

# The $\pi N \sigma$ term from pionic atoms

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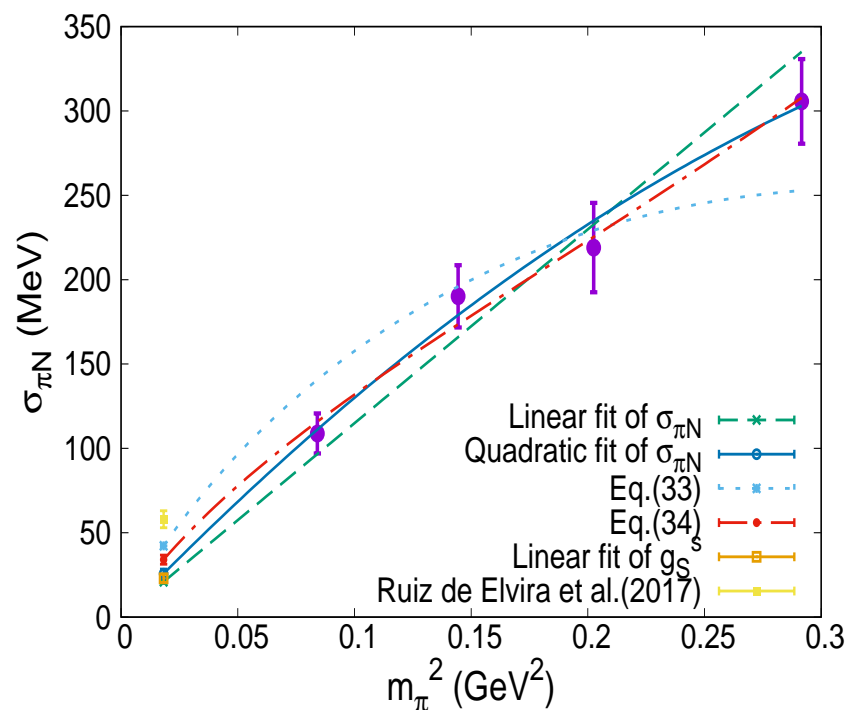
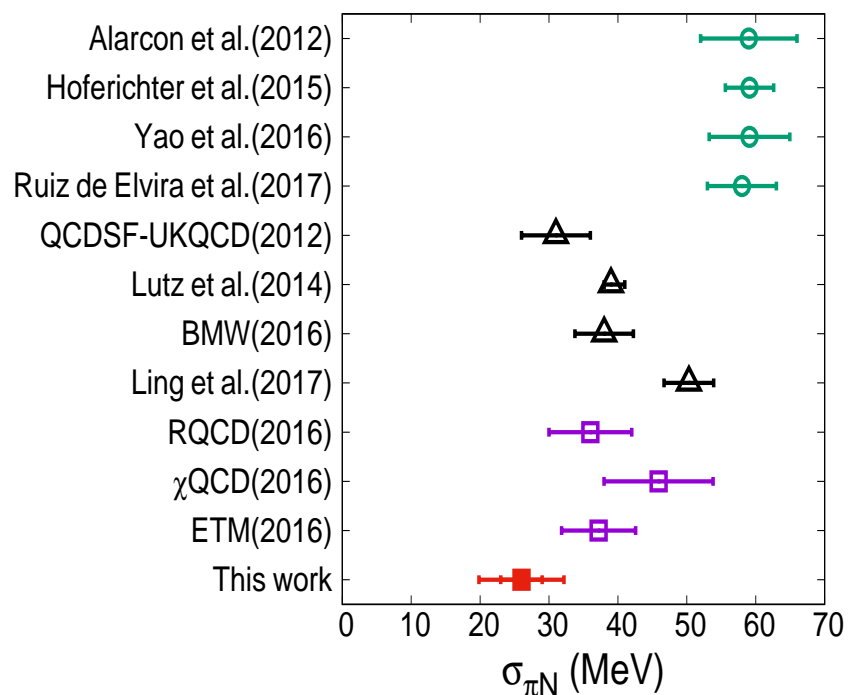
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- Partial restoration of chiral symmetry from pionic atoms. Last update: **NPA 928 (2014) 128.**
- Extracting  $\sigma_{\pi N}$  from pionic atom data  
Recent: **PLB 792 (2019) 340.**
- Comparison with other methods:
  - (i)  $b_0(\pi N)$  at  $m_\pi = 0$ :  **$\sigma_{\pi N} \sim 60 \text{ MeV}$**   
Hoferichter...Meißner, PRL 115 (2015) 092301
  - (ii) LQCD calculations:  **$\sigma_{\pi N} \sim 40 \pm 10 \text{ MeV}$**

# The pion-nucleon $\sigma$ term

$$\sigma_{\pi N} = \frac{\bar{m}_q}{2m_N} \Sigma_{u,d} \langle N | \bar{q}q | N \rangle, \quad \bar{m}_q = \frac{1}{2}(m_u + m_d)$$

records the contribution of explicit chiral symmetry breaking to the nucleon mass  $m_N$  arising from the non-zero value of the u and d quark masses in QCD.



various calcs. of  $\sigma_{\pi N}$

chiral extraps. of  $\sigma_{\pi N}$

N. Yamanaka et al. (JLQCD) PRD 98 (2018) 054516

# Partial restoration of chiral symmetry in/from pionic atoms

PANIC 02, Osaka: NPA 721 (2003)

Suzuki et al. 831c

Kolomeitsev-Kaiser-Weise 835c

Friedman-Gal 842c

Update: Friedman-Gal, NPA 928 (2014) 128

# Optical model analyses of hadronic atom data

- Handle large data sets across periodic table.
- Identify characteristic entities, thereby linking microscopic approaches to experiments.

## Tools of the trade: optical potential variants

- Make  $V_{\text{opt}}$  functional of the nuclear density  $\rho$ .
- Respect the low-density limit  $V_{\text{opt}}(\rho) \rightarrow t_{hN} * \rho$ .
- For pions, consider  $\rho_n - \rho_p$  dependence of  $b_1$  using  $r_n - r_p \approx \gamma \frac{N-Z}{A} + \delta$  with  $\gamma \approx 1.0 \pm 0.1$  fm.
- Introduce self consistently medium effects, particularly subthreshold hN kinematics.

