Leggett-Garg inequalities and neutrino oscillations

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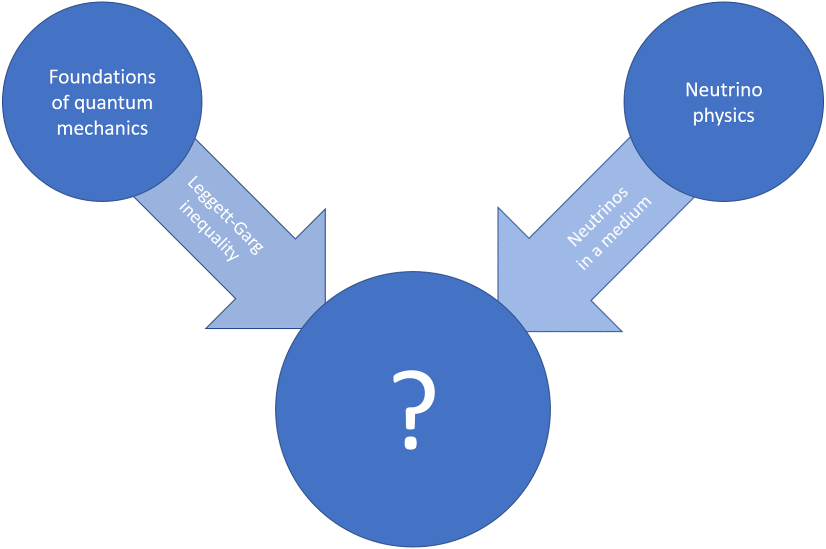
Frame Title


Foundations of quantum mechanics
Neutrino physics
Leggett-Garg inequality
Neutrinos in a medium
?

Historical overview

- 1930 Pauli predicts neutrinos under the name "neutron"

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen

- 1933 Fermi introduces the name "**neutrino**"
- 1956 Reines discovers the electron anti-neutrino
- theoretical works and experimental investigations continue until today

...

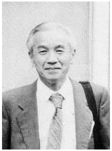
PMNS matrix

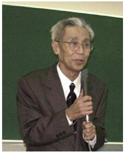
flavor basis $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$
mass basis $|\nu_i\rangle$, $i = 1, 2, 3$

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} \, |\nu_i\rangle$$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} \, |\nu_\alpha\rangle$$


Pontecorvo


Maki


Nakagawa


Sakata

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{accelerator}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{solar} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Dirac/Majorana}}$$

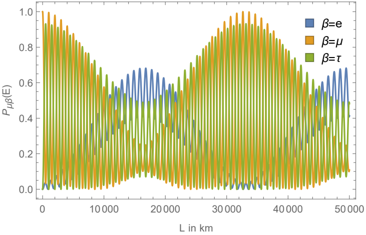
in this talk Dirac particles $\Rightarrow \alpha_1 = 0 = \alpha_2$

$\delta \ldots$ CP-violation angle
$c_{ij} = \cos(\theta_{ij}), s_{ij} = \sin(\theta_{ij})$

Ultrarelativistic limit $p_i \gg m_i$: $E_i \approx E + \frac{m_i^2}{2E}$ and also $t \approx L$

Oscillation probability

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right) + 2 \sum_{i>j} \mathrm{Im}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$



Oscillation probability

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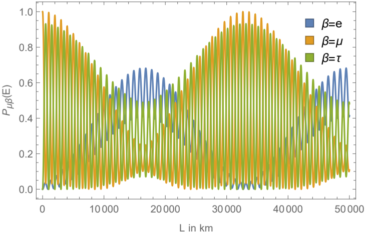
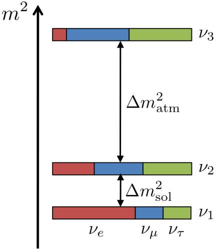
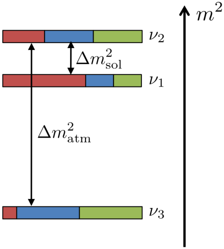


Mass hierarchy problem

normal hierarchy (NH)



inverted hierarchy (IH)



$P_{\nu_\alpha \to \nu_\beta}$ only dependent on $\Delta m^2_{ij} = m^2_i - m^2_j$
$\Rightarrow$ absolute masses and sign of Δm^2_{ij} non-trivial
$\Rightarrow$ 2 options:

