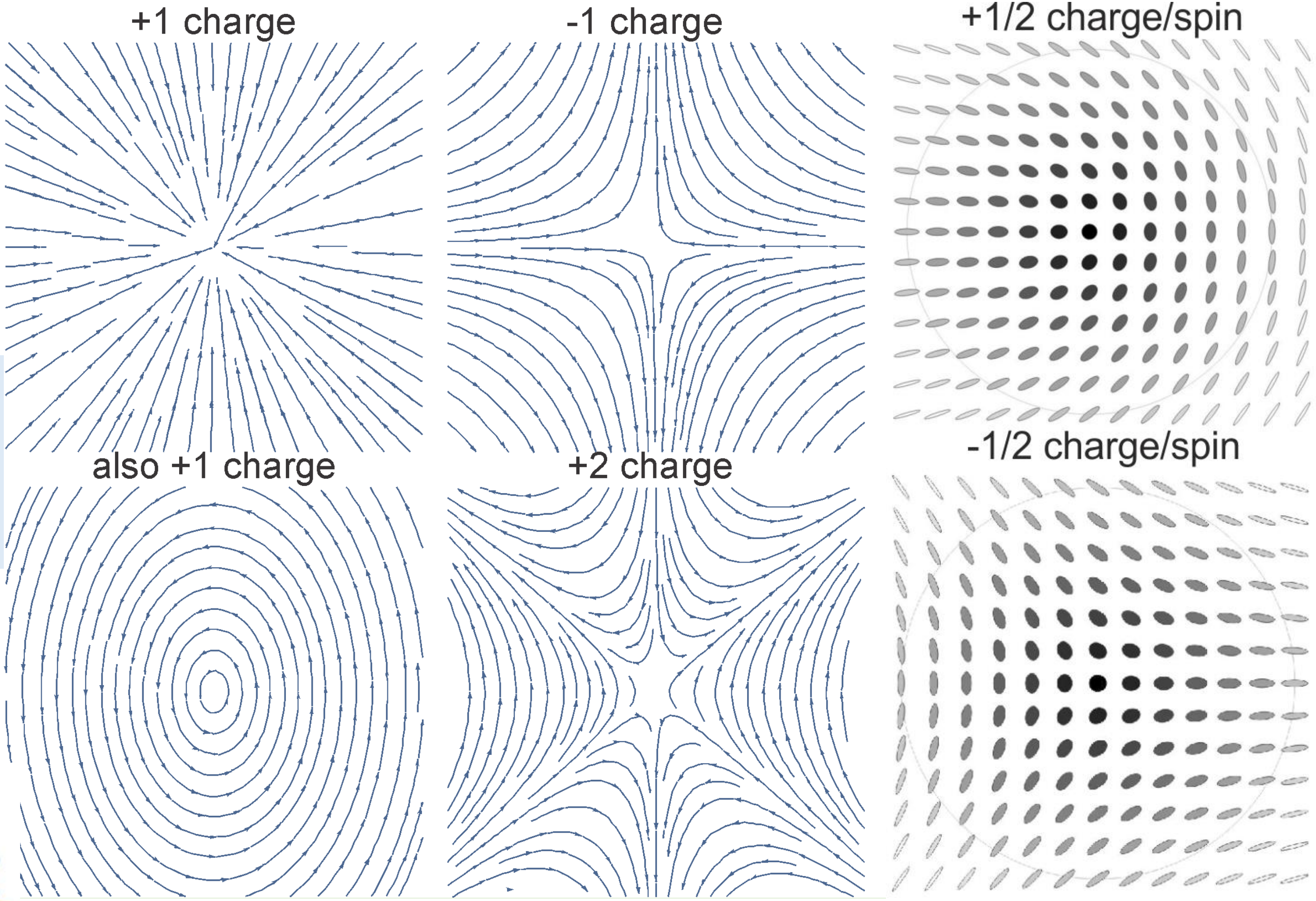
Can we get it with **biaxial nematic hedgehog**?

No naked charge – needed magnetic dipole

How to enforce clock? (leading to pilot wave)



2D topological charge



fluxon – **quant** of magnetic field

resembles **spin** ($\to \mu$) also **½**

$$2\pi k = \Delta\varphi = \frac{q}{\hbar}\oint_{\partial S} A\cdot dl = \frac{q}{\hbar}\oint_{S} B\cdot dS$$

quantum rotation operator

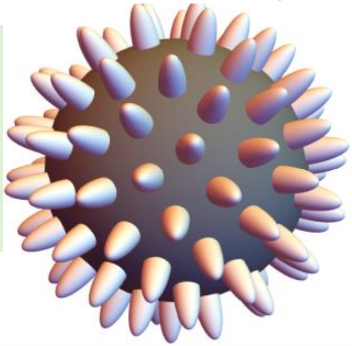
spin s particle by θ angle rotates

quantum phase: $\psi \to \psi\, e^{-is\theta}$

(½ spin) **bispinor rotation** by ϕ**:**

$$S[\Lambda_{\mathrm{rot}}] = \begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2} \end{pmatrix}$$

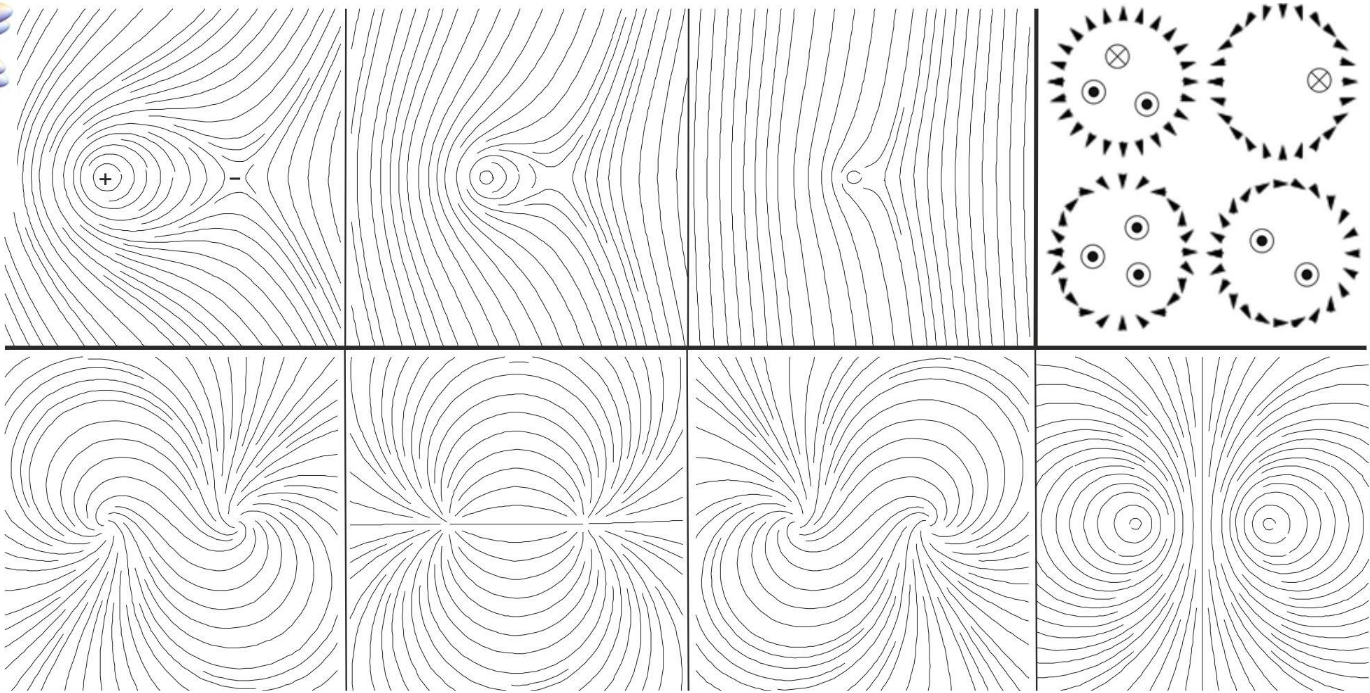
3D topological charge: electric



$V(r)$ distance dependence of field stress/energy gives **long-range attraction/repulsion**:

de Broglie clock/zitterbewegung

evolution of **quantum phase** around - forming pilot wave



Vacuum (long distance) dynamics:

1) EM: **quantized** electric charge with **Coulomb** in $V(r) \propto r^{-1}$ (+ magnetism from Lorentz invariance)

S^2: **Gauss law counts topological charge**

2) $\boldsymbol{S^1}$ **quantum phase**: **Berry**, **pilot wave**

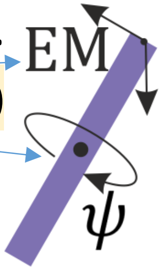
e.g. for Mach-Zehnder interfer.

Unify EM S^2 + QM S^1 → $SO(3)$

"extended phase"

governed by **wave equation**:

Maxwell $\square A_\mu \propto J_\mu$+~Klein-Gordon $\square\psi$



Momentum operator:

$$\hat{P} = -qA - i\hbar\nabla$$

suggests: A also hides derivative, describes local rotation

$$2\pi k = \Delta\varphi = \frac{q}{\hbar}\oint_{\partial S} A \cdot dl = \frac{q}{\hbar}\oint_S \ B \cdot dS$$

fluxon, GL order parameter

3×3 in 3D → 4×4 in 4D spacetime:

3) + gravity starting with GEM required for Newton + B_g for Lorentz invariance

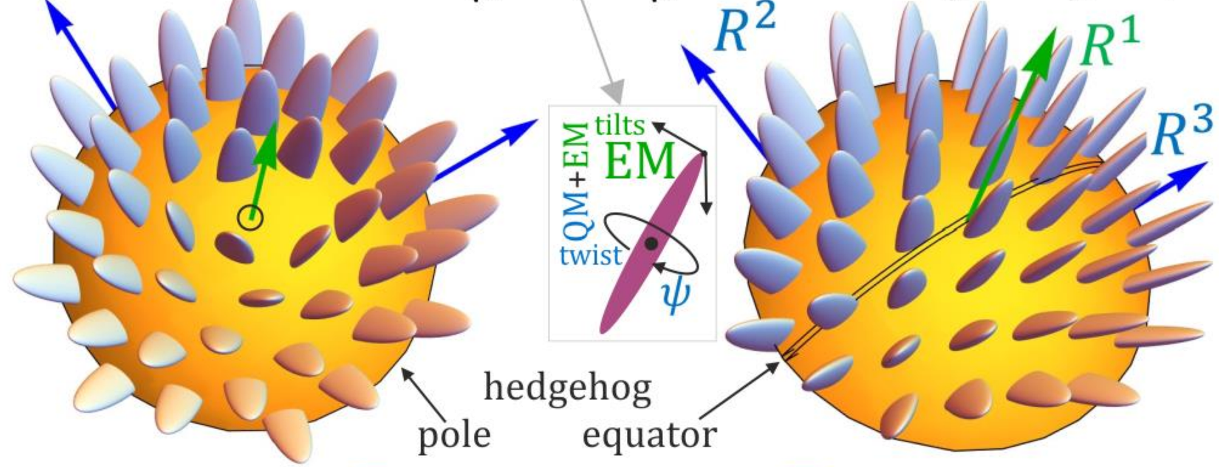
green: **tilt-tilt** of $\vec{n}$ main axis EM high energy **main curvature**

blue: **tilt-twist** QM phase low energy curvatures

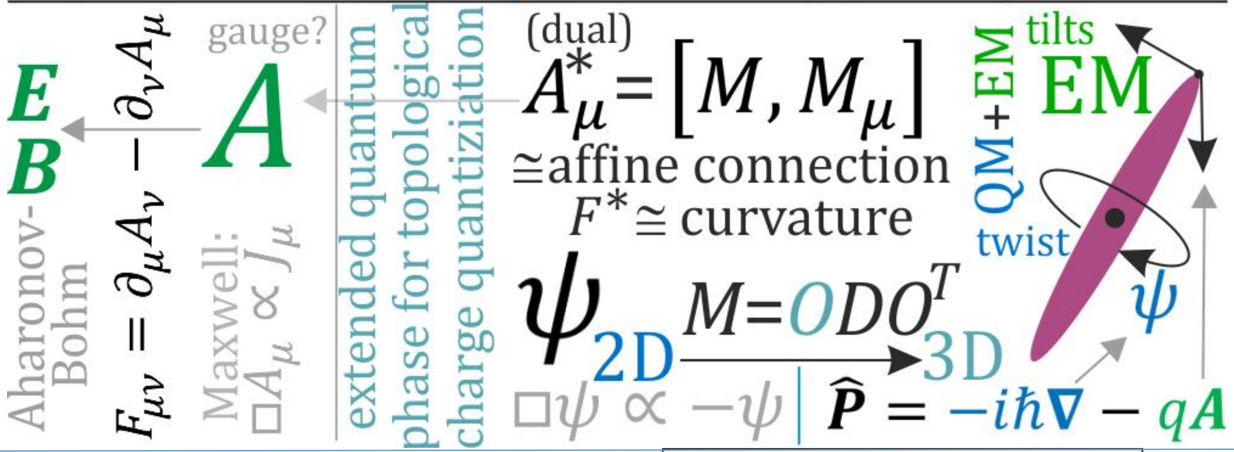
$$\Lambda = (\mathbf{1}, \boldsymbol{\delta}, 0) \Rightarrow \vec{A}_\mu \approx \left(\delta^2\Gamma^1_\mu, \Gamma^2_\mu, \Gamma^3_\mu\right), \ \ \vec{F}_{\mu\nu} \approx \left(R^1_{\mu\nu}, \delta R^2_{\mu\nu}, \delta R^3_{\mu\nu}\right)$$

