New concepts in tests of the Pauli Exclusion Principle in bulk matter

Edoardo Milotti (VIP Collaboration)

Dipartimento di Fisica and INFN-Sezione di Trieste

edoardo.milotti@ts.infn.it

Theoretical framework

- The Pauli Exclusion Principle (PEP) follows from the spin-symmetry connection (SSC).
- There are several proofs of the SSC, like, e.g., the one by Lüders and Zumino, based on the following assumptions (Phys. Rev. **110** (1958) 1450):

  - ✓ The theory is invariant with respect to the proper inhomogeneous Lorentz group (includes translations, does not include reflections)
  - ✓ Two operators of the same field at points separated by a spacelike interval either commute or anticommute (Locality)
  - ✓ The vacuum is the state of lowest energy
  - ✓ The metric of the Hilbert space is positive definite
  - ✓ The vacuum is not identically annihilated by a field

Both SSC and PEP arise naturally from the basic axioms of QFT, how could it be otherwise?

Small violations of PEP

- Straightforward QM model (Ignatiev & Kuzmin, Okun)

  Three-level Fermi oscillator:

$$\begin{array}{ll} a^\dagger|0\rangle = |1\rangle & a|0\rangle = 0 \\ a^\dagger|1\rangle = \beta|2\rangle & a|1\rangle = |0\rangle \\ a^\dagger|2\rangle = 0 & a|2\rangle = \beta|1\rangle \end{array}$$

  The model leads to transition probabilities proportional to $\beta^2/2$ and applies to all electrons (fermions)

- Quon-model of Greenberg and collaborators (Greenberg ...)

  The model is a true QFT, based on deformed commutators:

$$a_k a_l^+ - q a_l^+ a_k = \delta_{kl} \quad (-1 < q < 1)$$

Rare, pairwise violations of PEP (QM only)

$$
\begin{aligned}
\Psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = {} & |\Psi_1(\mathbf{x}_1),\Psi_2(\mathbf{x}_2),\Psi_3(\mathbf{x}_3)\rangle + |\Psi_1(\mathbf{x}_1),\Psi_3(\mathbf{x}_2),\Psi_2(\mathbf{x}_3)\rangle + |\Psi_3(\mathbf{x}_1),\Psi_1(\mathbf{x}_2),\Psi_2(\mathbf{x}_3)\rangle \\
& + |\Psi_3(\mathbf{x}_1),\Psi_2(\mathbf{x}_2),\Psi_1(\mathbf{x}_3)\rangle - |\Psi_2(\mathbf{x}_1),\Psi_3(\mathbf{x}_2),\Psi_1(\mathbf{x}_3)\rangle - |\Psi_2(\mathbf{x}_1),\Psi_1(\mathbf{x}_2),\Psi_3(\mathbf{x}_3)\rangle
\end{aligned}
$$

Wrong sign. A "small violation" means that such wrong signs happen with very low probability.

This is a property of given pairs, not a property of the individual electrons.

In this case the symmetry of the global wavefunction is described by a Young tableau such as the one in the paper by Rahal and Campa (PRA, 38 (1988) 3728).

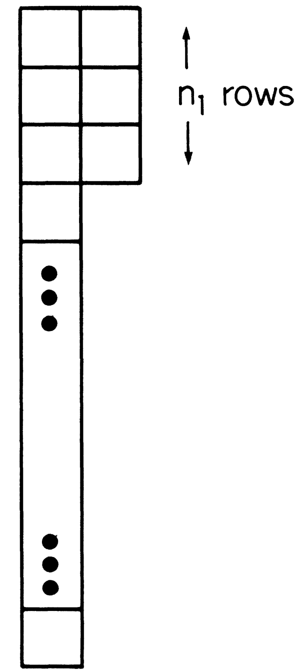


FIG. 2. Young tableau for an N-particle system violating the PEP. In the figure $n_1 \ll N$, N being the total number of boxes.

Both kinds of violations are ruled out by the postulates of QFT (they are incompatible with a relativistic QFT)

BUT

there are subtle differences in the experimental tests.

The probability of violating Pauli's Principle must be exceedingly small, but what are we really talking about?

- small violation of Fermi-Dirac statistics; quon theory is the best formalization, however, this theory is not relativistically correct (as shown by Govorkov); eventually quon theory boils down to the simple Ignatiev and Kuzmin's toy model (3-level model of Fermions)
  NOT A PROPERTY OF THE ELECTRON, BUT RATHER A TRANSIENT STATE THAT CAN BE REVERSED
  >>> TRANSITIONS CAN HAPPEN INDEPENDENTLY OF PAST HISTORY
- small number of electron pairs with "wrong" sign in global wavefunction; this is forbidden by Pauli's proof of the principle, however it is easy to treat and visualize
  GLOBAL, PERMANENT PROPERTY OF THE WHOLE SET OF ELECTRONS IN THE UNIVERSE
  >>> TRANSITIONS DEPEND ON PAST HISTORY OF SAMPLE

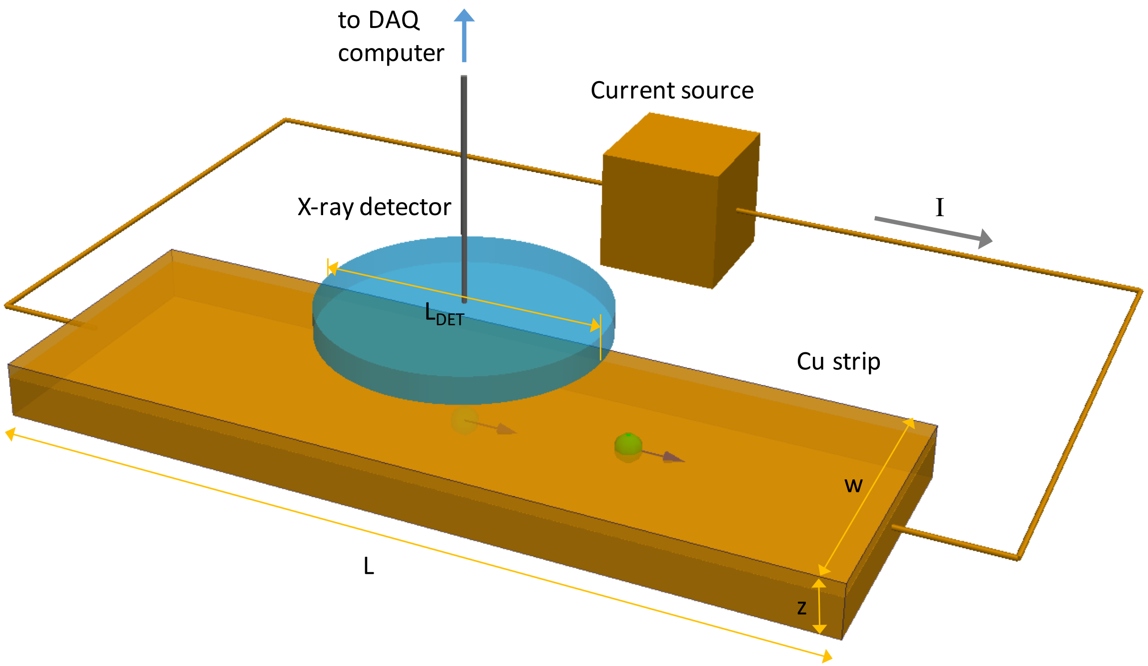
VIP experiment 101

The VIP experiment (and its upgraded version, VIP-2) uses the Ramberg and Snow (RS) method to search for violations of the Pauli Exclusion Principle in the Gran Sasso underground laboratory.