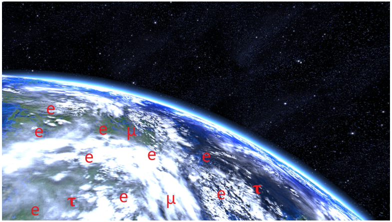
$+\Delta m^2_{31} \ldots$ normal mass hierarchy

$-\Delta m^2_{31} \ldots$ inverted mass hierarchy

Neutrinos propagating through matter

- 1978/79 Wolfenstein, 1985 Mikheyev, Smirnov: matter changes oscillation parameters (MSW effect)
- 1986: Parke, Bethe, Rosen, Gelb: analytic treatment

Matter contains much more electrons than muons and tauons!



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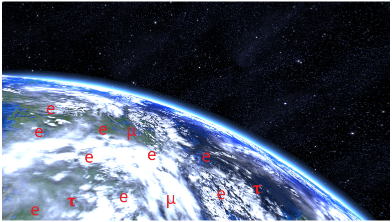


additional matter effect for electrons from matter density

$$V_f = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$A = \pm\sqrt{2} G_F N_e$
$G_F \ldots$ Fermi constant
$N_e \ldots$ electron number density

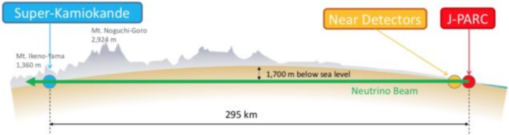
$\Rightarrow$

$$\tilde{H}_m = H_m + V_m = H_m + U^{-1} V_f U$$

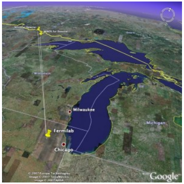
$$\tilde{U}(t) = U e^{-i\tilde{H}_m t} U^{-1}$$

universität wien

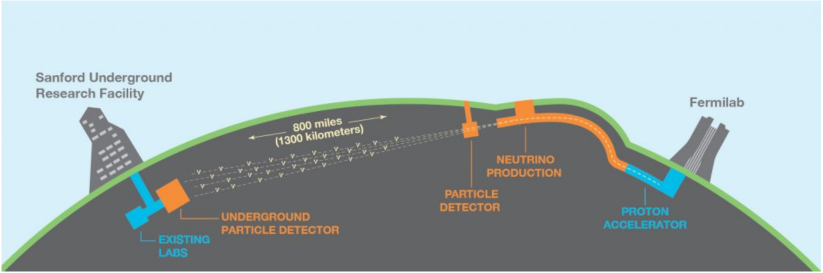
T2K



NOνA

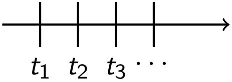


DUNE



Derivation of the Leggett-Garg inequality

Dichotomic quantity: $Q_i = \pm 1$



$$Q_1 Q_2 + Q_2 Q_3 - Q_1 Q_3 = \pm 1$$

Derivation of the Leggett-Garg inequality

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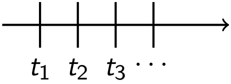


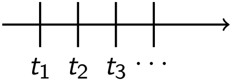
$$Q_1 Q_2 + Q_2 Q_3 - Q_1 Q_3 = \pm 1$$

Bell inequality: locality & realism

$$\langle A, B \rangle + \langle B, C \rangle - \langle A, C \rangle \leq 1$$

Derivation of the Leggett-Garg inequality

Dichotomic quantity: $Q_i = \pm 1$



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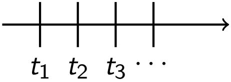
Leggett-Garg inequality (LGI):

- Macrorealism per se: $\left\langle Q_{t_i} Q_{t_j} \right\rangle = \left\langle Q_{t_i} \right\rangle \left\langle Q_{t_j} \right\rangle$
- Noninvasive measurability: $[Q_{t_i}, Q_{t_i}] = 0$

$$K_3 = \langle Q_{t_1} Q_{t_2} \rangle + \langle Q_{t_2} Q_{t_3} \rangle - \langle Q_{t_1} Q_{t_3} \rangle \leq 1$$

Derivation of the Leggett-Garg inequality



Dichotomic quantity: $Q_i = \pm 1$

$$Q_1 Q_2 + Q_2 Q_3 - Q_1 Q_3 = \pm 1$$

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Leggett-Garg inequality (LGI):