≈fixed far from charge

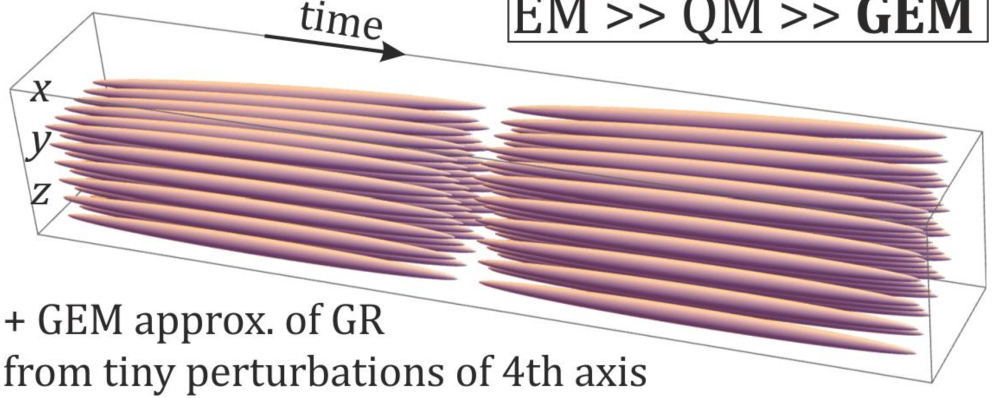
$$\Gamma_\mu = O^T \partial_\mu O \qquad \vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$$



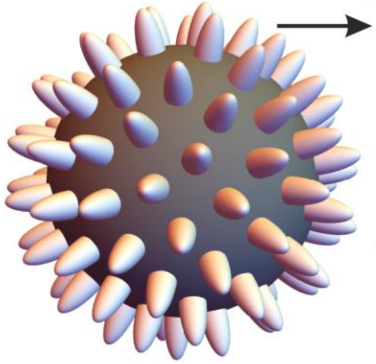
$$\mathcal{L}_{QED} = -\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi - F_{\mu\nu}F^{\mu\nu}/4$$



EM >> QM >> **GEM**



+ GEM approx. of GR
from tiny perturbations of 4th axis

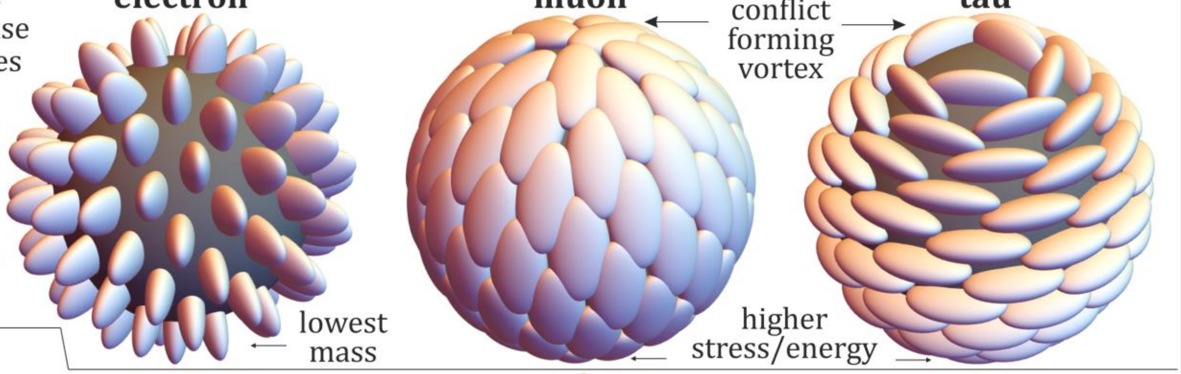


uniaxial nematic
unitary vector field
1 distingused axis

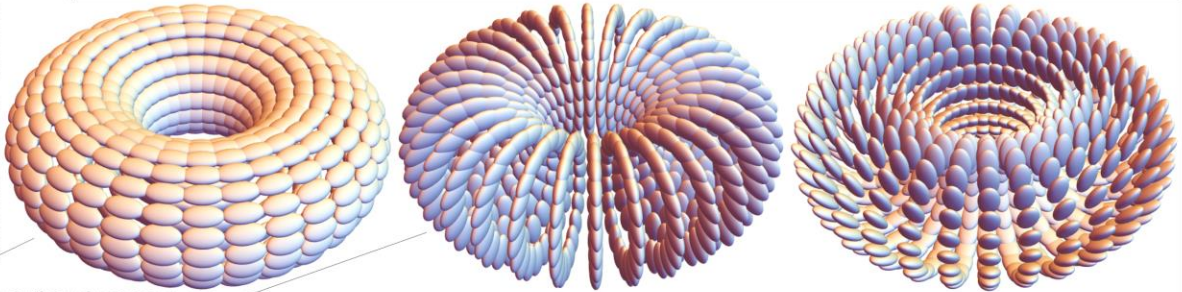
charge + Coulomb

biaxial nematic
vector field + phase
3 distingused axes

3 leptons? charged magnetic dipole

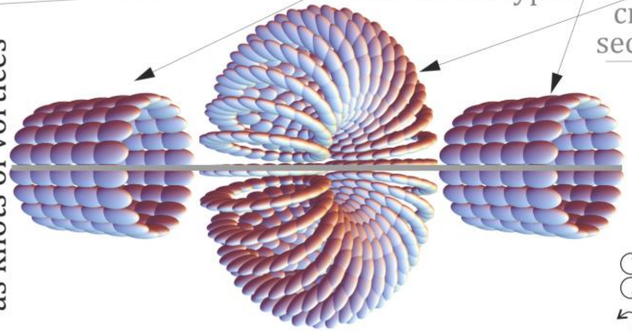


3 neutrinos? extremely stable „oscillations" between

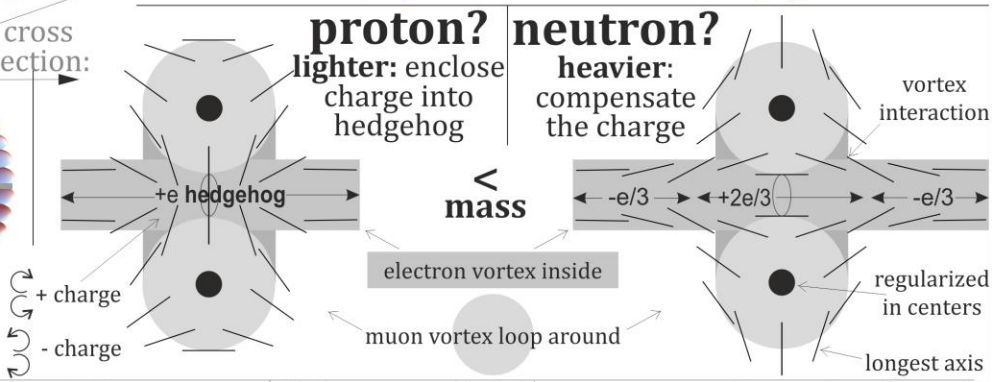


two vortex types

baryons? as knots of vortices



vortex loop around vortex, they interact



proton? **lighter:** enclose charge into hedgehog

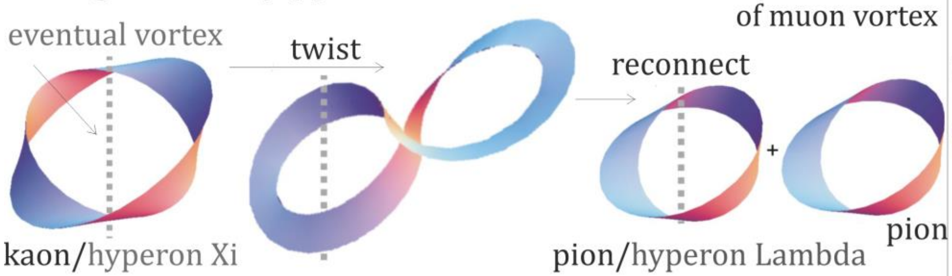
neutron? **heavier**: compensate the charge

< mass

electron vortex inside

muon vortex loop around

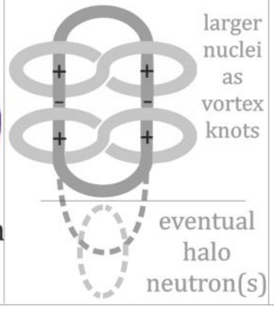
strangeness decay (?): as internal twists (half-rotations) of muon vortex



kaon/hyperon Xi

pion/hyperon Lambda

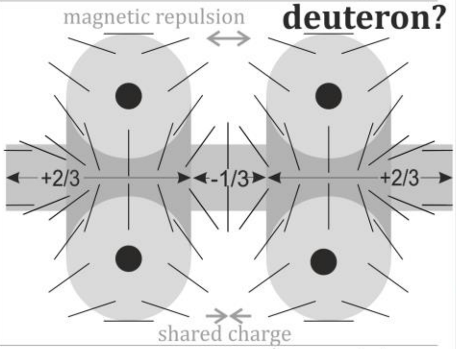
alpha particle(?)



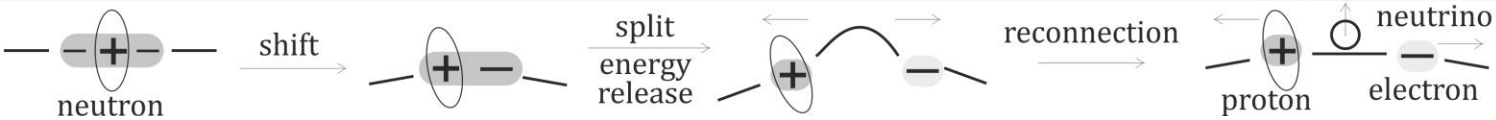
larger nuclei as vortex knots

eventual halo neutron(s)

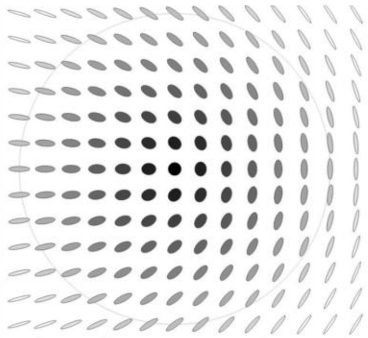
deuteron?