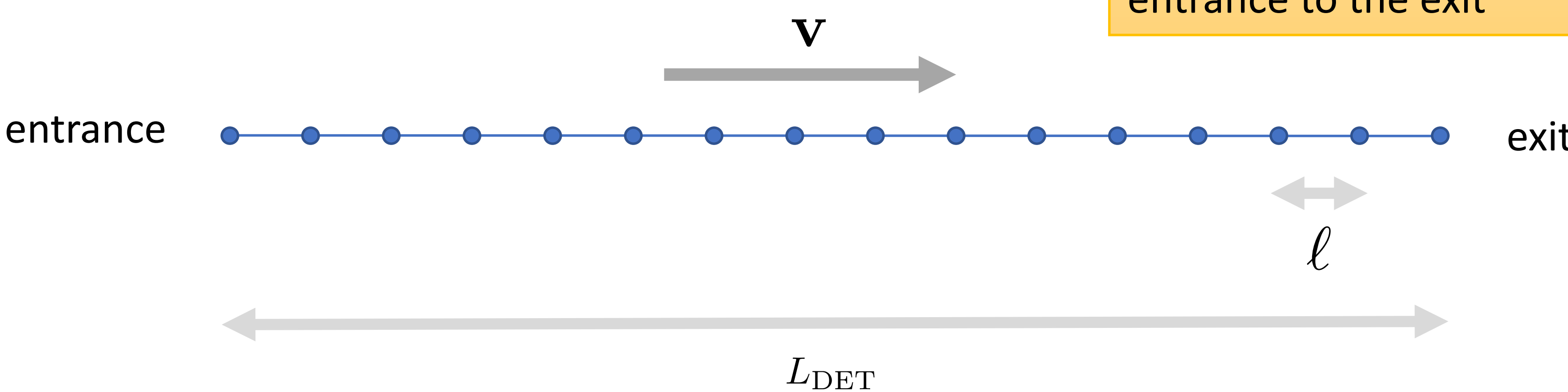
The RS method consists in feeding a copper conductor with a high DC current, so that the large number of newly-injected conduction electrons can interact with the copper atoms and possibly cascade electromagnetically to an already occupied atomic ground state if their wavefunction has the wrong symmetry with respect to the atomic electrons, emitting characteristic X-rays as they do so.

In order to evaluate the probability of finding such non-Paulian configurations, we need a model for the capture of the "wrong symmetry" electrons.

Ramberg and Snow provided such a model in their original experimental proposal in Phys. Lett. 238 (1990) 438.



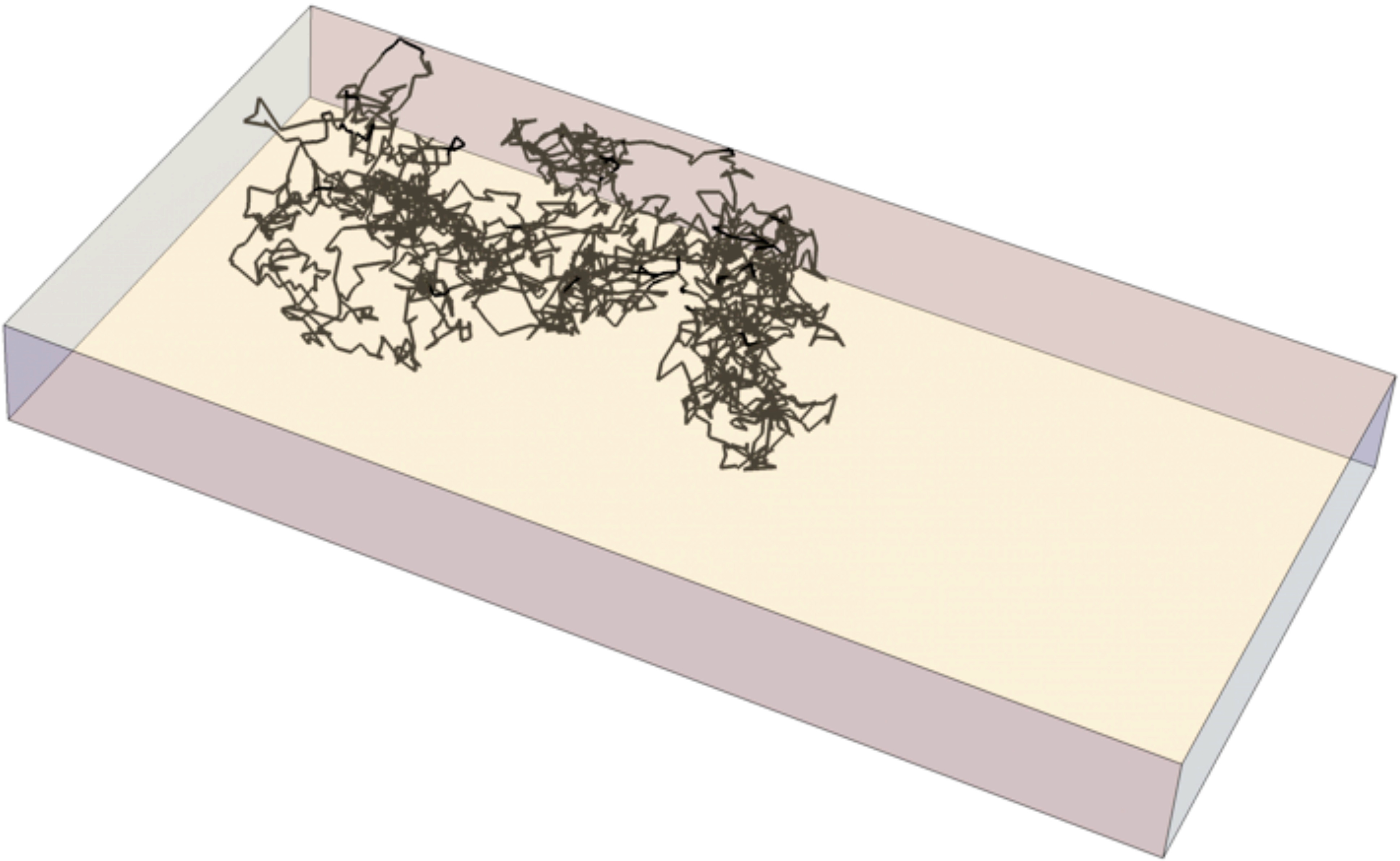
The RS model of electron transfer under the detector: a linear series of steps from the entrance to the exit