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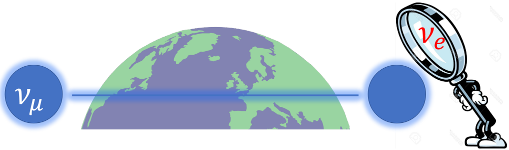
$$K_3 = \langle Q_{t_1} Q_{t_2} \rangle + \langle Q_{t_2} Q_{t_3} \rangle - \langle Q_{t_1} Q_{t_3} \rangle \leq 1$$

Leggett-Garg type inequality (LGtI):

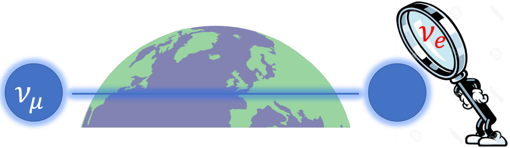
$$K_3^{\mathrm{LGI}} = \langle Q_{t_1} Q_{t_2} \rangle + \langle Q_{t_2} Q_{t_3} \rangle - \langle Q_{t_1} Q_{t_3} \rangle \leq 1$$

$$\xRightarrow{\text{stationarity}} K_3^{\mathrm{LGtI}} = \langle Q_{t_1} Q_{t_2} \rangle + \langle Q_{t_1} Q_{t_3 - t_2} \rangle - \langle Q_{t_1} Q_{t_3} \rangle \leq 1$$

Leggett-Garg (type) inequality for neutrinos

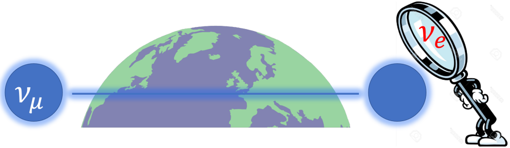


Dichotomic operator: $Q = P_+ - P_- = 2\,|\nu_e\rangle\langle\nu_e| - \mathbb{1}$
Correlation function: $C_{ij} = p_{i+}q_{j+|i+} + p_{i-}q_{j-|i-} - p_{i+}q_{j-|i+} - p_{i-}q_{j+|i-}$



Leggett-Garg inequality:

$$K_3^{\mathrm{LGI}} = C_{0,t_1} + C_{t_1,t_2} - C_{0,t_2}$$
$$= 1 - 2P_{\mu\to e}(t_1) - 2P_{\mu\to e}(t_{21})$$
$$\underbrace{+ 4\,\mathrm{Re}[\sum_{\kappa=\{e,\mu,\tau\}} \widetilde{U}_{ee}(t_{21})\widetilde{U}^*_{e\kappa}(t_{21})\widetilde{U}_{\mu\kappa}(t_2)\widetilde{U}^*_{\mu e}(t_2)]}_{\text{experimentally not directly measurable}} \leq 1$$



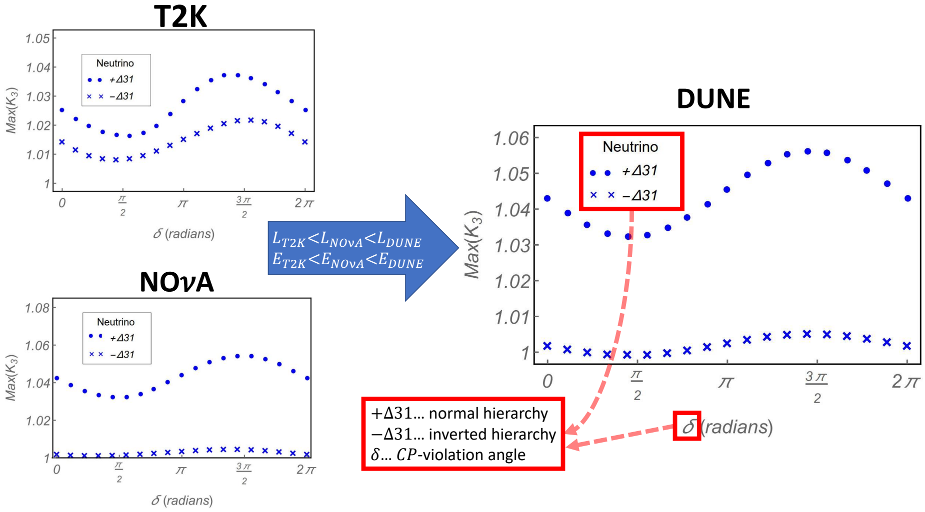
Leggett-Garg inequality:

$$
\begin{aligned}
K_3^{\mathrm{LGI}} &= C_{0,t_1} + C_{t_1,t_2} - C_{0,t_2} \\
&= 1 - 2P_{\mu\to e}(t_1) - 2P_{\mu\to e}(t_{21}) \\
&\quad + \underbrace{4\,\mathrm{Re}\Big[\sum_{\kappa=\{e,\mu,\tau\}} \widetilde{U}_{ee}(t_{21})\widetilde{U}^*_{e\kappa}(t_{21})\widetilde{U}_{\mu\kappa}(t_2)\widetilde{U}^*_{\mu e}(t_2)\Big]}_{\text{experimentally not directly measurable}} \leq 1
\end{aligned}
$$

Leggett-Garg type inequality (high energies):

$$
\begin{aligned}
K_3^{\mathrm{LGtI}} &= C_{0,t_1} + C_{0,t_{21}} - C_{0,t_2} \\
&= 1 - 2P_{\mu\to e}(t_1) - 2P_{\mu\to e}(t_{21}) + 2P_{\mu\to e}(t_2) \leq 1
\end{aligned}
$$

$$K_3^{\text{LGtI}} = 1 - 2P_{\mu \to e}(L_1) - 2P_{\mu \to e}(L_{21}) + 2P_{\mu \to e}(L_2) \leq 1$$



J.Naikoo, A.Alok, S.Banerjee, S.Sankar, G.Guarnieri, Ch.Schultze, B.Hiesmayr, http://lanl.arxiv.org/pdf/1710.05562

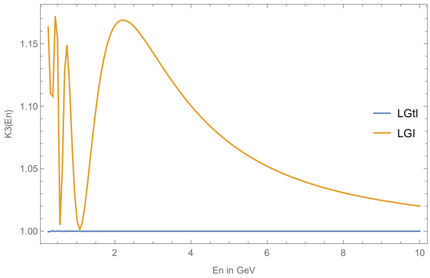
LGI and LGtI for neutrinos (DUNE)

Since $t \approx L$ distances L_1, L_2 instead of t_1, t_2

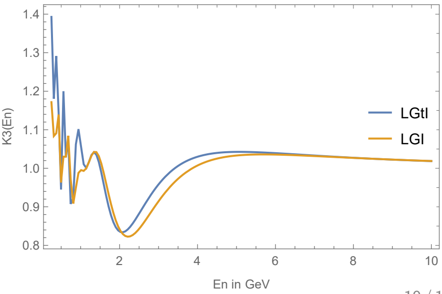
$$K_3^{\mathrm{LGI}} = 1 - 2P_{\mu\to e}(L_1) - 2P_{\mu\to e}(L_{21}) + 4\,\mathrm{Re}[\cdots] \leq 1$$

$$K_3^{\mathrm{LGtI}} = 1 - 2P_{\mu\to e}(L_1) - 2P_{\mu\to e}(L_{21}) + 2P_{\mu\to e}(L_2) \leq 1$$

Experimental baseline distances L_1, L_2



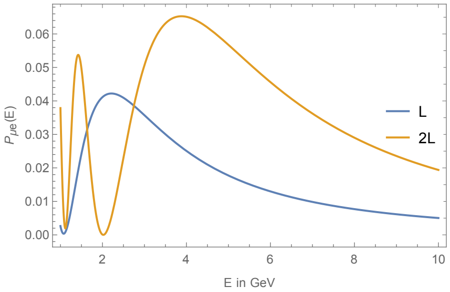
Only one baseline distance $L_1 = L$, $L_2 = 2L$



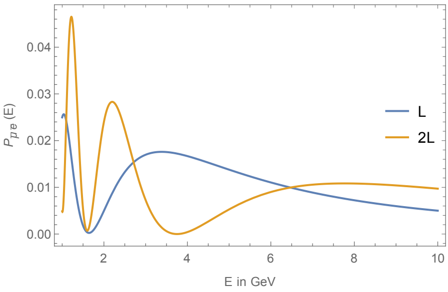
$$K_3^{\text{LGtI}} = 1 - 4P_{\mu\to e}(L) - 2P_{\mu\to e}(2L) \leq 1$$