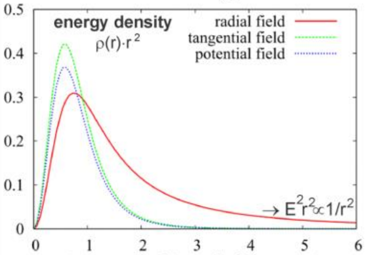
beta decay:



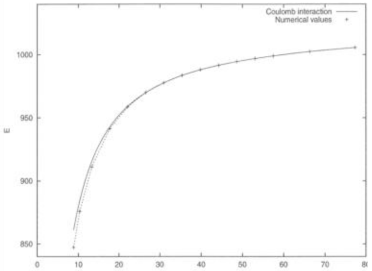
vortex crosssection:



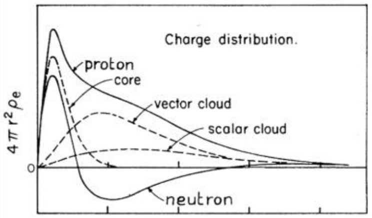
Faber electron **regularized:**



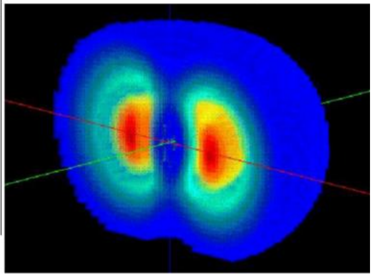
running coupling deformation:



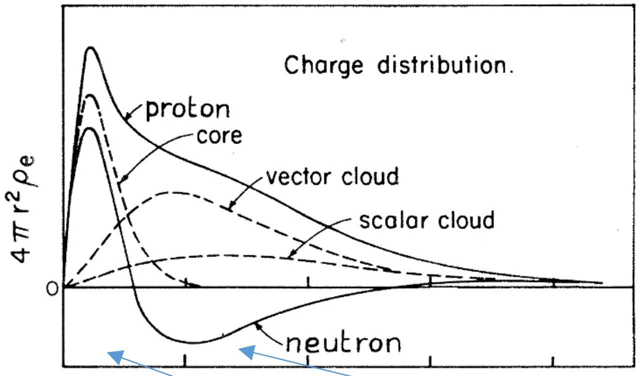
charge distributions (Wilson):

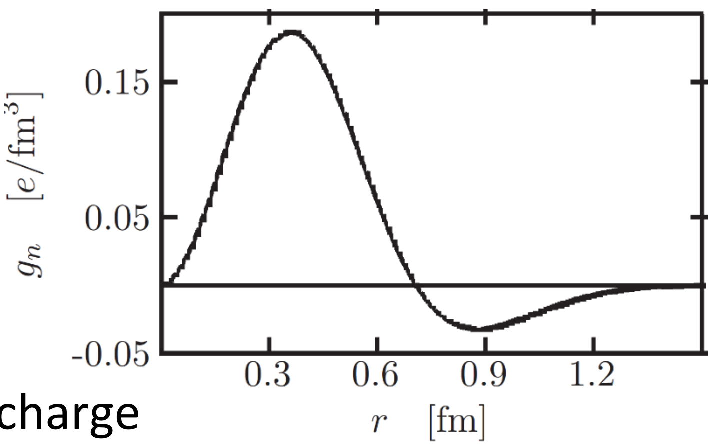


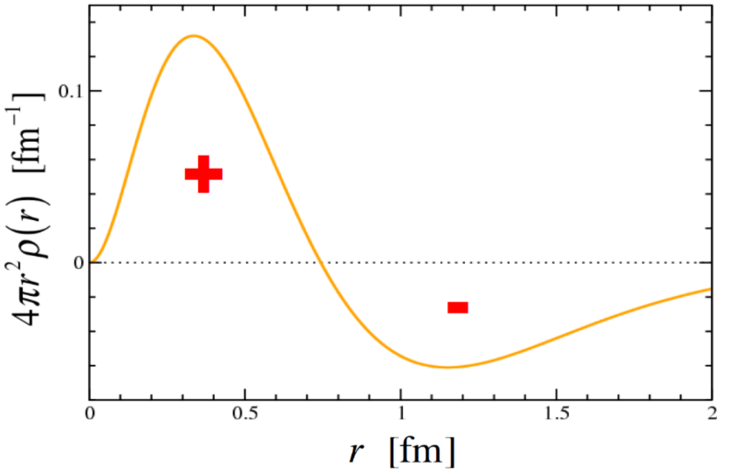
deuteron densities (Garçon):



Baryons: Wilson 1962 Acta. Phys. Pol. 1999: Greene 2015





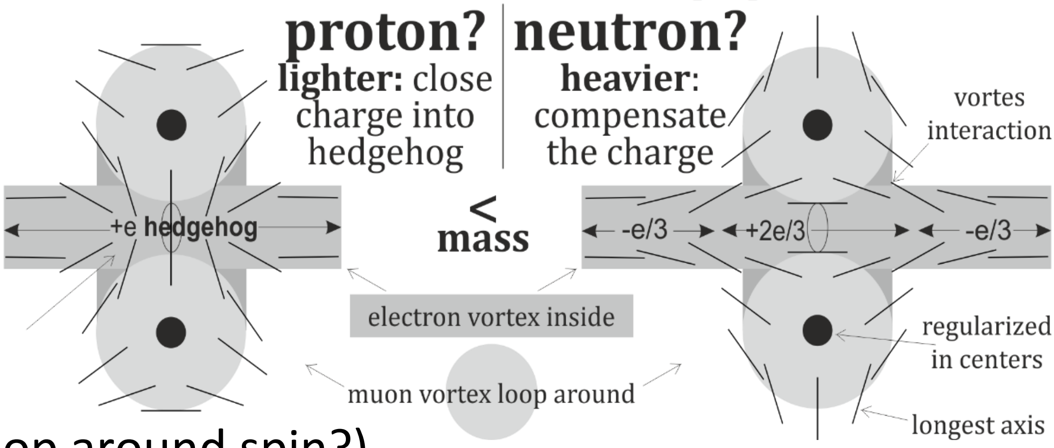


Neutron: "+" core, "-" shell charge

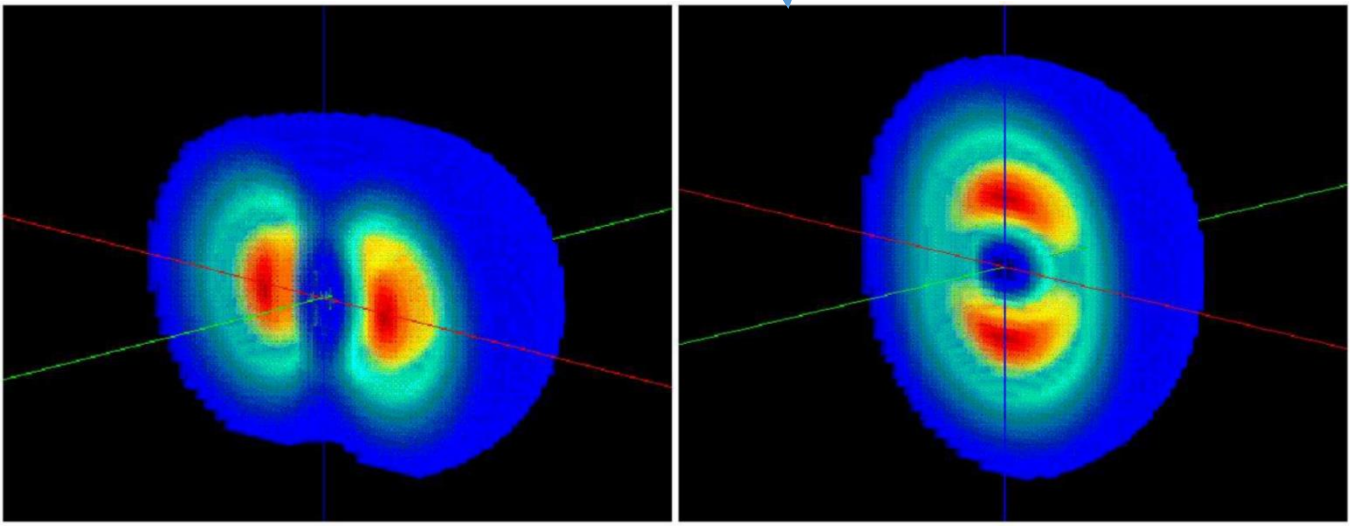
Deuteron: **large electric quadrupole moment**
like "**+ - +**" (how to get it for p+n ???)

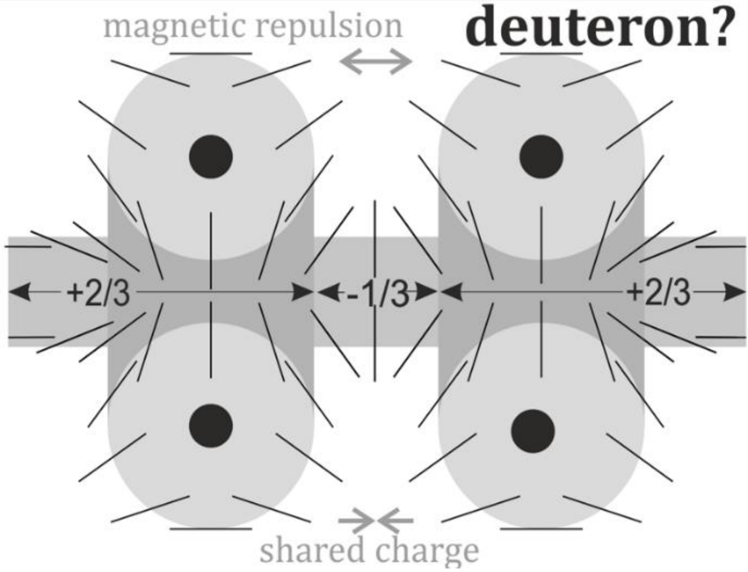
$\mu_d \approx \mu_p + \mu_n$ – **aligned spins?**

0.857 vs 0.879 μ_N magnetic dipole moments



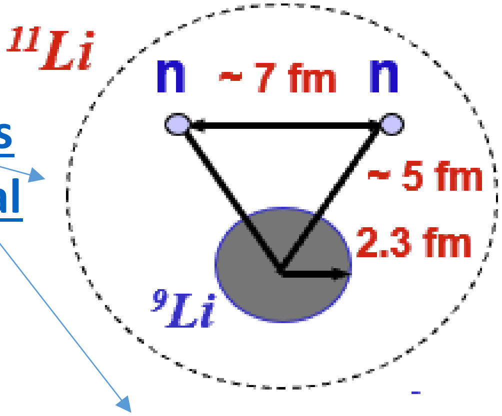
The deuteron: structure and form factors, Advances in Nuclear Physics, 2001 (energy in loop around spin?)