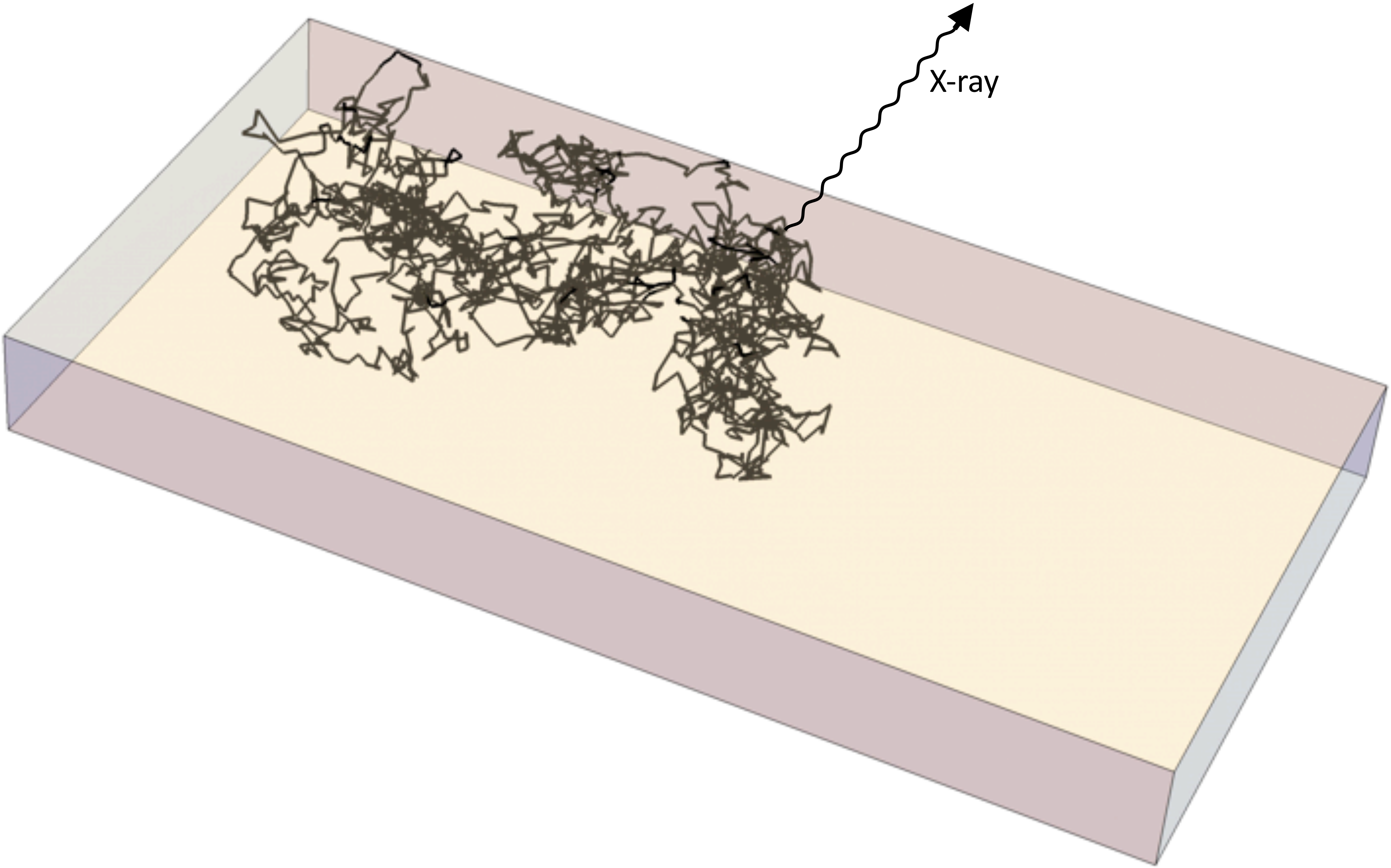


A current is useful if one wants to control the anomalous X-ray rate, but in principle one can detect remnant signals from the already-present electrons.

We must analyze in greater detail the behavior of the individual electrons in the metal target

First, it is important to observe that the electrons' motion cannot be a set of straight lines, but rather a set of random walks

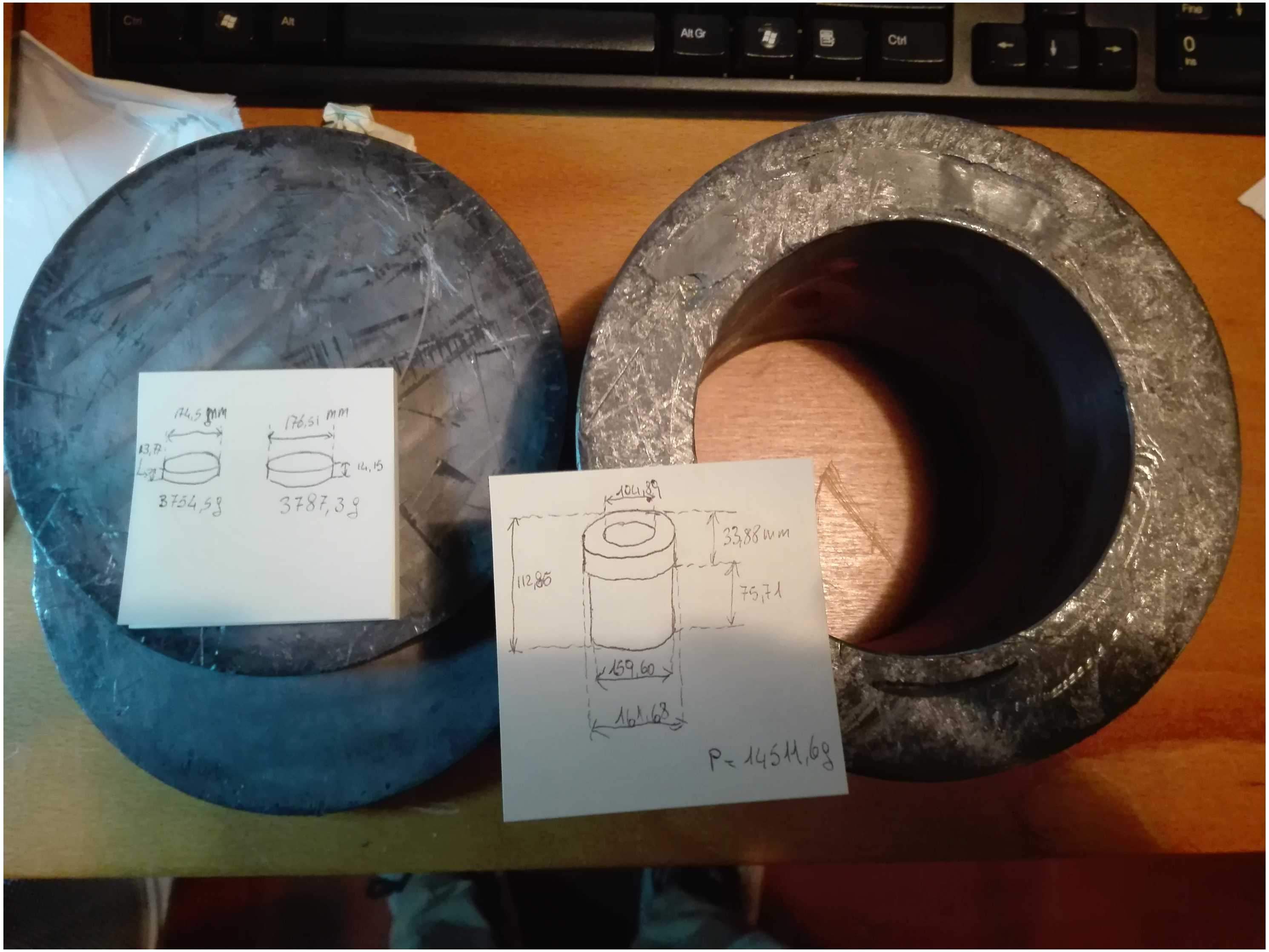



X-ray

Considering Cu (Pb) and taking the average scattering time from conduction theory $\tau \approx 2.5 \times 10^{-14}$ ($\tau \approx 1.3 \times 10^{-15}$), one finds about 1.21×10^{21} (2.42×10^{22}) scatterings per year, and therefore it would take at least 480 years (25 years) for a single electron to scatter off all of the atoms in a mole of Cu (Pb).