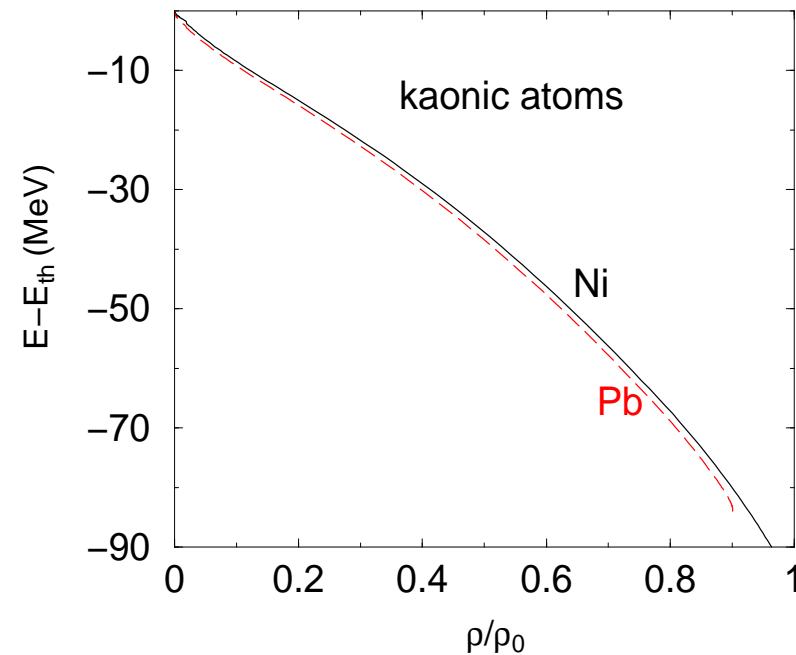
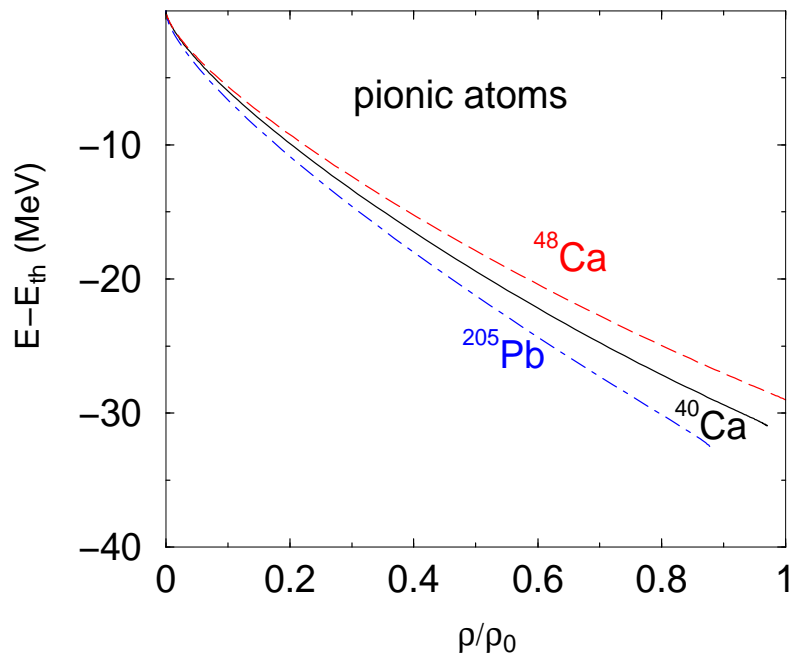
# Self-consistency in mesic-atom & nuclear calculations

Cieplý-Friedman-Gal-Gazda-Mareš, PLB 702 (2011) 402

$$s_{hN} = (\sqrt{s_{\text{th}}} - B_h - B_N)^2 - (\vec{p}_h + \vec{p}_N)^2 < s_{\text{th}}$$

$$\sqrt{s_{hN}} \rightarrow E_{\text{th}} - B_N - B_h - \xi_N \frac{p_N^2}{2m_N} - \xi_h \frac{p_h^2}{2m_h}$$

$$\xi_{N(h)} = \frac{m_{N(h)}}{(m_N + m_h)} \quad \frac{p_h^2}{2m_h} \sim -V_h - B_h$$



in-medium  $\pi^-$  energy shift

in-medium  $K^-$  energy shift

# In-medium $hN$ amplitudes

Friedman-Gal-Mareš, PLB 725 (2013) 334

Cieplý-Friedman-Gal-Mareš, NPA 925 (2014) 126

- KG equation and self-energies:

$$[ \nabla^2 + \tilde{\omega}_h^2 - m_h^2 - \Pi_h(\omega_h, \rho) ] \psi = 0$$

$$\tilde{\omega}_h = \omega_h - i\Gamma_h/2, \quad \omega_h = m_h - B_h$$

$$\Pi_h(\omega_h, \rho) \equiv 2\omega_h V_h = -4\pi \frac{\sqrt{s}}{m_N} f_{hN}(\sqrt{s}, \rho) \rho$$

- Pauli blocking (Waas-Rho-Weise NPA 617 (1997) 449):

$$f_{hN}^{\text{WRW}}(\sqrt{s}, \rho) = \frac{f_{hN}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/m_N)f_{hN}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2} I(\tilde{\omega}_h)$$

- $\sqrt{s}$ :  $\Lambda^*(1405) \Rightarrow f_{K-N}(\sqrt{s})$ ,  $N^*(1535) \Rightarrow f_{\eta N}(\sqrt{s})$ .