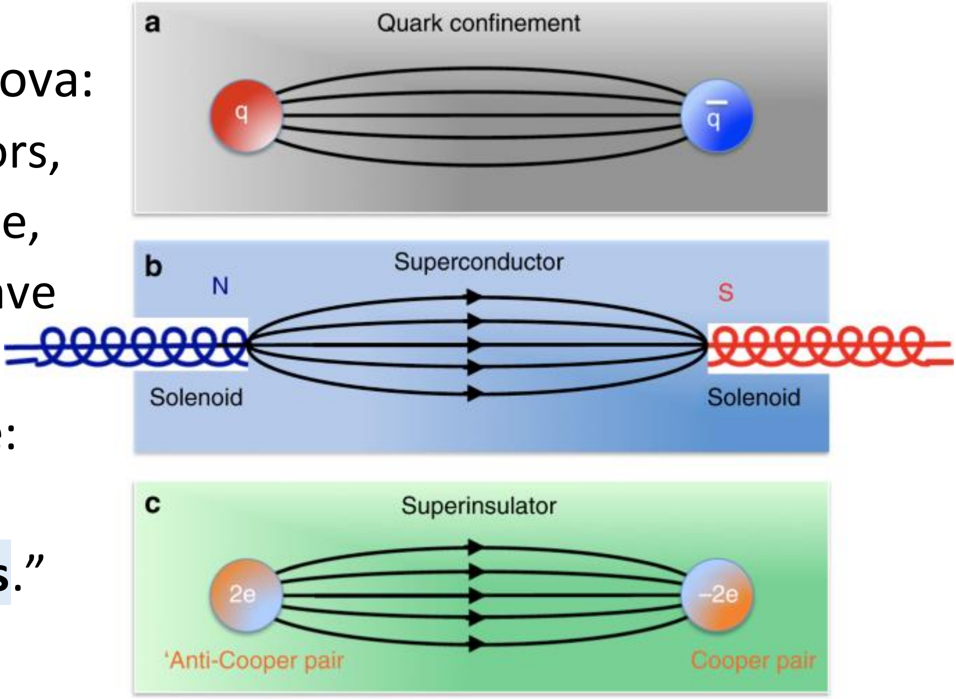


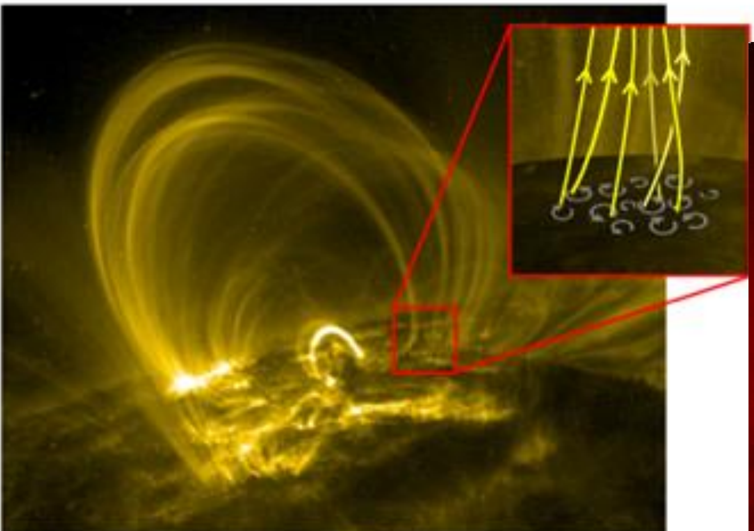
Required: **“fluxons” in vacuum** – magnetic field lines with energy density:

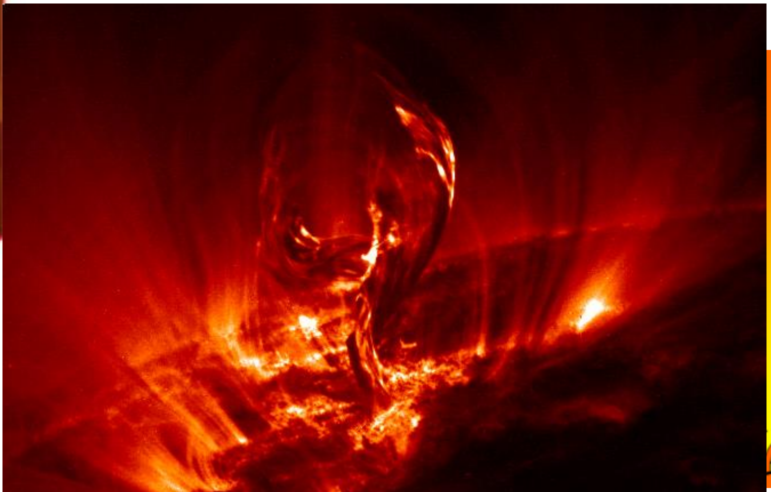
- **Holding nucleus together against Coulomb repulsion (?), halo neutrons**
- **popular quark string model, Nature article suggesting being topological**
- **Coronal** heating problem (surface 6000K, corona 10^7K), reconnections
- Holding electrons in parallel or anti-parallel alignment (spin of photon)
- Brawley et. al., “Electron-like scattering of positronium”, Science 2010

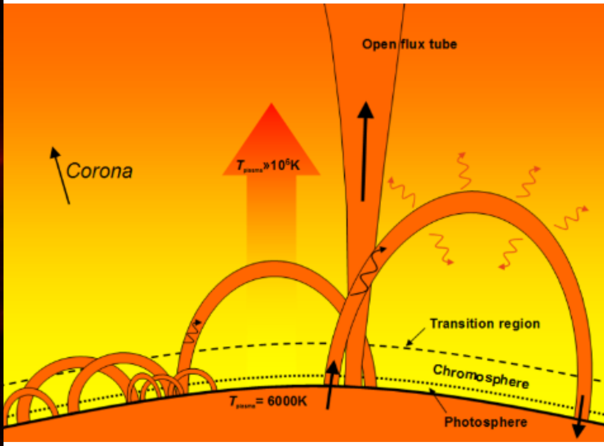


"Physics of **Magnetic Flux Tubes**" book by Ryutova: “Vortices in superfluid Helium and superconductors, magnetic flux tubes in solar atmosphere and space, filamentation process in biology and chemistry have probably a common ground, which is to be yet established. One conclusion can be made for sure: **formation of filamentary structures in nature is energetically favorable and fundamental process**.”









Intermediate step: transform unitary "**director**" field → **rotation** matrix field O

Affine **connection**: $O \to O(I + \epsilon\Gamma_\mu)$ for $\boldsymbol{\Gamma_\mu} = \boldsymbol{O^T O_\mu} = O^T \partial_\mu O$ anti-symmetric

Denote its coordinates with 2 vectors:

EM: $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$

GEM: $\vec{\Gamma}_\mu^g := (\Gamma_{\mu,01}, \Gamma_{\mu,02}, \Gamma_{\mu,03})$ tiny tilts of 0th axis

$$\Gamma_\mu = O^T O_\mu = \begin{pmatrix} 0 & \vec{\Gamma}^g_{\mu,1} & \vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}^g_{\mu,3} \\ -\vec{\Gamma}^g_{\mu,1} & 0 & -\vec{\Gamma}_{\mu,3} & \vec{\Gamma}_{\mu,2} \\ -\vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}_{\mu,3} & 0 & -\vec{\Gamma}_{\mu,1} \\ -\vec{\Gamma}^g_{\mu,3} & -\vec{\Gamma}_{\mu,2} & \vec{\Gamma}_{\mu,1} & 0 \end{pmatrix}$$

3D 1st axis **curvature**: $\vec{R}_{\mu\nu} \equiv \vec{R}^{ee}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ - **electromagnetism** we will focus on

4D 0th axis curvature: $\vec{R}^{gg}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}^g_\nu$ - **GEM** approximation of general relativity

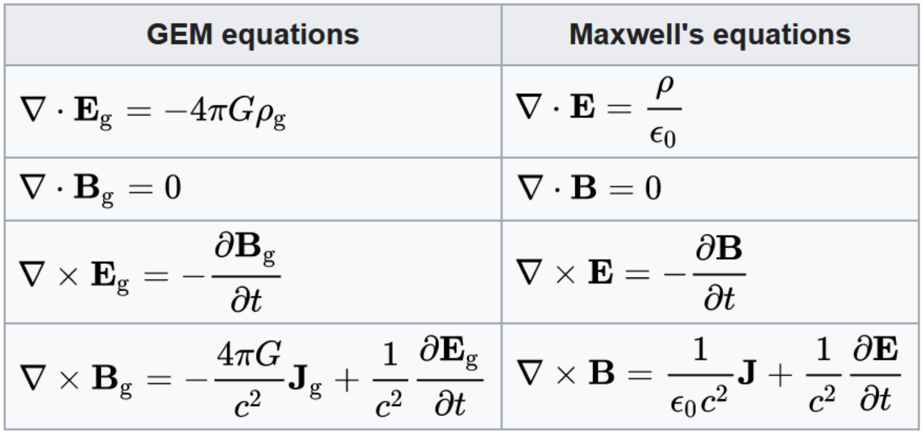
4D EM-GEM interaction: $\vec{R}^{eg}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}^g_\nu$ $\vec{R}^{ge}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}_\nu = -\vec{R}^{eg}_{\nu\mu}$ e.g. light bending by Sun

---

$F_{\mu\nu} = [\Gamma_\mu, \Gamma_\nu]$ in place of curvature for EM with topologically quantized charge?

$$[\Gamma_\mu, \Gamma_\nu] = \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu} + \vec{R}^{eg}_{\nu\mu} \\ \vec{R}^{eg}_{\mu\nu} - \vec{R}^{eg}_{\nu\mu} & \vec{R}^{ee}_{\mu\nu} + \vec{R}^{gg}_{\mu\nu} \end{pmatrix} := \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu,1} + \vec{R}^{eg}_{\nu\mu,1} & -\vec{R}^{eg}_{\mu\nu,2} + \vec{R}^{eg}_{\nu\mu,2} & -\vec{R}^{eg}_{\mu\nu,3} + \vec{R}^{eg}_{\nu\mu,3} \\ \vec{R}^{eg}_{\mu\nu,1} - \vec{R}^{eg}_{\nu\mu,1} & 0 & -\vec{R}^{ee}_{\mu\nu,3} - \vec{R}^{gg}_{\mu\nu,3} & \vec{R}^{ee}_{\mu\nu,2} + \vec{R}^{gg}_{\mu\nu,2} \\ \vec{R}^{eg}_{\mu\nu,2} - \vec{R}^{eg}_{\nu\mu,2} & \vec{R}^{ee}_{\mu\nu,3} + \vec{R}^{gg}_{\mu\nu,3} & 0 & -\vec{R}^{ee}_{\mu\nu,1} - \vec{R}^{gg}_{\mu\nu,1} \\ \vec{R}^{eg}_{\mu\nu,3} - \vec{R}^{eg}_{\nu\mu,3} & -\vec{R}^{ee}_{\mu\nu,2} - \vec{R}^{gg}_{\mu\nu,2} & \vec{R}^{ee}_{\mu\nu,1} + \vec{R}^{gg}_{\mu\nu,1} & 0 \end{pmatrix}$$

GEM – confirmed by Gravity Probe B approximation of **general relativity:** https://en.wikipedia.org/wiki/Gravitoelectromagnetism:

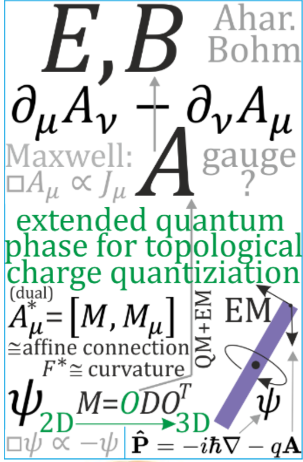


GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

$[\Gamma_\mu, \Gamma_\nu]$ vanishes in flat spacetime

GEM causes spatial curvature:

$$0 = \partial_\mu \partial_\nu O - \partial_\nu \partial_\mu O = \partial_\mu (O\Gamma_\nu) - \partial_\nu (O\Gamma_\mu) = O[\Gamma_\mu, \Gamma_\nu]$$

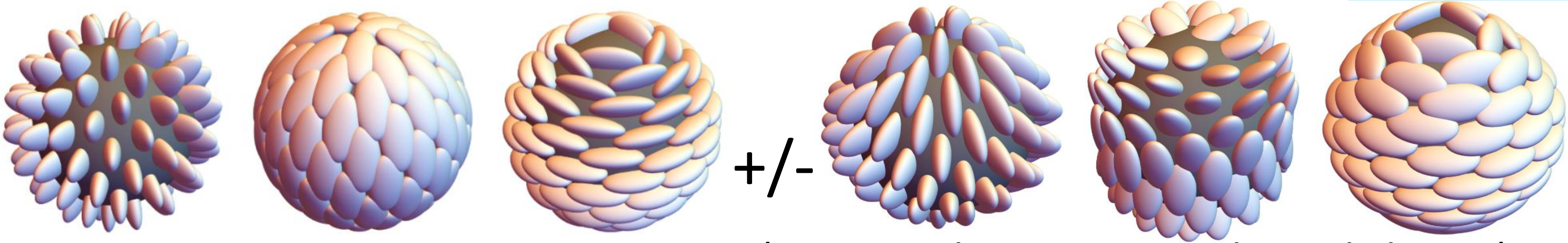


Let O matrix **rotate some objects**, e.g. **ellipsoid** in biaxial nematic

$M(x) \equiv \boldsymbol{M = ODO^T}$ field for $OO^T = I$ rotation

$D = \mathrm{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ with **Higgs-like e.g.** $\boldsymbol{V(M) = \sum_i (\lambda_i - \Lambda_i)^2}$