This suggests that it may be possible to detect a remnant signal of anomalous X-rays in blocks of metal that have been forged as long as several hundreds of years ago.

VIP test of a remnant violation in Roman lead



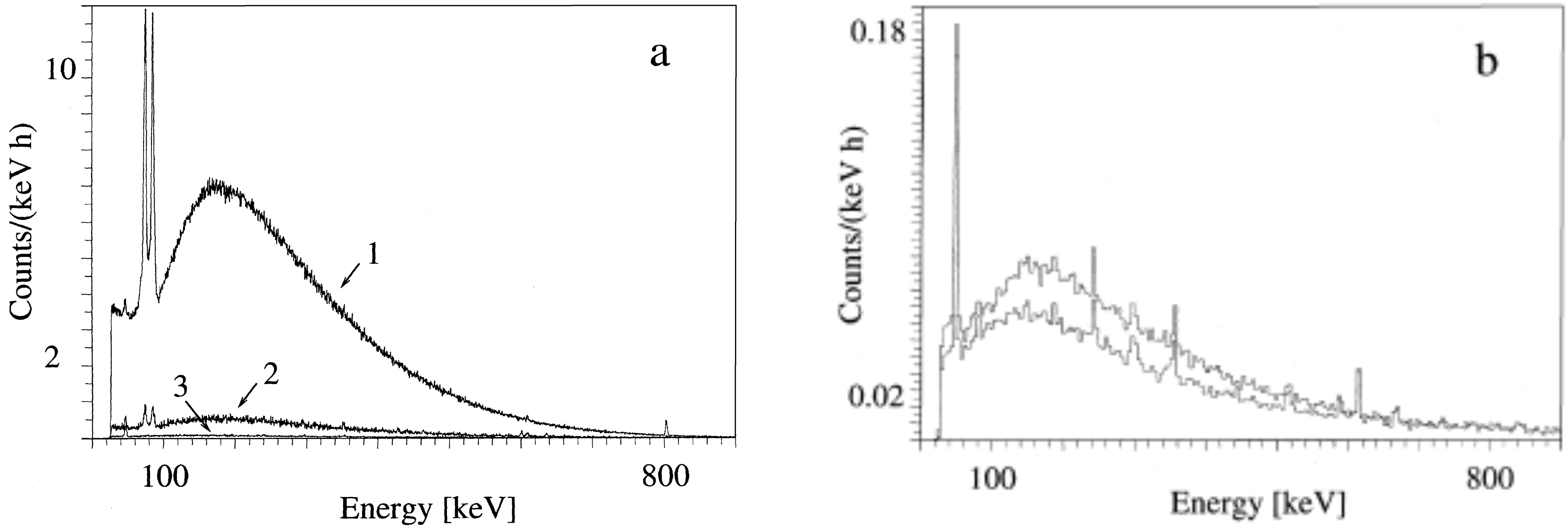
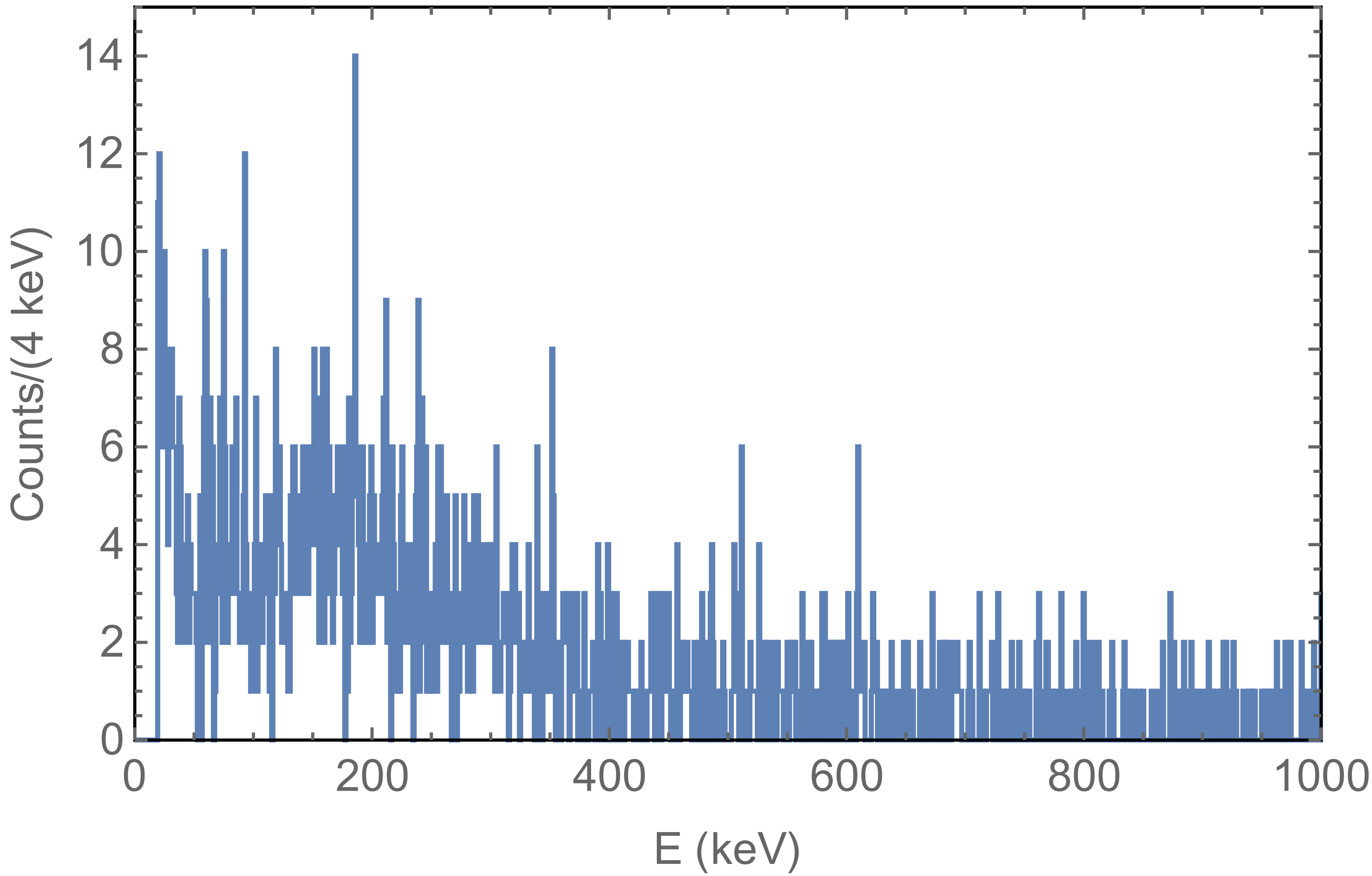