In medium  $\Rightarrow$  go subthreshold:  $\delta\sqrt{s} = \sqrt{s} - \sqrt{s_{\text{th}}}$

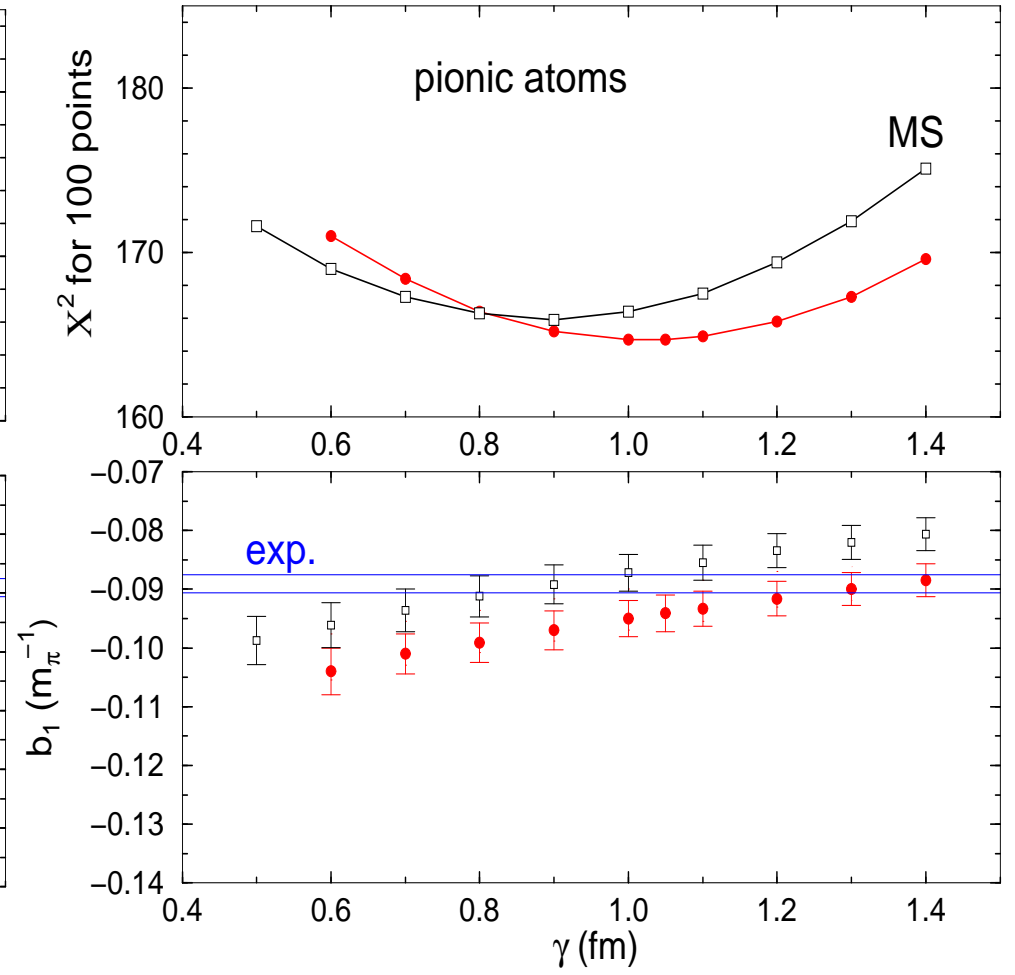
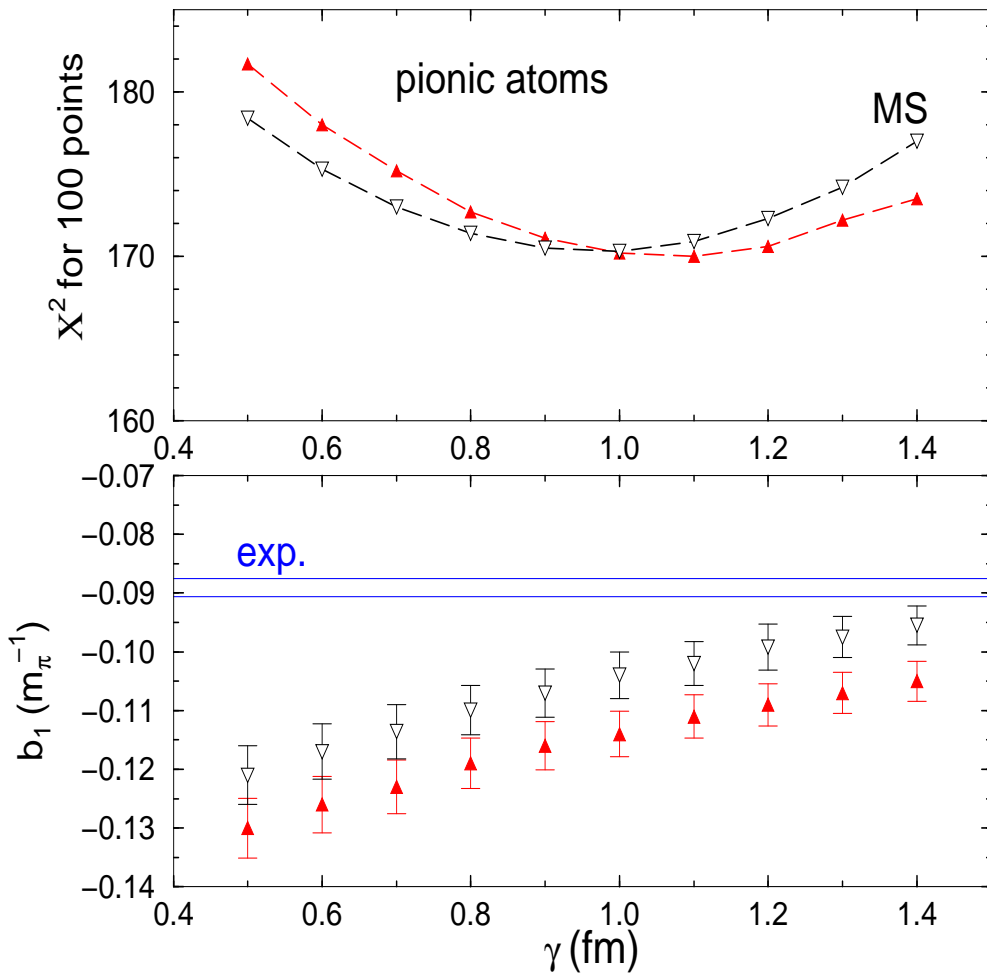
$$\delta\sqrt{s} \approx -B_N \frac{\rho}{\rho_0} - \xi_N B_h \frac{\rho}{\rho_0} - \xi_N T_N \left(\frac{\rho}{\rho_0}\right)^{2/3} + \xi_h \text{Re } V_h(\sqrt{s}, \rho)$$