For $\Lambda_0 > \Lambda_1 > \Lambda_2 > \Lambda_3$ fixed preferred shape – **vacuum state**



(3D) F tensor containing curvature (so Gauss law gives topological charge)

Let us postulate: $\boldsymbol{F_{\mu\nu} = [M_\mu, M_\nu]} = \partial_\mu M\, \partial_\nu M - \partial_\nu M\, \partial_\mu M$, getting **vacuum**:

Curvatures:

Gauss law ∝ topological charge

$\Lambda_1 \gg \Lambda_2 \approx \Lambda_3$

High energy: **EM**

Low energy: **QM**

QED, ψ Lorentz group

$$O^T F_{\mu\nu} O = O^T [M_\mu, M_\nu] O \approx [\Gamma_\mu D - D\Gamma_\mu, \Gamma_\nu D - D\Gamma_\nu] =$$

$$(\Lambda_1 - \Lambda_2)(\Lambda_3 - \Lambda_1)(\Lambda_2 - \Lambda_3) \begin{pmatrix} 0 & \frac{-\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & \frac{\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} \\ \frac{\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & 0 & \frac{-\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} \\ \frac{-\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} & \frac{\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} & 0 \end{pmatrix}$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

for $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ and $\boldsymbol{\Gamma_\mu = O^T O_\mu}$, $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$ as previously

Postulate **Lagrangian** as EM:

and **four-potential $\boldsymbol{A_\mu}$**:

$$\mathcal{L}=\sum_{\mu=1}^{3}\|F_{\mu 0}\|_F^2-\sum_{1\leq\mu<\nu\leq 3}\|F_{\mu\nu}\|_F^2-V$$

$2F_{\mu\nu}=2[\boldsymbol{M_\mu},\boldsymbol{M_\nu}]=\partial_\mu A_\nu-\partial_\nu A_\mu$ for $A_\mu=M\,M_\mu-M_\mu\,M\approx$ (**3D** vacuum)

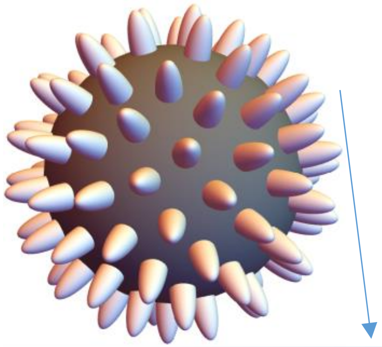
$\Lambda_1\gg\Lambda_2\approx\Lambda_3$

High energy: **EM**

Low energy: **QM**

$P=-i\hbar\nabla-qA$

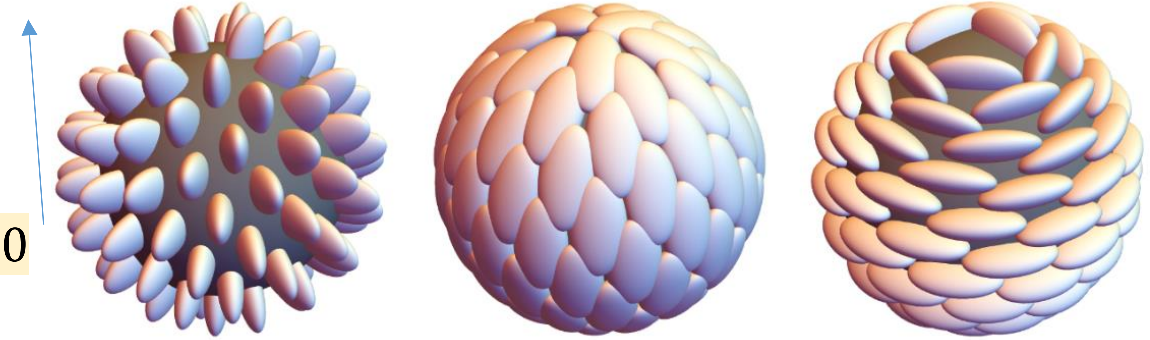
$$\approx O\begin{pmatrix}0 & \vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & -\vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2\\ -\vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & 0 & \vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2\\ \vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2 & -\vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2 & 0\end{pmatrix}O^T$$



shape Λ

dependence

perturbation $\Lambda_2>0$



In **uniaxial nematic case** e.g. simplest $(\Lambda_1=1,\Lambda_2=0,\Lambda_3=0)$: $\boldsymbol{M}=\vec{n}\,\vec{n}^T$

$$A_\mu=[M,M_\mu]=\|\vec{n}\|^2\begin{pmatrix}0 & (\vec{n}\times\vec{n}_\mu)_3 & -(\vec{n}\times\vec{n}_\mu)_2\\ -(\vec{n}\times\vec{n}_\mu)_3 & 0 & (\vec{n}\times\vec{n}_\mu)_1\\ (\vec{n}\times\vec{n}_\mu)_2 & -(\vec{n}\times\vec{n}_\mu)_3 & 0\end{pmatrix}$$

leading to EM $F_{\mu\nu}$ curvature as Faber: $\partial_\mu(\vec{n}\times\vec{n}_\nu)-\partial_\nu(\vec{n}\times\vec{n}_\mu)=2\,\vec{n}_\mu\times\vec{n}_\nu$

General: small perturbation $\Lambda_2>0$, **shape Λ dependence** in $\Gamma_\mu\overset{\Lambda}{\to}A_\mu,\ R_{\mu\nu}\overset{\Lambda}{\to}F_{\mu\nu}$

$$\frac{\partial\|F_{\mu\nu}\|_F^2}{\partial(\partial_\alpha A_\beta)} = \frac{1}{2}\frac{\partial\|\partial_\mu A_\nu - \partial_\nu A_\mu\|_F^2}{\partial(\partial_\alpha A_\beta)} = (\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha})F_{\alpha\beta}$$

Euler-Lagrange equation: **extended EM** with **topological charge quantization**

(integration by parts - last term should vanish as in Lorentz gauge condition):

Maxwell's equations:

F: E, B fields

EM: $V \sim AJ$

\+ Klein-Gordon: $V \sim m^2 A^2$

\+ GEM: $V \sim AJ^g$

J, J^g: four-currents

$$\frac{\partial\mathcal{L}}{\partial A_\alpha} = \frac{d}{dx_0}\frac{\partial\mathcal{L}}{\partial(\partial_0 A_\alpha)} + \sum_{i=1}^{3}\frac{d}{dx_i}\frac{\partial\mathcal{L}}{\partial(\partial_i A_\alpha)}$$

$$\frac{\partial V}{\partial A_\alpha} = \partial_0 F_{0\alpha} - \sum_{i=1}^{3}\partial_i F_{i\alpha} = \Box A_\alpha - \partial_\alpha\left(\partial_0 A_0 - \sum_i \partial_i A_i\right)$$

All 3x3 or 4x4 anti-symmetric matrices, $\Box = \partial_{00} - \partial_{11} - \partial_{22} - \partial_{33}$

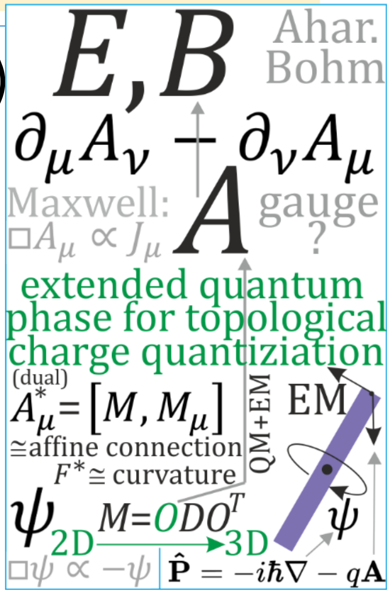
How to choose $V(M)$ or $V(A)$ $\sum_i(\lambda_i - \Lambda_i)^2$? $\sum_\mu\left(||A_\mu||_F^2 - 1\right)^2$? $\det M = 1$?

$A_\mu = [M, \partial_\mu M] \approx \left(\epsilon_{ijk}(\Lambda_i - \Lambda_j)\,\vec{\Gamma}_{\mu,k}\right)_{ij}$ contains velocity – **Higgs $V(A)$ would enforce M clock**

Energy density/Hamiltonian ($||A||_F^2 = \mathrm{Tr}(AA^T)$, last term should vanish)

$$\mathcal{H} = \sum_{\mu=1}^{3}\frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)}\partial_0 A_\mu - \mathcal{L} = \sum_{\mu=1}^{3} F_{0\mu}\bullet(2F_{0\mu} + \partial_\mu A_0) - \mathcal{L}$$

$$\mathcal{H} = \sum_{0\le\mu<\nu\le 3}\|F_{\mu\nu}\|_F^2 + V + \sum_{\mu=1}^{3} F_{0\mu}\bullet\partial_\mu A_0$$

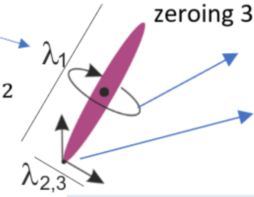


Model: field $M(t,x,y,z) \equiv \boldsymbol{M = ODO^T}$ of real symmetric 3x3 matrices,
$OO^T = I$ describes **local rotation**, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ **shape** of "molecule"
(to be extended to 4x4 tensor field – adding gravitoelectromagnetism)
$(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ is **shape** preferring (Λ_i) shape fixed by model, e.g. $V = \sum_i (\lambda_i - \Lambda_i)^2$

With **Lagrangian**: $\mathcal{L} = \sum_{\mu=1}^{3} ||F_{\mu 0}||_F^2 - \sum_{1\le\mu<\nu\le 3} ||F_{\mu\nu}||_F^2 - V$

for $F_{\mu\nu} = [\partial_\mu M, \partial_\nu M] = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad A_\mu = [M, \partial_\mu M]$

$(V \approx 0)$ Assume **vacuum case:** $(\lambda_i) \approx (\Lambda_i) = (1, \delta, 0)$ for tiny δ related with Planck $\hbar$



zeroing 3x3 M **variations: 3 rotations**, and 3 axis elongations – zeroing the lowest δ:

twist: ~Klein-Gordon $\quad X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$

tilt1: $X^1 \cdot \Gamma^3 = 0$

tilt2: $X^1 \cdot \Gamma^2 = 0$

all 3 **elongations**: $|B^1| = |E^1|$ (dominated in 4D)

for $X^i := \left(-\nabla \cdot B^i,\ \overrightarrow{\partial_0 B^\iota + \nabla \times E^\iota}\right)$ as in **Maxwell equations**

$\Gamma_\mu = O^T O_\mu$

$\vec{\Gamma}_\mu = (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$

$\vec{B}_i = \vec{R}_{0i}$

$\vec{E}_{1,2,3} = (\vec{R}_{32}, \vec{R}_{13}, \vec{R}_{23})$

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

Variating Lagrangian (vacuum $V = 0$) leads to evolution equation:

$$\boxed{0 = \sum_{\mu\nu} d_{\mu\nu} \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\Gamma_\nu, [\overline{M}_\mu, G']] - [\Gamma_\mu, [\overline{M}_\nu, G']]\right) + \overline{F}_{\mu\nu,\nu}[\overline{M}_\mu, G'] - \overline{F}_{\mu\nu,\mu}[\overline{M}_\nu, G']\right)}$$

Which for **3 rotation generators** give

$$X^i := \left(-\nabla \cdot B^i, \overrightarrow{\partial_0 B^\iota + \nabla \times E^\iota}\right)$$

~Maxwell for 2 tilts (high energy) as:

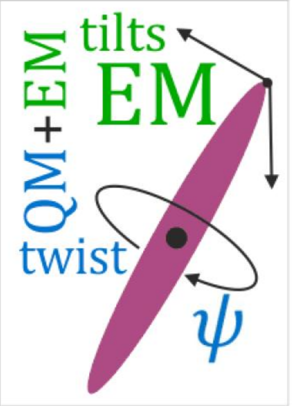
$$X^1 \cdot \Gamma^3 = 0 = X^1 \cdot \Gamma^2$$