$$P_{\mu\to e}(2L, E) = P_{\mu\to e}(L, \tilde{E})$$

Normal mass hierarchy

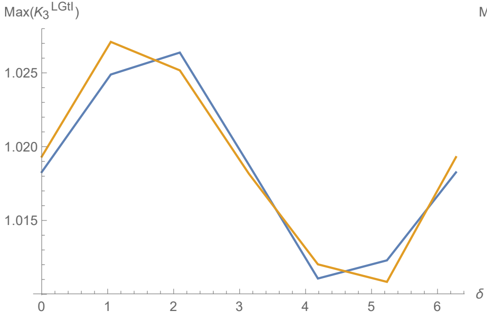


Inverted mass hierarchy

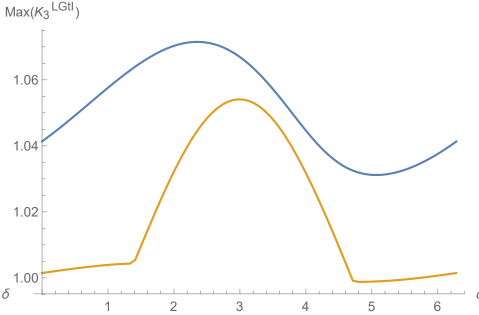


Maximal violation: the process (DUNE)

no matter effect $\left(\rho = 0\frac{g}{cm^3}\right)$

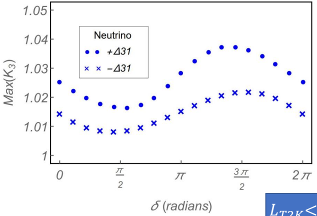


matter effect $\left(\rho = 2.8\frac{g}{cm^3}\right)$

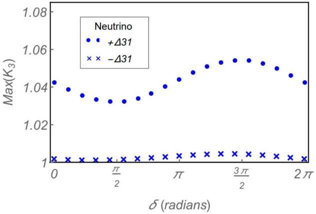


$$K_3^{\text{LGtI}} = 1 - 2P_{\mu\to e}(L_1) - 2P_{\mu\to e}(L_{21}) + 2P_{\mu\to e}(L_2) \leq 1$$

T2K

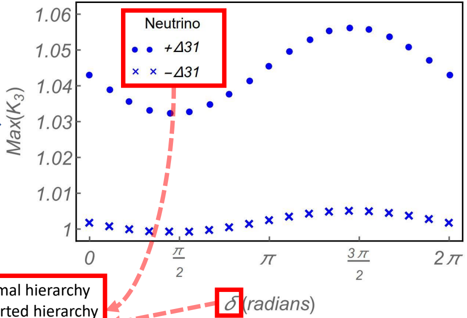


NOνA



$L_{T2K} < L_{NO\nu A} < L_{DUNE}$
$E_{T2K} < E_{NO\nu A} < E_{DUNE}$

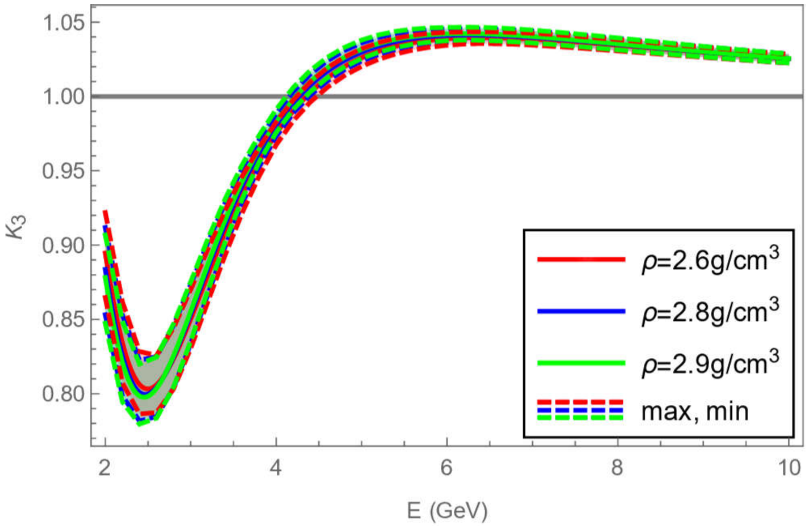
DUNE



$+\Delta31$… normal hierarchy
$-\Delta31$… inverted hierarchy
δ… CP-violation angle