- **A self-consistency cycle in  $\delta\sqrt{s}$  for given  $\rho$ .**

# Pion-nucleus optical potential

## Ericson-Ericson form (1966)

- $2\mu V_{\text{opt}}(r) = q(r) + \vec{\nabla} \alpha(r) \vec{\nabla}$
- **s-wave**  $q(\mathbf{r}) \sim b_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)] + 4B_0\rho_n(r)\rho_p(r)$ ,  $b_0 \rightarrow b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)p_F$
- **On-shell values from  $\pi^-$ -H &  $\pi^-$ -D atoms (PSI)**  
 Baru...Phillips (2011): PLB 694, 473; NPA 872, 69  
 $(b_0^{\text{free}}, b_1^{\text{free}}) = (0.0076(31), -0.0861(9)) m_\pi^{-1}$ .  
 Hoferichter...(2015):  $(-0.0009, -0.0854) m_\pi^{-1}$ .
- **LO  $\chi$  limit:**  $b_0^{\text{TW}} = 0$ ,  $b_1^{\text{TW}} = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.079 m_\pi^{-1}$   
 GMOR:  $\frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \simeq 1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2} \Rightarrow \sim b_1^{-1}(\rho)$ .
- **p-wave  $\alpha(r)$ :**  $b_0 \rightarrow c_0$ ,  $b_1 \rightarrow c_1$ , &LL;  $B_0 \rightarrow C_0$   
 altogether 8 parameters.



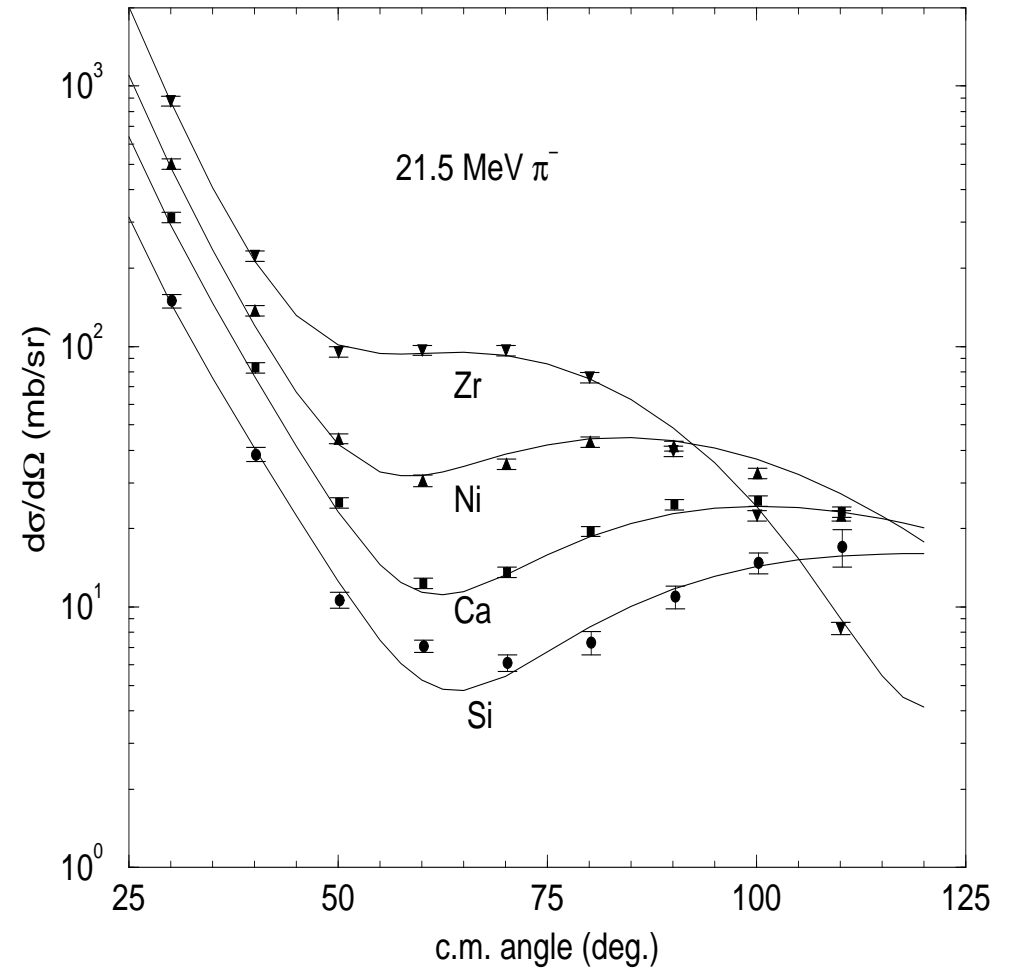
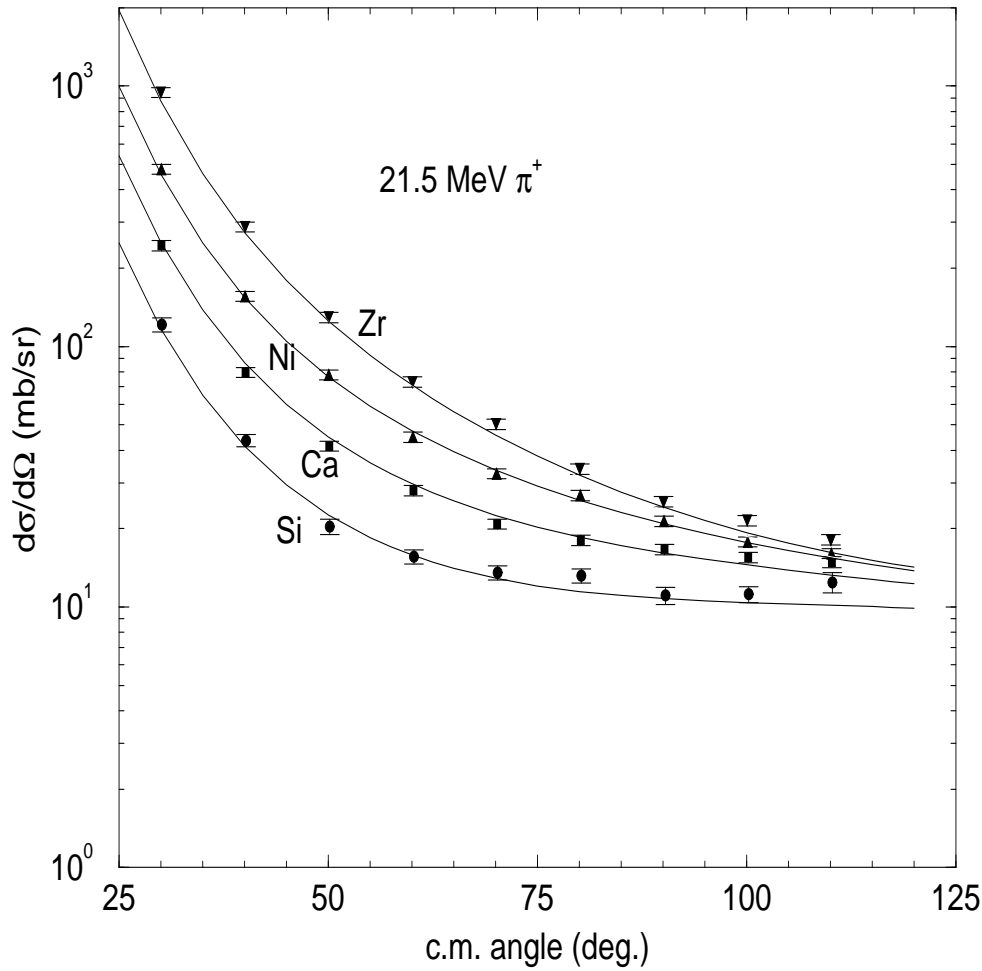
empirical  $b_0(E)$  using  
 $b_1$  independent of  $\rho$