~Klein-Gordon for twist (low energy) as:

$$X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$$

for hedgehog:

$$2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|} \cdot \nabla\right)^2\right)\psi$$



```
d = DiagonalMatrix[{1, δ, 0}];                       (* ellipsoid shape, δ ~ ℏ *)
Gx = {{0, 0, 0}, {0, 0, -1}, {0, 1, 0}};   (* twist generator *)
Gy = {{0, 0, 1}, {0, 0, 0}, {-1, 0, 0}};   (* tilt1 generator *)
Gz = {{0, -1, 0}, {1, 0, 0}, {0, 0, 0}};   (* tilt2 generator *)
Ga = {{1, 0, 0}, {0, 0, 0}, {0, 0, 0}}; (* 3 elongation generators *)
Gb = {{0, 0, 0}, {0, 1, 0}, {0, 0, 0}};
Gc = {{0, 0, 0}, {0, 0, 0}, {0, 0, 1}}; (* Gpt of G'=Gd + dGᵀ *)
Gpt = Join[Table[G.d + d.Transpose[G], {G, {Gx, Gy, Gz}}], {Ga, Gb, Gc}];
com[A_, B_] := A.B - B.A;                         (* commutator *)
cd = {{3, 2}, {1, 3}, {2, 1}};                         (* ∈ijk*)
vect[m_] := Table[m[[cd[[i, 1]], cd[[i, 2]]]], {i, 3}]; (* → rotation vector *)
Γμ = {{0, -Γμ^3, Γμ^2}, {Γμ^3, 0, -Γμ^1}, {-Γμ^2, Γμ^1, 0}};     (* its matrix form *)
sub = Table[Cross[{Γμ^1, Γμ^2, Γμ^3}, {Γν^1, Γν^2, Γν^3}][[i]] == R^i_{μ,ν}, {i, 3}];
Mμ = com[Γμ, d]; Γν = Γμ /. μ → ν; Mν = com[Γν, d]; Fμν = Simplify[com[Mμ, Mν], sub];
vrip = Table[Simplify[Tr[Fμν.(com[Γν, com[Mμ, Gp]] - com[Γμ, com[Mν, Gp]]) +
      (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,ν}, {i, 3}]).com[Mμ, Gp] - (*integrate by parts*)
      (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,μ}, {i, 3}]).com[Mν, Gp]]], {Gp, Gpt}];
vr = Simplify[Series[vrip / {2 δ^2, 2, 2, -4, 2, 2}, {δ, 0, 0}] // Normal, sub]
```

$$\{R^3_{\{\mu,\nu,\nu\}}\Gamma^2_\mu - R^2_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^3_{\{\mu,\nu,\mu\}}\Gamma^2_\nu + R^2_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,$$
$$R^1_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^1_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,\ -R^1_{\{\mu,\nu,\nu\}}\Gamma^2_\mu + R^1_{\{\mu,\nu,\mu\}}\Gamma^2_\nu,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2\}$$

```
fin = Table[Sum[v /. μ → 0, {ν, 1, 3}] - Sum[v /. {μ → cd[[i, 1]], ν → cd[[i, 2]]},
    {i, 3}], {v, vr}];                             (* Lagrangian = Σμν ±||Fμν||^2 *)
sub1 =                                          (* rename R curvatures as BE fields *)
 Flatten[Table[{R^i_{0,j} → B^i_j, R^i_{cd[[j,1]],cd[[j,2]]} → E^i_j,
    Table[{R^i_{0,j,k} → B^i_{j,k}, R^i_{cd[[j,1]],cd[[j,2]],k} → E^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
Column[FullSimplify[fn = fin /. sub1], Dividers → All]
```

$$\left(B^3_{\{1,1\}} + B^3_{\{2,2\}} + B^3_{\{3,3\}}\right)\Gamma^2_0 - \left(B^2_{\{1,1\}} + B^2_{\{2,2\}} + B^2_{\{3,3\}}\right)\Gamma^3_0 + \Gamma^3_3\left(B^2_{\{3,0\}} + E^2_{\{1,2\}} - E^2_{\{2,1\}}\right) - \Gamma^2_3\left(B^3_{\{3,0\}} + E^3_{\{1,2\}} - E^3_{\{2,1\}}\right) + \Gamma^3_2\left(B^2_{\{2,0\}} - E^2_{\{1,3\}} + E^2_{\{3,1\}}\right) - \Gamma^2_2\left(B^3_{\{2,0\}} - E^3_{\{1,3\}} + E^3_{\{3,1\}}\right) + \Gamma^3_1\left(B^2_{\{1,0\}} + E^2_{\{2,3\}} - E^2_{\{3,2\}}\right) - \Gamma^2_1\left(B^3_{\{1,0\}} + E^3_{\{2,3\}} - E^3_{\{3,2\}}\right)$$

~Klein-Gordon

$$\left(B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}}\right)\Gamma^3_0 - \Gamma^3_3\left(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}\right) - \Gamma^3_2\left(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}\right) - \Gamma^3_1\left(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}}\right)$$

~Maxwell1

$$-\left(\left(B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}}\right)\Gamma^2_0\right) + \Gamma^2_3\left(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}\right) + \Gamma^2_2\left(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}\right) + \Gamma^2_1\left(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}}\right)$$

~Maxwell2

$$\left(B^1_1\right)^2 + \left(B^1_2\right)^2 + \left(B^1_3\right)^2 - \left(E^1_1\right)^2 - \left(E^1_2\right)^2 - \left(E^1_3\right)^2$$

electric field enforces magnetic (?)

$$\left(B^1_1\right)^2 + \left(B^1_2\right)^2 + \left(B^1_3\right)^2 - \left(E^1_1\right)^2 - \left(E^1_2\right)^2 - \left(E^1_3\right)^2$$

-> de Broglie clock/zitterbewegung?

$$\left(B^1_1\right)^2 + \left(B^1_2\right)^2 + \left(B^1_3\right)^2 - \left(E^1_1\right)^2 - \left(E^1_2\right)^2 - \left(E^1_3\right)^2$$

in 4D dominated by 0-th: gravity?

$E^{2}_{\{1,3\}}$ - type: 1 - high energy (EM tilt-tilt), 2,3 low energy (QM tilt-twist)
- spatial coordinate (1,2,3), derivative (0,1,2,3)

Hedgehog ansatz for longest axis (± 1 charge), with $\psi(t,x,y,z)$ phase/twist function

In vacuum ($A=0$) leads to **Klein-Gordon-like**: $\left(\hat{E}-q\phi\right)^2\psi=(\hat{p}-qA)^2\psi+m^2\psi$

dual formulation $(E \leftrightarrow B)$: $A^{hedg}=\frac{1}{r^2}(x,y,z)$ $\qquad (\hat{E}=i\hbar\partial_t \;\;,\;\; \hat{p}=-i\hbar\nabla)$

$\Psi=\exp(i\psi)$ $\qquad \hat{p}\Psi=-i\nabla\Psi=\Psi\nabla\psi$ $\qquad$ here from: $X^2\cdot\Gamma^3=X^3\cdot\Gamma^2$

we get wave-like: $2\partial_{tt}\psi=\left(\left(\nabla-\vec{A}^{hedg}\right)^2+\left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

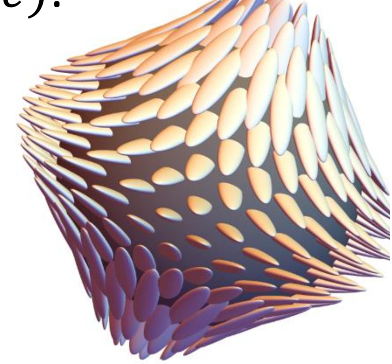
Dirac equation? (also zitterbewegung)

Bispinor for **electron** (up), **positron** (down) with **spin direction** (a,b,c):

$$\frac{1}{4}\begin{bmatrix} 1+c & a-ib & \pm(1+c) & \pm(a-ib) \\ a+ib & 1-c & \pm(a+ib) & \pm(1-c) \\ \pm(1+c) & \pm(a-ib) & 1+c & a-ib \\ \pm(a+ib) & \pm(1-c) & a+ib & 1-c \end{bmatrix}$$

$$S[\Lambda_{\text{boost}}]=\begin{pmatrix} e^{+\chi\cdot\sigma/2} & 0 \\ 0 & e^{-\chi\cdot\sigma/2}\end{pmatrix}$$

$$S[\Lambda_{\text{rot}}]=\begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2}\end{pmatrix}$$



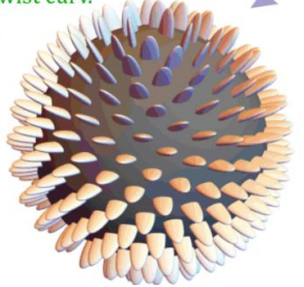
```
sph = {x → r * Cos[θ] * Cos[ϕ], y → r * Cos[θ] * Sin[ϕ], z → r * Sin[θ]}; (*spherical*)
Q0 = FullSimplify[MatrixExp[ϕ * Gz].MatrixExp[θ * Gy].MatrixExp[ψ * Gx] /.
   {ϕ → ArcTan[x, y], θ → -ArcTan[Sqrt[x^2 + y^2], z]}]; (* hedgehog *)
Q = Q0; tQ = Transpose[Q]; M = Simplify[Q.d.tQ];
fBE := Table[Simplify[vect[tQ.com[D[M, c⟦1⟧], D[M, c⟦2⟧]].Q]],
   {c, {{t, x}, {t, y}, {t, z}, {z, y}, {x, z}, {y, x}}}];
M0 = 0.001 * IdentityMatrix[3] + 0.05 * M /. {δ → 0.1, ψ → 0}; (* shape to draw *)
points = SpherePoints[300];
Row[{Column[{"B"₁, "B"₂, "B"₃, "E"₁, "E"₂, "E"₃}], "=", fBE /. ψ → 0 // MatrixForm,
  Graphics3D[{Table[Ellipsoid[p, M0 /. {x → p⟦1⟧, y → p⟦2⟧, z → p⟦3⟧}], {p, points}],
   Gray, Sphere[{0, 0, 0}, 1]}, Boxed → False, ImageSize → Small]}]
```

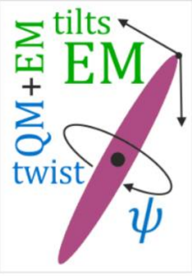
type: EM: high energy tilt-tilt curvature

QM: low energy, tilt-twist curv.

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} B_1 \\ B_2 \\ B_3 \\ E_1 \\ E_2 \\ E_3 \end{array} = & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{x\,(-1+\delta)}{\left(x^2+y^2+z^2\right)^{3/2}} & 0 & -\frac{x\,z\,\delta}{\sqrt{x^2+y^2}\,\left(x^2+y^2+z^2\right)^{3/2}} \\ -\frac{y\,(-1+\delta)}{\left(x^2+y^2+z^2\right)^{3/2}} & 0 & -\frac{y\,z\,\delta}{\sqrt{x^2+y^2}\,\left(x^2+y^2+z^2\right)^{3/2}} \\ -\frac{z\,(-1+\delta)}{\left(x^2+y^2+z^2\right)^{3/2}} & 0 & -\frac{z^2\,\delta}{\sqrt{x^2+y^2}\,\left(x^2+y^2+z^2\right)^{3/2}} \end{pmatrix} \end{array}$$

coordinate





```
Q = Q0 /. {ψ → ψ[t, x, y, z]};                          (* assume phase dependence *)
Γs = Simplify[Table[vect[Transpose[Q].D[Q, v]], {v, {t, x, y, z}}], r > 0];
BE = Simplify[Table[Cross[Γs⟦c⟦1⟧⟧, Γs⟦c⟦2⟧⟧],          (* find BE fields *)
     {c, {{1, 2}, {1, 3}, {1, 4}, {4, 3}, {2, 4}, {3, 2}}}]];
BEd = Simplify[Table[D[BE, v], {v, {t, x, y, z}}]];      (* BE derivatives *)
sub2 = Flatten[Join[Table[Γ^j_{k-1} → Γs⟦k, j⟧, {k, 4}, {j, 3}],
     Table[{B^j_i → BE⟦i, j⟧, E^j_i → BE⟦i + 3, j⟧,
       Table[{B^j_{i,k-1} → BEd⟦k, i, j⟧, E^j_{i,k-1} → BEd⟦k, i + 3, j⟧}, {k, 4}]},
      {i, 3}, {j, 3}]]];
(fne = FullSimplify[fn⟦1 ;; 3⟧ /. sub2] * (x^2 + y^2 + z^2)^2) // Column (*equations:*)
```

$$-2z\,\psi^{(0,0,0,1)}[t,x,y,z]+\left(x^2+y^2+2z^2\right)\psi^{(0,0,0,2)}[t,x,y,z]-$$
$$2y\,\psi^{(0,0,1,0)}[t,x,y,z]+2yz\,\psi^{(0,0,1,1)}[t,x,y,z]+x^2\,\psi^{(0,0,2,0)}[t,x,y,z]+$$
$$2y^2\,\psi^{(0,0,2,0)}[t,x,y,z]+z^2\,\psi^{(0,0,2,0)}[t,x,y,z]-2x\,\psi^{(0,1,0,0)}[t,x,y,z]+$$
$$2xz\,\psi^{(0,1,0,1)}[t,x,y,z]+2xy\,\psi^{(0,1,1,0)}[t,x,y,z]+2x^2\,\psi^{(0,2,0,0)}[t,x,y,z]+$$
$$y^2\,\psi^{(0,2,0,0)}[t,x,y,z]+z^2\,\psi^{(0,2,0,0)}[t,x,y,z]-2\left(x^2+y^2+z^2\right)\psi^{(2,0,0,0)}[t,x,y,z]$$