Fig. 1. Spectra in the region to 900 keV recorded with shields made with various types of lead. (a) 1. common modern lead; 2. modern lead with a certified content of less that 20 Bq kg^{-1} ^{210}Pb; 3. Roman lead from Oristano (the spectrum with Silvia lead is undistinguishable from it). (b) Comparison between the spectra with Roman lead from Oristano (lower curve) and the blank where the shield was made with OFHC copper (upper curve).

from Alessandrello et al., NIM B 142 (1998) 163. OFHC = Oxygen-free, high conductivity



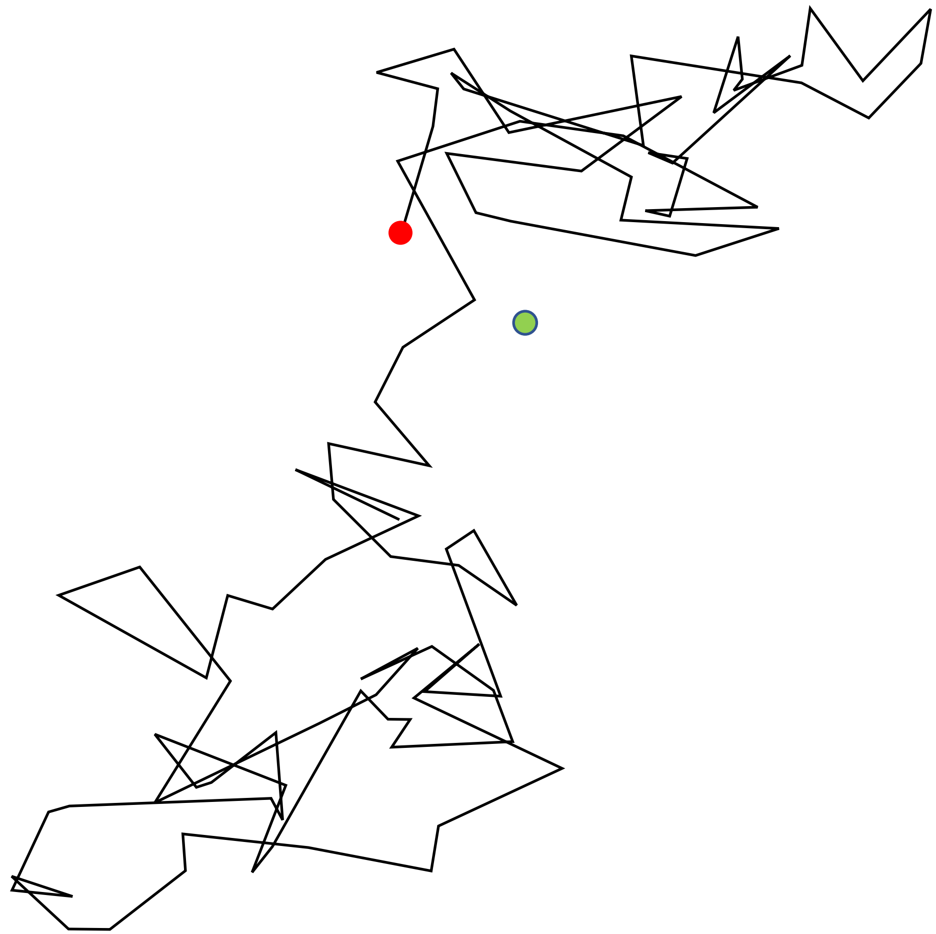
Measured X-ray spectrum (data-taking time ≈ 42 d ≈ 1000 h)

How many relic X-rays are expected?

... we need a statistical model for the capture process.

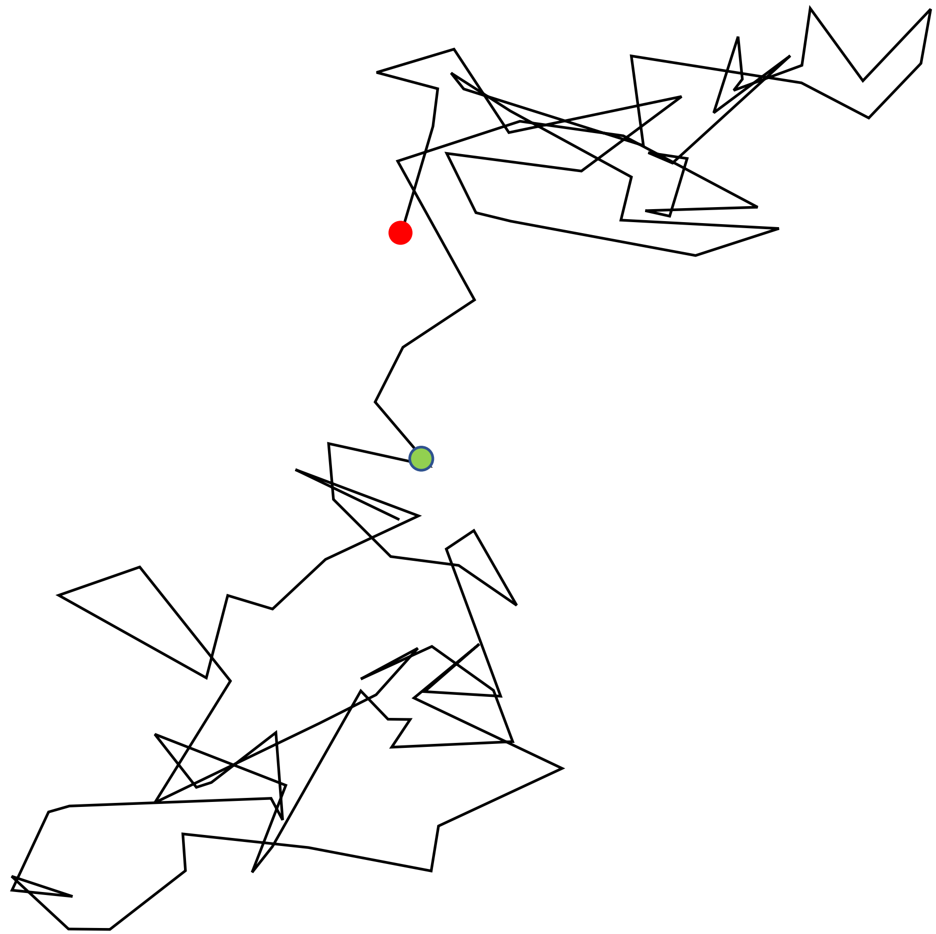
No interaction between electron and atom in past history.