empirical  $b_0(E)$  using  
 $b_1(\rho)$  with  $\sigma_{\pi N}=50$  MeV

$\gamma$  dependence of fits to 100 data points, Ne to U  
 More precise determination of  $b_1$  than thru DBS

E. Friedman, A. Gal, NPA 928 (2014) 128





**E. Friedman et al., PRL 93 (2004) 122302, PRC 72 (2005) 034609**  
**PSI results reproduced with  $b_1(\rho)$  ansatz (Weise, 2000)**

$$b_1(\rho) = -\frac{\mu_{\pi N}}{8\pi f_\pi^2(\rho)}, \quad \frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \simeq 1 - \frac{\sigma\rho}{m_\pi^2 f_\pi^2}, \text{ applied for } \sigma=50 \text{ MeV}$$

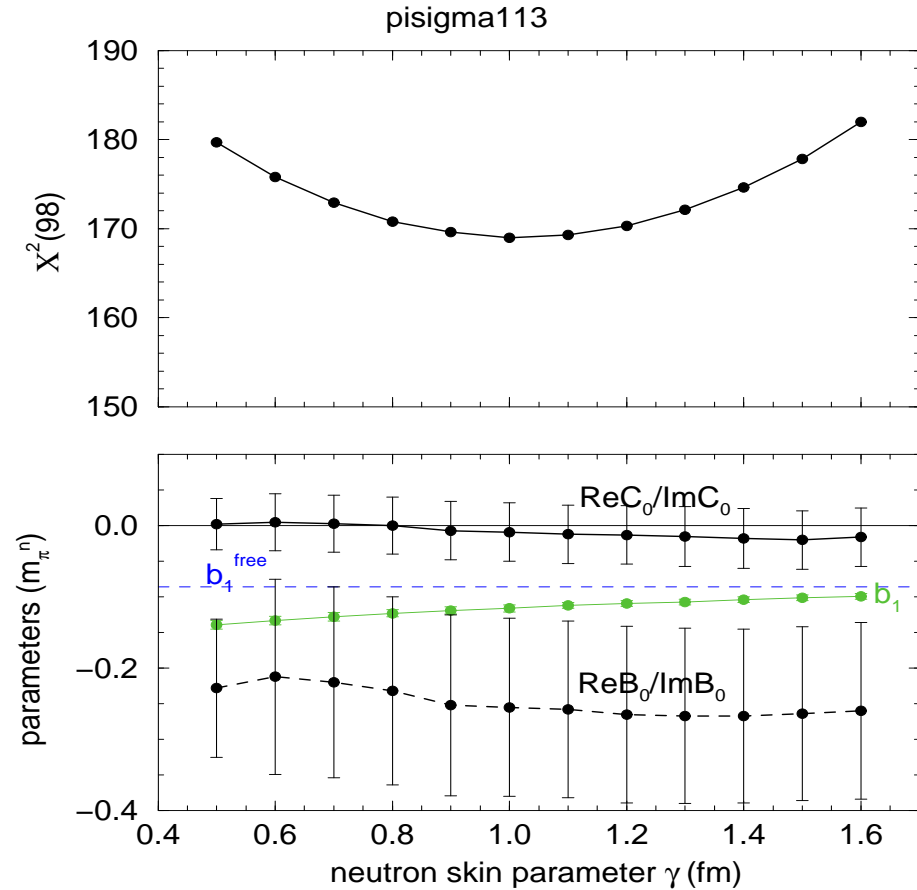
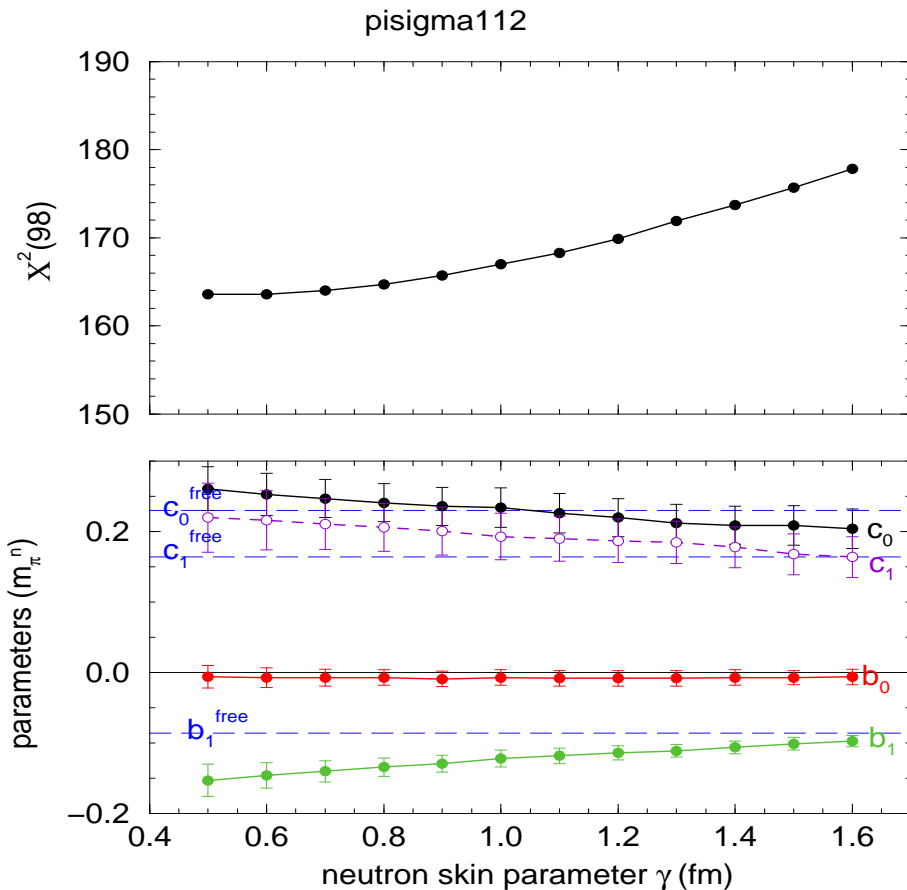
**Consistency between  $\pi^-$  atoms &  $\pi^\pm$  scattering deductions.**