0

0,0 for 2nd, 3rd equation - they are satified, the first equation equalized to 0:

which turns out Klein-Gordon-like: $2\partial_{tt}\psi=\left(\left(\nabla-\vec{A}^{hedg}\right)^2+\left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

for dual: $\vec{A}^{hedg}(x,y,z)=(x,y,z)/r^2$ $\qquad \Psi=\exp(i\psi)$ $\qquad \hat{p}\Psi=-i\nabla\Psi=\Psi\nabla\psi$

```
r = Sqrt[x^2 + y^2 + z^2]; A = {x, y, z} / r^2;
gmA[f_] := Grad[f, {x, y, z}] - A * f; Adg[f_] := (A * r).Grad[f, {x, y, z}];
Simplify[fne⟦1⟧ / r^2 - Sum[gmA[gmA[ψ[t, x, y, z]]]⟦i, i⟧, {i, 3}] -
  Adg[Adg[ψ[t, x, y, z]]]]
```

$$-2\,\psi^{(2,0,0,0)}[t,x,y,z]$$

Derivation of Maxwell-like equations for gravity (GEM), in $E = B = 0$ case

```
d = DiagonalMatrix[{g, 1, δ, 0}]; cd = {{3, 2}, {1, 3}, {2, 1}}; com[A_, B_] := A.B - B.A;
ξ = DiagonalMatrix[{-1, 1, 1, 1}]; (* signature *)  coms[A_, B_] := A.ξ.B - B.ξ.A;
Γ_μ = {{0, Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}, {-Γ̃^1_μ, 0, -Γ^3_μ, Γ^2_μ}, {-Γ̃^2_μ, Γ^3_μ, 0, -Γ^1_μ}, {-Γ̃^3_μ, -Γ^2_μ, Γ^1_μ, 0}}; (*4D rot*)
G4 = Table[Coefficient[Γ_μ, v], {v, {Γ^1_μ, Γ^2_μ, Γ^3_μ, Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}}];  (* rotation generators *)
dg = Table[tm = Table[0, 4, 4]; tm[[i, i]] = 1; tm, {i, 4}];          (* elongation generators *)
Gpt = Join[Table[coms[G, d], {G, G4}], dg];                      (* G' size 3+3+4=10 tables *)
sub = Flatten[Table[{Cross[{Γ^1_μ, Γ^2_μ, Γ^3_μ}, {Γ^1_ν, Γ^2_ν, Γ^3_ν}][[i]] == R^i_{μ,ν},      (* EM curvatures *)
      Cross[{Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}, {Γ̃^1_ν, Γ̃^2_ν, Γ̃^3_ν}][[i]] == R̃^i_{μ,ν}}, {i, 3}]];
ds[a_] := Flatten[Table[{R^i_{μ,ν} → R^i_{μ,ν,a}, R̃^i_{μ,ν} → R̃^i_{μ,ν,a}}, {i, 3}]];     (* derivatives *)
cs = Flatten[Table[{Γ^i_μ → 0, Γ^i_ν → 0}, {i, 3}]];                     (* assume EM E=B=0 here  *)
M_μ = coms[Γ_μ, d] /. cs; Γ_ν = Γ_μ /. μ → ν; M_ν = coms[Γ_ν, d] /. cs; F_μν = Simplify[coms[M_μ, M_ν], sub];
vr = Table[Tr[F_μν.ξ.(coms[Γ_ν, coms[M_μ, Gp]] - coms[Γ_μ, coms[M_ν, Gp]]).ξ +     (* evolution equations *)
    (F_μν /. ds[ν]).ξ.coms[M_μ, Gp].ξ - (F_μν /. ds[μ]).ξ.coms[M_ν, Gp].ξ], {Gp, Gpt}] /.
sub1 = Flatten[Table[{R̃^i_{0,j} → B̃^i_j, R̃^i_{cd[[j,1]],cd[[j,2]]} → Ẽ^i_j,              (* GEM EB fields *)
    Table[{R̃^i_{0,j,k} → B̃^i_{j,k}, R̃^i_{cd[[j,1]],cd[[j,2]],k} → Ẽ^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
fin = Simplify[Table[Sum[v /. μ → 0, {ν, 1, 3}] - Sum[v /. {μ → cd[[i, 1]], ν → cd[[i, 2]]}, {i, 3}], {v, vr}]];
Column[fnl = Limit[(fin[[4 ;; 6]] /. sub1) / 2 / g^4, g → Infinity] // FullSimplify, Dividers → All]
```

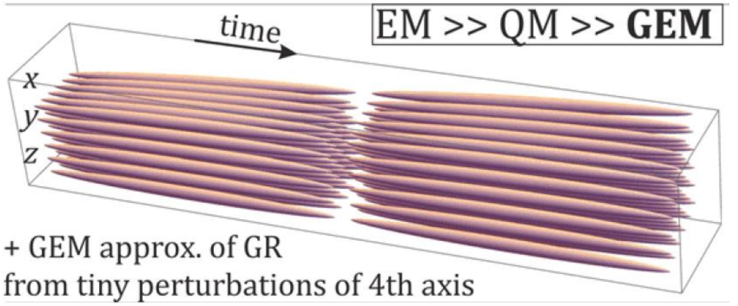
$$\begin{pmatrix} 0 & \tilde\Gamma^1_\mu & \tilde\Gamma^2_\mu & \tilde\Gamma^3_\mu \\ -\tilde\Gamma^1_\mu & 0 & -\Gamma^3_\mu & \Gamma^2_\mu \\ -\tilde\Gamma^2_\mu & \Gamma^3_\mu & 0 & -\Gamma^1_\mu \\ -\tilde\Gamma^3_\mu & -\Gamma^2_\mu & \Gamma^1_\mu & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

$$\left(\tilde B^3_{\{1,1\}}+\tilde B^3_{\{2,2\}}+\tilde B^3_{\{3,3\}}\right)\tilde\Gamma^2_0-\left(\tilde B^2_{\{1,1\}}+\tilde B^2_{\{2,2\}}+\tilde B^2_{\{3,3\}}\right)\tilde\Gamma^3_0+\tilde\Gamma^3_3\left(\tilde B^2_{\{3,0\}}+\tilde E^2_{\{1,2\}}-\tilde E^2_{\{2,1\}}\right)-\tilde\Gamma^2_3\left(\tilde B^3_{\{3,0\}}+\tilde E^3_{\{1,2\}}-\tilde E^3_{\{2,1\}}\right)+\tilde\Gamma^3_2\left(\tilde B^2_{\{2,0\}}-\tilde E^2_{\{1,3\}}+\tilde E^2_{\{3,1\}}\right)-\tilde\Gamma^2_2\left(\tilde B^3_{\{2,0\}}-\tilde E^3_{\{1,3\}}+\tilde E^3_{\{3,1\}}\right)+\tilde\Gamma^3_1\left(\tilde B^2_{\{1,0\}}+\tilde E^2_{\{2,3\}}-\tilde E^2_{\{3,2\}}\right)-\tilde\Gamma^2_1\left(\tilde B^3_{\{1,0\}}+\tilde E^3_{\{2,3\}}-\tilde E^3_{\{3,2\}}\right)$$

$$-\left(\left(\tilde B^3_{\{1,1\}}+\tilde B^3_{\{2,2\}}+\tilde B^3_{\{3,3\}}\right)\tilde\Gamma^1_0\right)+\left(\tilde B^1_{\{1,1\}}+\tilde B^1_{\{2,2\}}+\tilde B^1_{\{3,3\}}\right)\tilde\Gamma^3_0-\tilde\Gamma^3_3\left(\tilde B^1_{\{3,0\}}+\tilde E^1_{\{1,2\}}-\tilde E^1_{\{2,1\}}\right)+\tilde\Gamma^1_3\left(\tilde B^3_{\{3,0\}}+\tilde E^3_{\{1,2\}}-\tilde E^3_{\{2,1\}}\right)-\tilde\Gamma^3_2\left(\tilde B^1_{\{2,0\}}-\tilde E^1_{\{1,3\}}+\tilde E^1_{\{3,1\}}\right)+\tilde\Gamma^1_2\left(\tilde B^3_{\{2,0\}}-\tilde E^3_{\{1,3\}}+\tilde E^3_{\{3,1\}}\right)-\tilde\Gamma^3_1\left(\tilde B^1_{\{1,0\}}+\tilde E^1_{\{2,3\}}-\tilde E^1_{\{3,2\}}\right)+\tilde\Gamma^1_1\left(\tilde B^3_{\{1,0\}}+\tilde E^3_{\{2,3\}}-\tilde E^3_{\{3,2\}}\right)$$

$$\left(\tilde B^2_{\{1,1\}}+\tilde B^2_{\{2,2\}}+\tilde B^2_{\{3,3\}}\right)\tilde\Gamma^1_0-\left(\tilde B^1_{\{1,1\}}+\tilde B^1_{\{2,2\}}+\tilde B^1_{\{3,3\}}\right)\tilde\Gamma^2_0+\tilde\Gamma^2_3\left(\tilde B^1_{\{3,0\}}+\tilde E^1_{\{1,2\}}-\tilde E^1_{\{2,1\}}\right)-\tilde\Gamma^1_3\left(\tilde B^2_{\{3,0\}}+\tilde E^2_{\{1,2\}}-\tilde E^2_{\{2,1\}}\right)+\tilde\Gamma^2_2\left(\tilde B^1_{\{2,0\}}-\tilde E^1_{\{1,3\}}+\tilde E^1_{\{3,1\}}\right)-\tilde\Gamma^1_2\left(\tilde B^2_{\{2,0\}}-\tilde E^2_{\{1,3\}}+\tilde E^2_{\{3,1\}}\right)+\tilde\Gamma^2_1\left(\tilde B^1_{\{1,0\}}+\tilde E^1_{\{2,3\}}-\tilde E^1_{\{3,2\}}\right)-\tilde\Gamma^1_1\left(\tilde B^2_{\{1,0\}}+\tilde E^2_{\{2,3\}}-\tilde E^2_{\{3,2\}}\right)$$



Newton attraction? $\sim 10^{-36} \times$ of Coulomb???

reduced energy for closer curvature sources???

reduced distance - increased curvature^2 in energy

- $r \to 0$ Coulomb, $r \to \infty$ Newton – maybe $r \to r^{-1}$, "**repulsion in infinity**"?

- maybe somehow **opposite curvatures**? E.g. for protons – neutrons?

- due to **spacetime signature**? (also "negative" energy to propel the clock?)