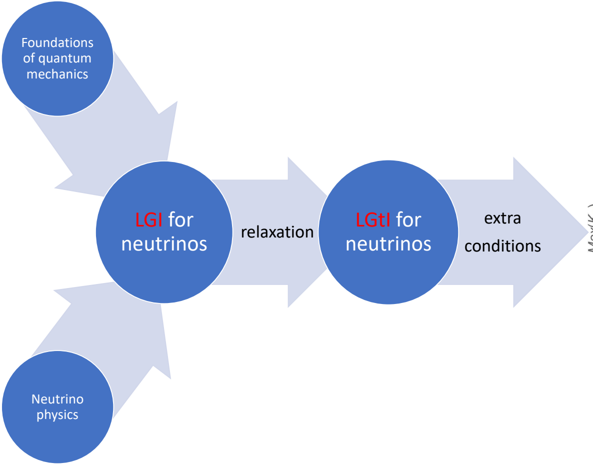
J.Naikoo, A.Alok, S.Banerjee, S.Sankar, G.Guarnieri, Ch.Schultze, B.Hiesmayr, http://lanl.arxiv.org/pdf/1710.05562

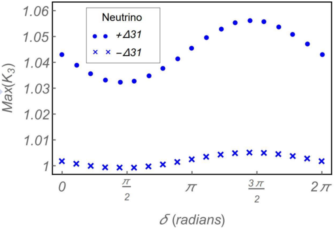
Matter density dependence (DUNE)



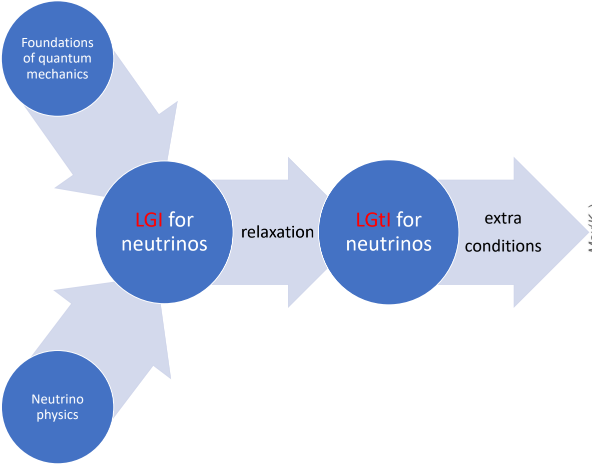
J.Naikoo, A.Alok, S.Banerjee, S.Sankar, G.Guarnieri, Ch.Schultze, B.Hiesmayr, http://lanl.arxiv.org/pdf/1710.05562

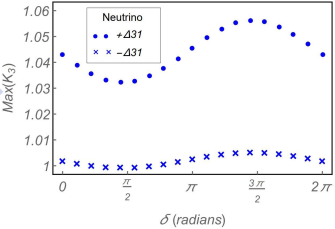
Summary and Outlook





universität wien

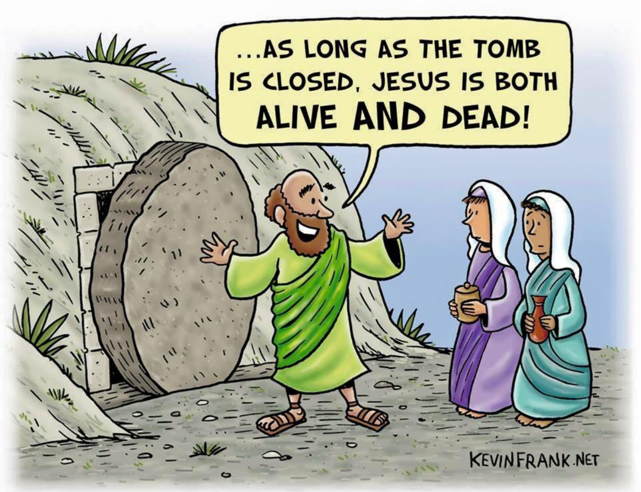




Possible questions for the future

- further investigation into structure of problematic term in Leggett-Garg inequality
- range of possible errors in experiments

Thank you very much for your attention!



Saint Schrodinger, the forgotten disciple.