# Constraining the pion-nucleon $\sigma$ term from pionic atoms

Friedman-Gal, PLB 792 (2019) 340

# Multi-parameter fits to $\pi^-$ atom data (I)

$\approx 100$  data point, shifts and widths, across the periodic table,  
 $\rho$ -independent  $b_1$  misses  $b_1^{\text{free}}$  by 5-6 error bars near  $\gamma \approx 1$ .



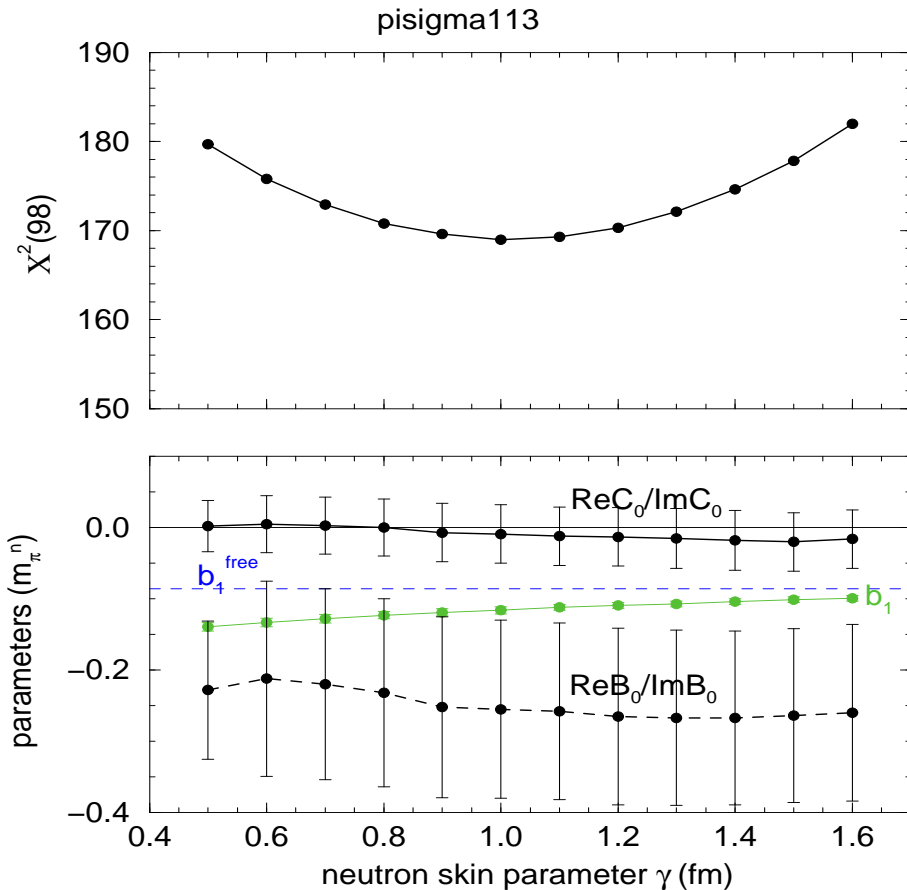
$b_0, c_0, c_1 \approx$  free-space values  
 no  $\chi_{\min}^2$  for 8 parameters