$$p = P(0) = \exp(-T/N\tau)$$



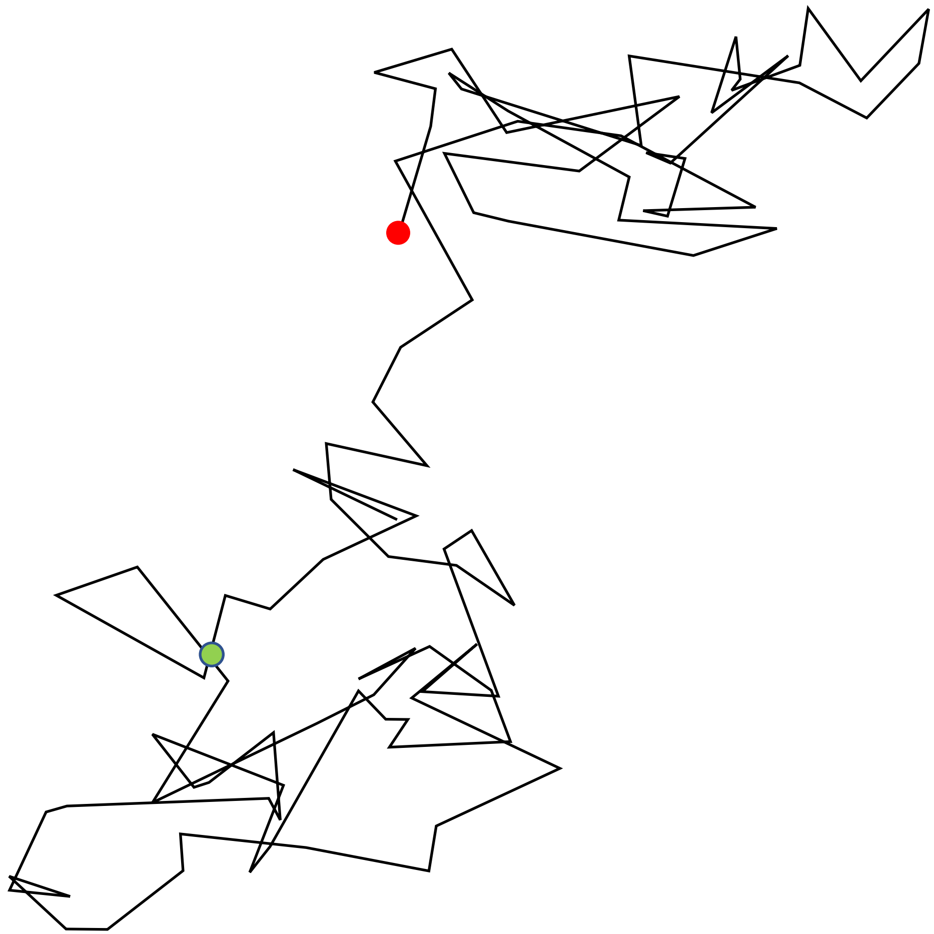
1 interaction between electron and atom in past history.

$$p = P(1) = (1 - P_{\mathrm{cpt}}) \frac{(T/N\tau)}{1!} \exp(-T/N\tau)$$



2 interactions between electron and atom in past history.

$$p = P(2) = (1 - P_{\mathrm{cpt}})^2 \, \frac{(T/N\tau)^2}{2!} \exp(-T/N\tau)$$



Probability of that the electron pair (free electron, electron bound in atom) meet and the free electron is NOT captured a the k-th encounter

$$(1 - P_{\rm cpt})^k \times \frac{(T/N\tau)^k}{k!} e^{-T/N\tau}$$

Therefore, the probability that the electron is NOT captured during time T is

$$\sum_{k=0}^{\infty} (1 - P_{\rm cpt})^k \times \frac{(T/N\tau)^k}{k!} e^{-T/N\tau} = \exp\left(-P_{\rm cpt} T/N\tau\right)$$

The time constant is $N\tau/P_{\rm cpt}$

The effect causes a gradual reduction in time of the number of effectively available electron pairs.

Estimate of the relic signal (average number of expected X-rays in the ROI)

$$N_X = N_{\text{free}} N_{\text{int}} P_{\text{cpt}} \epsilon_{\text{tot}} \frac{\beta^2}{2} \overbrace{\exp(-P_{\text{cpt}} T/N\tau)}^{\text{Time-dependent reduction factor}}$$

Number of expected X-rays

Total number of conduction electrons

Number of interactions per electron

$$N_{\text{int}} = \Delta t/\tau$$

Total measurement time

Capture probability per encounter

X-ray detection efficiency

Fraction of "wrongly paired" conduction electrons

Age of metal block

$$N_{\text{free}} = n_e \times V$$

Electron density from conduction theory

Target volume

From the measured excess of X-rays in the region of interest we find an estimate of the violation parameter

$$\frac{\beta^2}{2}\exp(-P_{\mathrm{cpt}}T/N\tau) \approx \frac{N_X^m}{P_{\mathrm{cpt}}\epsilon_{\mathrm{tot}}N_{\mathrm{free}}N_{\mathrm{int}}}$$

However, because of the actual smallness of the excess when compared with the statistical fluctuations, finally we obtain only an upper bound (e.g., at the 3 sigma level)

$$\frac{\beta^2}{2}\exp(-P_{\mathrm{cpt}}T/N\tau) < \frac{N_{3\sigma}}{P_{\mathrm{cpt}}\epsilon_{\mathrm{tot}}N_{\mathrm{free}}N_{\mathrm{int}}}$$

Further experimental considerations

- an old metal brick may have been forged so long ago that all the existing pairs may already have met
- a new block may be the result of the forging of several old bricks (n) (it is reasonable to assume that each brick has roughly the same mass and therefore the same number of conduction electrons, and also the same age)
- an experiment (such as VIP-Lead) can contain several blocks (ν)

This implies a few modifications in the previous equations.

• Estimate of the relic signal for several independent blocks:

$$N_X = \frac{\beta^2}{2} \times (N_{\rm int} P_{\rm cpt} \epsilon_{\rm tot}) \times \left[\sum_{i=1}^{i=\nu} N_{\rm free}^i \exp(-P_{\rm cpt} T_i / N\tau) \right]$$

• Effective number of electrons in each block, obtained from *n* nearly identical "spent" bricks. There is a correction factor associated with the age of the brick at the time of forging the blocks (all the bricks are assumed to have the same age)

$$N_{\rm free,eff}^i \approx [(n-1) + \exp(-P_{\rm cpt} T_{\rm brick}^i / N\tau)] N_{\rm free}^{\rm brick}$$

If the bricks were actually made long before the molding of the block, we can neglect the exponentials and simplify the effective number

$$N^i_{\mathrm{free,eff}} \approx \left[(n-1) + \exp(-P_{\mathrm{cpt}} T^i_{\mathrm{brick}}/N\tau)\right] N^{\mathrm{brick}}_{\mathrm{free}}$$

$$\Big\Downarrow \quad T_i \gg N\tau/P_{\mathrm{cpt}}$$

$$N^i_{\mathrm{free,eff}} \approx (n-1) N^{\mathrm{brick}}_{\mathrm{free}}$$

The reduction is largest for two same-size bricks, and here we assume, conservatively, n=2, so that

$$N^i_{\mathrm{free,eff}} \approx N^{\mathrm{brick}}_{\mathrm{free}} \approx \frac{1}{2} N^i_{\mathrm{free}}$$

Finally, we write

$$N_X \gtrapprox \frac{\beta^2}{2} \times \frac{N_{\rm int} P_{\rm cpt} \epsilon_{\rm tot}}{2} \times \left[\sum_{i=1}^{i=\nu} N^i_{\rm free} \exp(-P_{\rm cpt} T_i / N\tau) \right]$$

from which we obtain

$$\frac{\beta^2}{2} < \frac{2N_{3\sigma}}{(N_{\rm int} P_{\rm cpt} \epsilon_{\rm tot}) \times \left[\sum_{i=1}^{i=\nu} N^i_{\rm free} \exp(-P_{\rm cpt} T_i / N\tau) \right]}$$

Actual data (DAQ time $\Delta T = 42\text{d} \approx 3.6 \times 10^6$ s)

Table 1. Values of the parameters which characterise the Roman lead target.