$\xi = \mathrm{diag}(-1,1,1,1)$ $\quad AB \to A\xi B \quad$ $A_\mu^\nu = \sum_\alpha A_{\mu\alpha}\xi^{\alpha\nu}$

time has opposite sign

Lorentz-invariant Lagrangian?

$$\mathcal{L} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + V(M)$$

for $R_{\alpha\beta\mu\nu} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

$[A,B] \to A\xi B - B\xi A$

$\mathrm{Tr}(AA^T) \to \mathrm{Tr}(A\xi A^T\xi)$

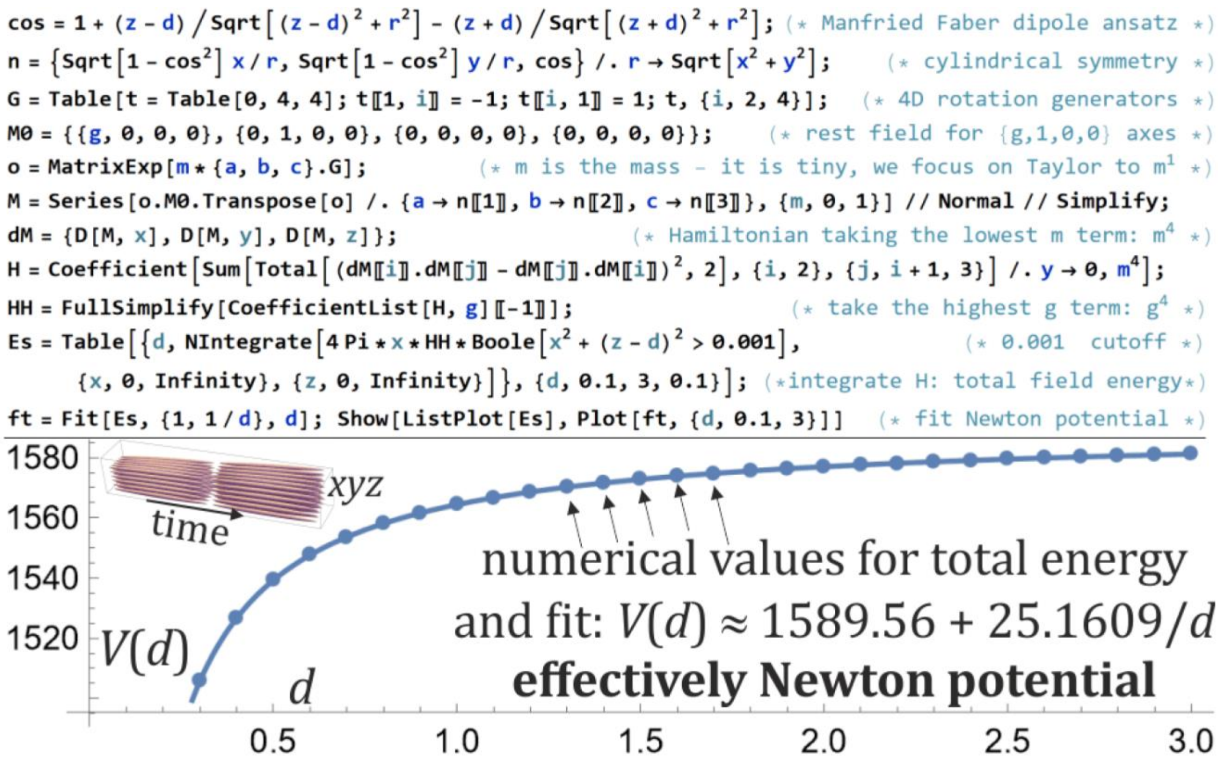
General relativity Lagrangian

(Einstein-Hilbert action):

$$\mathcal{L} = R\sqrt{-g} \qquad \left(R = R_{\alpha\beta\mu\nu}\xi^{\alpha\beta}\xi^{\mu\nu}\right)$$

First power of (contracted) curvature,

so which is proper 1 vs 2? Mixture?



```
cos = 1 + (z - d) / Sqrt[(z - d)^2 + r^2] - (z + d) / Sqrt[(z + d)^2 + r^2]; (* Manfried Faber dipole ansatz *)
n = {Sqrt[1 - cos^2] x / r, Sqrt[1 - cos^2] y / r, cos} /. r → Sqrt[x^2 + y^2];      (* cylindrical symmetry *)
G = Table[t = Table[0, 4, 4]; t[[1, i]] = -1; t[[i, 1]] = 1; t, {i, 2, 4}];   (* 4D rotation generators *)
M0 = {{g, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}};          (* rest field for {g,1,0,0} axes *)
o = MatrixExp[m * {a, b, c}.G];                 (* m is the mass - it is tiny, we focus on Taylor to m^1 *)
M = Series[o.M0.Transpose[o] /. {a → n[[1]], b → n[[2]], c → n[[3]]}, {m, 0, 1}] // Normal // Simplify;
dM = {D[M, x], D[M, y], D[M, z]};                        (* Hamiltonian taking the lowest m term: m^4 *)
H = Coefficient[Sum[Total[(dM[[i]].dM[[j]] - dM[[j]].dM[[i]])^2, 2], {i, 2}, {j, i + 1, 3}] /. y → 0, m^4];
HH = FullSimplify[CoefficientList[H, g][[-1]]];                         (* take the highest g term: g^4 *)
Es = Table[{d, NIntegrate[4 Pi * x * HH * Boole[x^2 + (z - d)^2 > 0.001],                  (* 0.001  cutoff *)
      {x, 0, Infinity}, {z, 0, Infinity}]}, {d, 0.1, 3, 0.1}]; (*integrate H: total field energy*)
ft = Fit[Es, {1, 1 / d}, d]; Show[ListPlot[Es], Plot[ft, {d, 0.1, 3}]]   (* fit Newton potential *)
```

Figure 3. A simplified calculation of Newton effective potential (in GitHu
- analogously as in Fig. 2, but this time instead of large spatial rotations, usi
tiny tilts of 0th time axis for gravity (no mass quantization). Spherically sy

Standard model – 26+ parameters, Lagrangian

Noncompatible with **gravity** (“too big infinity”)

Could it be **expansion of some simpler model?**

Where to search for it? Predictability vs freedom

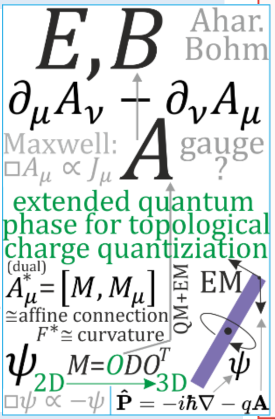
Maybe in liquid crystals? Available in physics …

charge quantization + Coulomb-like interaction …

Postulate skrymion-like Hamiltonian for M

$$\mathcal{H} = \sum_{\mu\nu=0..3} ||\partial_\mu A_\nu - \partial_\nu A_\mu||_F^2 + V$$

$A_\mu = [M, M_\mu]$ clock from A Higgs-like V?



EM >> quantum phase >> GEM vacuum dynamics

EM with missing **charge quantization**, **regularization**

3 leptons: same charge – different mass, $\mu \neq 0$

3 neutrinos: very stable, oscillations, ↑beta decay

3 families of **baryons**: $m_n > m_p, m_d < m_p + m_n$

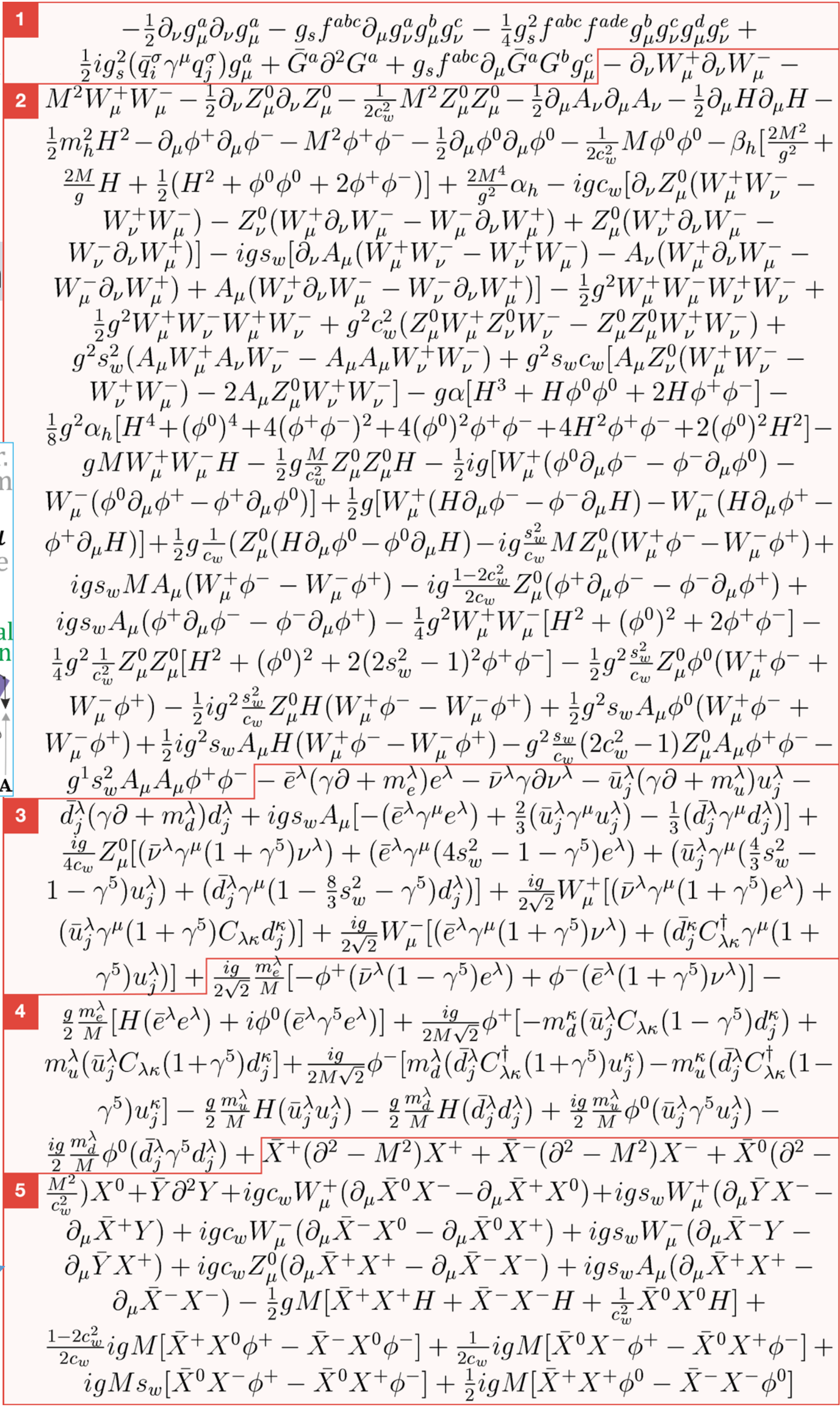
deuteron with quadrupole moment, $\mu_d \approx \mu_p + \mu_n$

nuclei as knots – also halo neutrons in e.g. ~5 fm

strangeness, decaying to mesons …

effective?

Feynman ensemble of topological defects → QFT



$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu$$

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - \frac{1}{2}\partial_\mu H\partial_\mu H - \frac{1}{2}m_h^2H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2}g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3+H\phi^0\phi^0+2H\phi^+\phi^-] - \frac{1}{8}g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^1s_w^2A_\mu A_\mu\phi^+\phi^-$$

$$- \bar{e}^\lambda(\gamma\partial+m^\lambda_e)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial+m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial+m^\lambda_d)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda)+\frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j)-\frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda)+(\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda)+(\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)u^\lambda_j)+(\bar{d}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2-\gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda)+(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda)+(\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)]$$

$$+ \frac{ig}{2\sqrt{2}}\frac{m^\lambda_e}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda)+\phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m^\lambda_e}{M}[H(\bar{e}^\lambda e^\lambda)+i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m^\kappa_d(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j)+m^\lambda_u(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m^\lambda_d(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j)-m^\kappa_u(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j) +$$

$$\bar{X}^+(\partial^2-M^2)X^+ + \bar{X}^-(\partial^2-M^2)X^- + \bar{X}^0(\partial^2-\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]$$