$c_0 = 0.23, c_1 = 0.17 m_\pi^{-3}$   
 $\chi_{\min}^2$  for 6 parameters

# Multi-parameter fits to $\pi^-$ atom data (II)

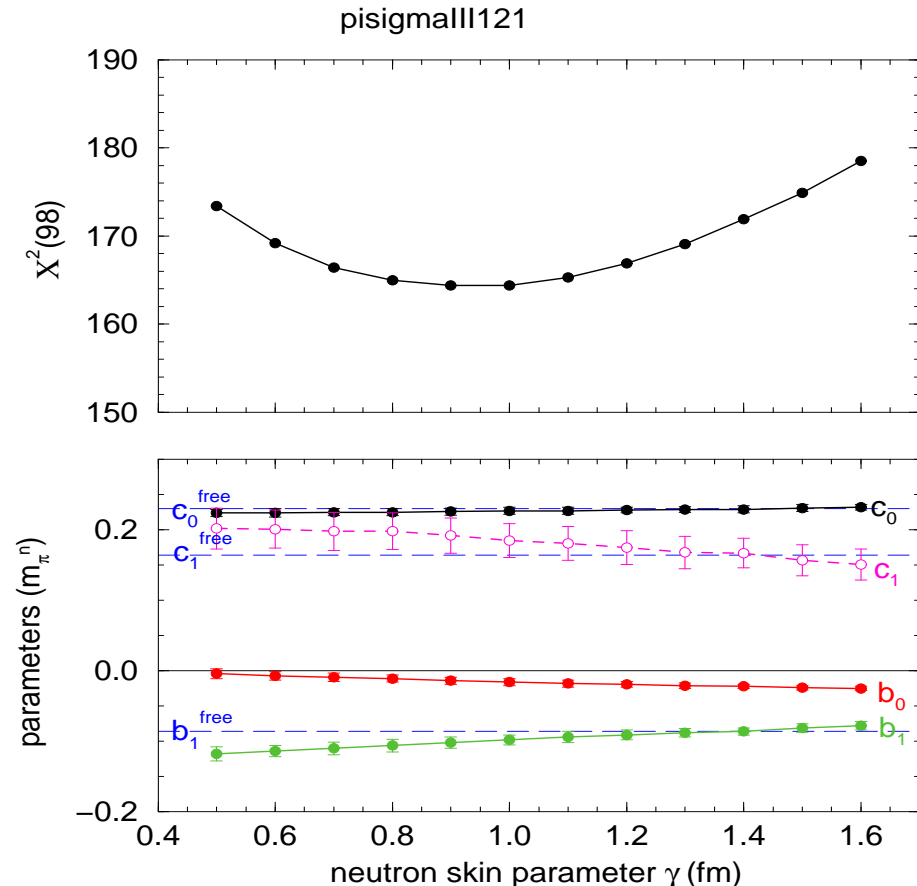
$\approx 100$  data point, shifts and widths, across the periodic table,

$\rho$ -independent  $b_1 \Rightarrow b_1(\rho)$  with  $\sigma=50$  MeV.



$$c_0 = 0.23, c_1 = 0.17 m_\pi^{-3}$$

$\chi_{\min}^2$  for 6 parameters

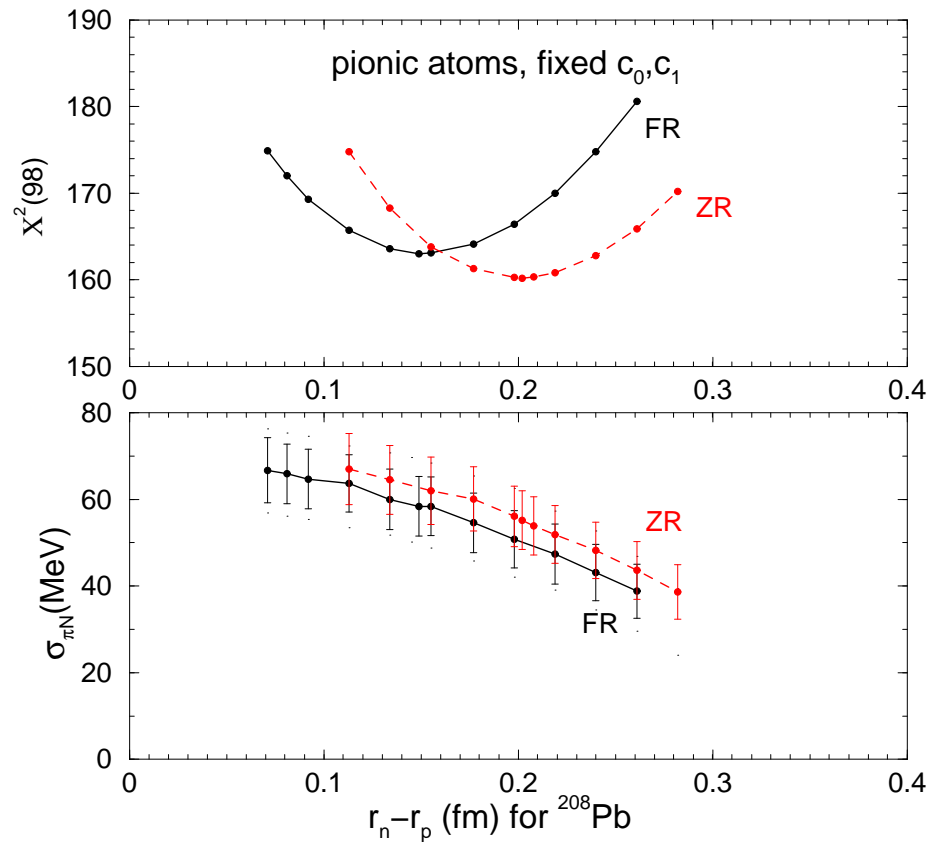
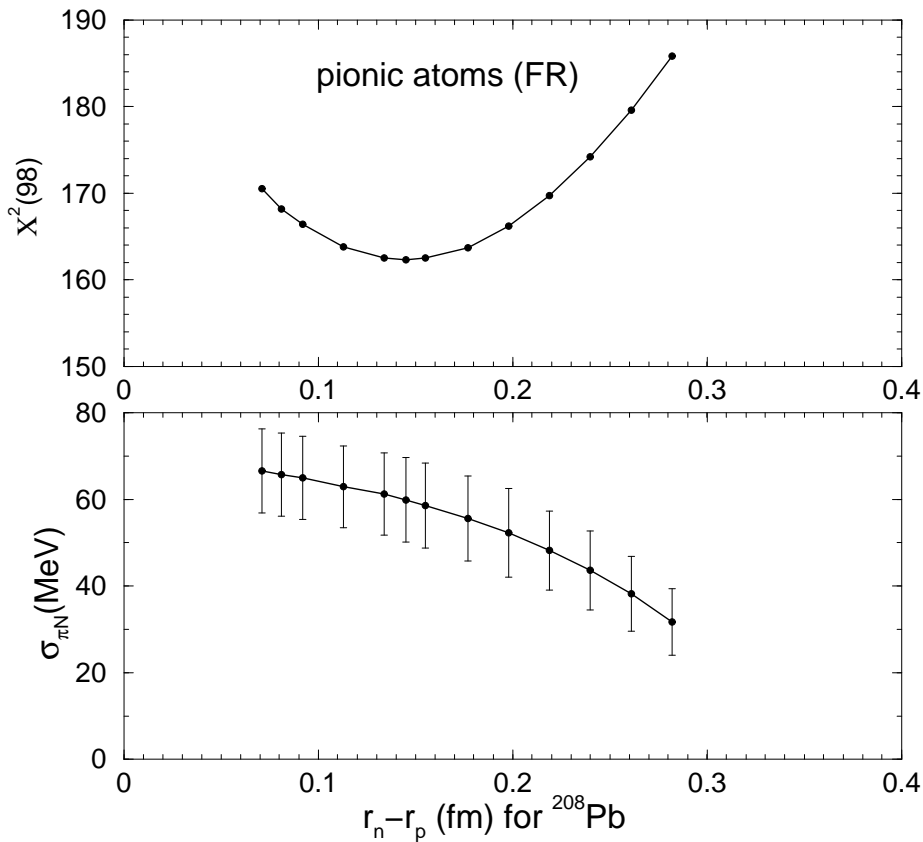