τ	$n_e(\text{m}^{-3})$	N_{free}	$\Delta t/\tau$
$1.30 \cdot 10^{-15}$s	$1.33 \cdot 10^{29}$	$2.89 \cdot 10^{26}$	$2.78 \cdot 10^{21}$

Table 3. The table summarizes the ROI intervals for the forbidden K_α atomic transitions, the corresponding efficiency factors, the expected number of counts with the average value of $b = 4.44$ counts/keV, and our X-ray counts in the same regions.

Forb. transitions	**ROI**	ϵ_{ROI}	ϵ_{BR}	ϵ_x	ϵ_{tot}	expected	counts
$K_{\alpha 1}$	$(73.4 \div 74.1)$ keV	0.811	0.47	$5.0 \cdot 10^{-5}$	$1.9 \cdot 10^{-5}$	3.1	1
$K_{\alpha 2}$	$(71.3 \div 72.0)$ keV	0.834	0.23	$3.6 \cdot 10^{-5}$	$7.0 \cdot 10^{-6}$	3.1	7

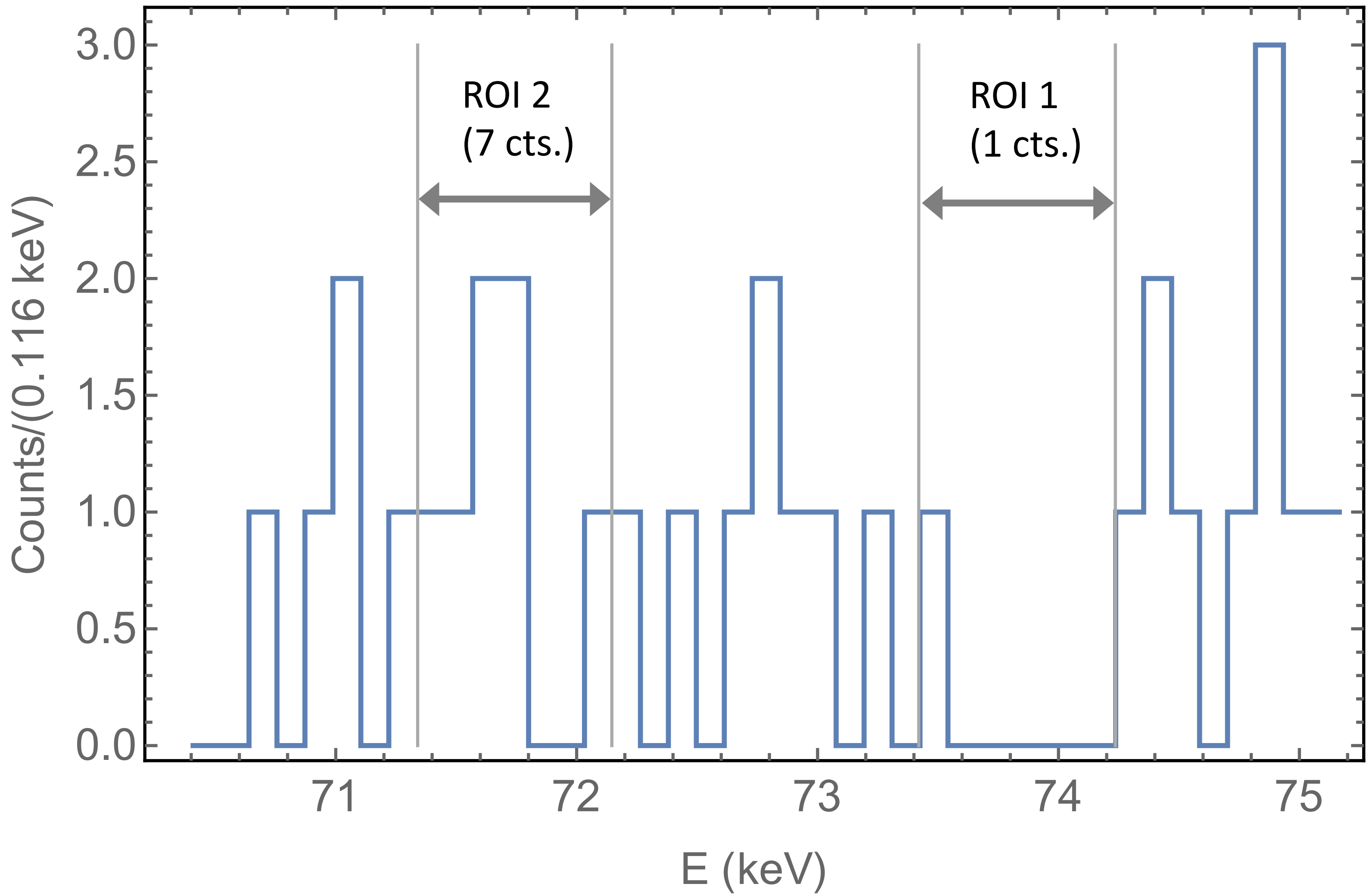
fraction of the line in the ROI

branching ratio

detection efficiency

combined efficiency

number of X-rays actually detected in the ROI


ROI 2
(7 cts.)
ROI 1
(1 cts.)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Counts/(0.116 keV)
71
72
73
74
75
E (keV)

Data analysis

1. Validity of null hypothesis (no violation)

2. Set a (frequentist) bound to the violation parameter

For the first step we need the Likelihood

$$\mathcal{L}(z_1, z_2) = \frac{b^{z_1}}{z_1!} \exp(-b) \frac{b^{z_2}}{z_2!} \exp(-b)$$

counts in the ROI's

background level

The p-value corresponding to the observed values can be calculated from the Likelihood

$$
\begin{aligned}
p &= \sum_{j=z_1+1}^{\infty} \frac{b^j}{j!} \exp(-b) \sum_{k=z_2+1}^{\infty} \frac{b^k}{k!} \exp(-b) \\
&= \left[1 - \sum_{j=0}^{z_1} \frac{b^j}{j!} \exp(-b)\right] \left[1 - \sum_{j=0}^{z_2} \frac{b^k}{k!} \exp(-b)\right]
\end{aligned}
$$

$$
\boxed{p = 0.0118 \quad \Rightarrow \quad 2.26\ \sigma}
$$

The 3sigma upper limit can be obtained from an approximation of the formula derived earlier

$$\frac{\beta^2}{2} < \frac{2N_{3\sigma}}{(\epsilon_{\mathrm{tot}} P_{\mathrm{cpt}} N_{\mathrm{int}}) \times \left[\sum_{i=1}^{i=\nu} N_{\mathrm{free}}^i \exp(-P_{\mathrm{cpt}} T_i / N\tau)\right]}$$

$$\approx \frac{2N_{3\sigma}}{\epsilon_{\mathrm{tot}} P_{\mathrm{cpt}} N_{\mathrm{int}} N_{\mathrm{free}}}$$