Bell inequality for entangled kaons

strangeness: $S\left|K^0\right\rangle = +\left|K^0\right\rangle$

$S\left|\bar{K}^0\right\rangle = -\left|\bar{K}^0\right\rangle$

lifetime: $|K_S\rangle = \frac{1}{N}(p\left|K^0\right\rangle - q\left|\bar{K}^0\right\rangle)$

$|K_L\rangle = \frac{1}{N}(p\left|K^0\right\rangle + q\left|\bar{K}^0\right\rangle)$

CP: $\left|K_1^0\right\rangle = \frac{1}{\sqrt{2}}(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle)$

$\left|K_2^0\right\rangle = \frac{1}{\sqrt{2}}(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle)$

$CP\left|K_1^0\right\rangle = +\left|K_1^0\right\rangle$

$CP\left|K_2^0\right\rangle = -\left|K_2^0\right\rangle$

Bell inequality

$$P(K_S, \bar{K}^0) \leq P(K_S, K_1^0) + P(K_1^0, \bar{K}^0)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\left|K^0\bar{K}^0\right\rangle - \left|\bar{K}^0K^0\right\rangle)$$

$$\Downarrow$$

$$|p| \leq |q| \rightarrow \delta \leq 0$$

$$(\delta_{\text{exp}} = (3.27 \pm 0.12) \cdot 10^3 \geq 0)$$

R. Bertlmann, W. Grimus and B. Hiesmayr. Bell inequality and CP violation in the neutral kaon system.Physics Letters A, 289(1):21 26, 2001.

characteristic equation of $N \times N$ matrix M:

$\chi(\lambda) \equiv \det(M - \lambda I) = \lambda + c_{N-1}\lambda^{N-1} + \cdots + c_1\lambda + c_0 I$

$\xRightarrow{\textbf{Cayley-Hamilton}} e^M = \sum_{n=0}^{\infty} \frac{M^n}{n!} = \sum_{n=0}^{N-1} a_n M^n$

also $\quad M = M_0 + \frac{1}{N}(\mathrm{Tr}\, M)I$

$$\Downarrow$$

$$e^{-i\tilde{H}_m L} = \Phi[a_0 I + a_1(-iLT) + a_2(-iLT)^2]$$

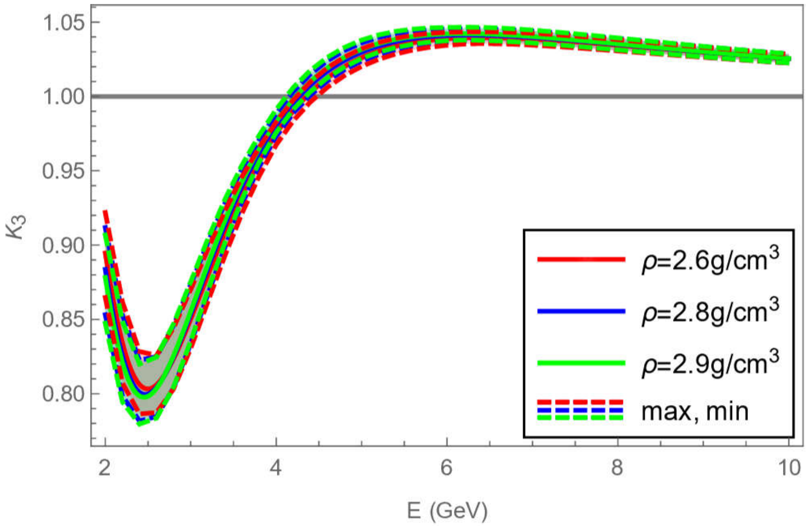
$$\Phi \equiv e^{-iL\,\mathrm{Tr}\,\tilde{H}_m/3},\ T \equiv \tilde{H}_m - (\mathrm{Tr}\,\tilde{H}_m)I/3$$

$$\Downarrow$$

$$\tilde{U}(L) = Ue^{-i\tilde{H}_m t}U^{-1} = \Phi \sum_{a=1}^{3} e^{-iL\lambda_a} \frac{1}{3\lambda_a^2 + c_1}[(\lambda_a^2 + c_1)I + \lambda_a \tilde{T} + \tilde{T}^2]$$

$$\tilde{T} = UTU^{-1}, c_1 = \det T \cdot \mathrm{Tr}\, T^{-1}$$

Matter density dependence (DUNE)



J.Naikoo, A.Alok, S.Banerjee, S.Sankar, G.Guarnieri, Ch.Schultze, B.Hiesmayr,
http://lanl.arxiv.org/pdf/1710.05562