$$\text{Re}B_0 = \text{Re}C_0 = 0$$

$\chi_{\min}^2$  for 6 parameters

# Fitting $\sigma_{\pi N}$ to $\pi^-$ atom data

$\approx 100$  data point, shifts and widths, across the periodic table,  
about 10 are from deeply bound pionic atoms.

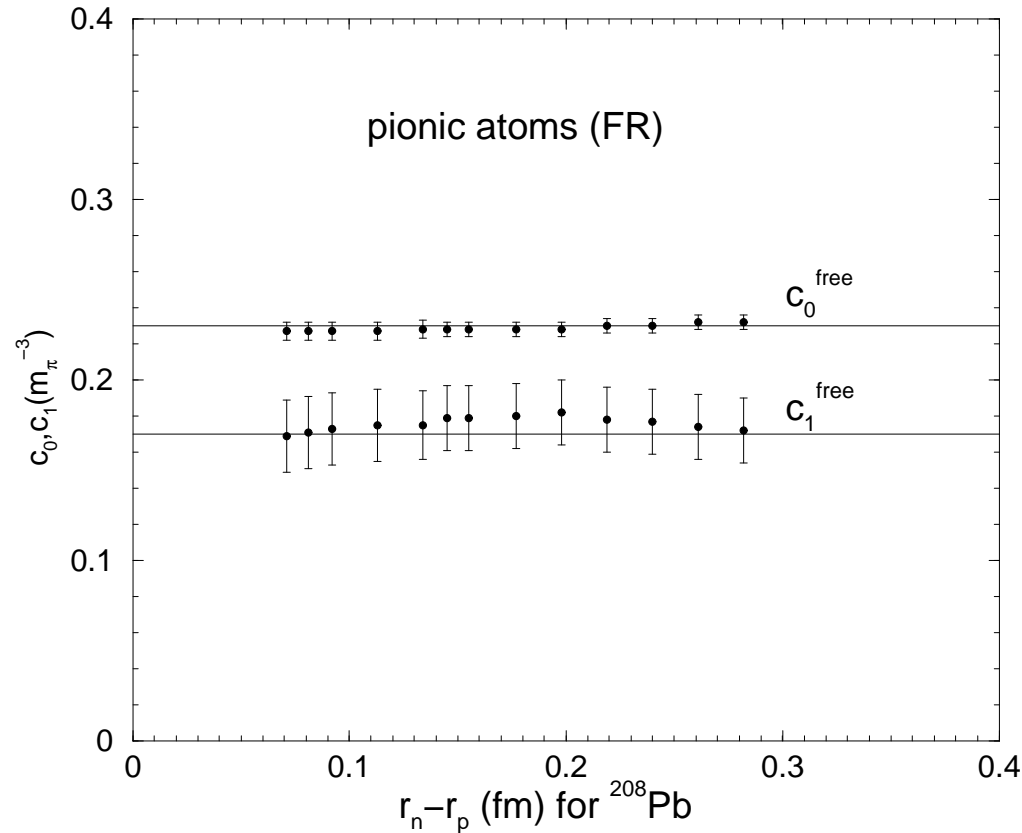


Upper:  $\chi^2$ , Lower:  $\sigma_{\pi N}$   
6 parameters

Finite & Zero range p waves  
4 parameters

Best-Fit  $\sigma_{\pi N} = 57 \pm 7 \text{ MeV}$

# Stability to $\pi N$ p-wave parameters



Horizontal lines mark the SAID free-space values of the  $\pi N$  scattering volumes.

Resulting  $\sigma_{\pi N}$  is robust to fit details

# Discussion & Summary

- Corrections for  $m_\pi \rightarrow m_\pi(\rho)$  &  $\sigma_{\pi N} \rightarrow \sigma_{\pi N}(\rho)$  at  $\rho_\pi \approx 0.1 \text{ fm}^{-3}$  are only a few percent.
- Our  $\sigma_{\pi N} = 57 \pm 7 \text{ MeV}$  agrees with Hoferichter et al. PRL 115 (2015) 092301 & PLB 760 (2016) 74 value  $\sigma_{\pi N} = 59.1 \pm 3.5 \text{ MeV}$  which depends primarily on extrapolating  $b_0$  to the  $m_\pi=0$  Cheng-Dashen point.
- Note that the model dependence of  $b_0$  is fairly large compared to that of  $b_1$  upon which our pionic-atom determination relies.
- Need to improve LQCD derivations...

Thanks for your attention!