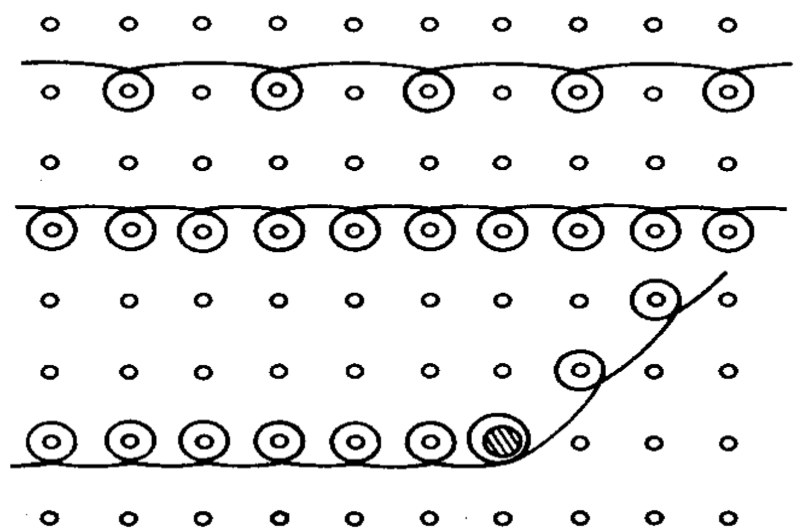
$$\frac{N_{3\sigma}}{\epsilon_{\mathrm{tot}}} = 3\sqrt{\frac{z_1}{\epsilon_{\mathrm{tot},1}^2} + \frac{z_2}{\epsilon_{\mathrm{tot},2}^2}}$$

$$\boxed{\frac{\beta^2}{2} < 2.7 \times 10^{-40}}$$

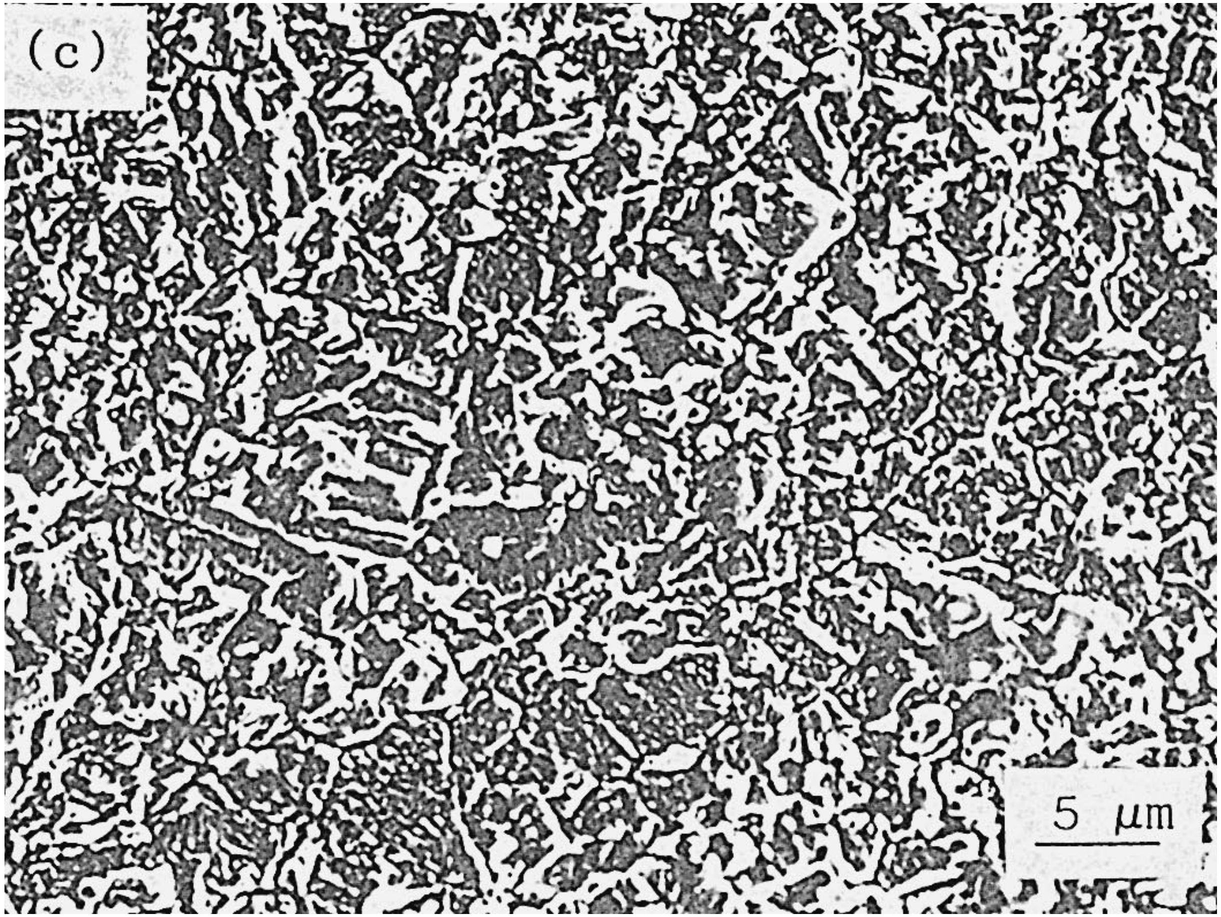
Is the computed number of scatterings the correct one?

The scatterings have been computed in the approximation of the Lorentz-Drude-Sommerfeld model of conduction. If we turn to quantum mechanics, we find that a more refined description of the electrons in metals is provided by the Bloch wavefunctions. But what does a scattering mean in this context?

An intuitive view was provided by Seitz, as shown in the figure. The motion in periodic crystals is mapped onto periodic orbits. The orbits are broken by any imperfection.



So, in this view scatterings are only interruptions in the periodic motion. We should instead resort to close encounters to estimate the probability of capture.



The complex microstructure of annealed Cu, from Sadykov, Barykin and Aslanyan, Wear **225-229** (1999) 649.

The number of close encounters

The number of close encounters is obviously related to the number density of atoms in the sample. Taking the standard values of density and atomic weight of copper one finds

$$n_{\mathrm{Cu}} \approx 8.5 \times 10^{22} \ \mathrm{atoms/cm}^3$$

which corresponds to an average atom-atom distance

$$\delta \approx 2 \times 10^{-10} \ \mathrm{m}$$

This distance is traveled in a time

$$\delta / v_{\mathrm{F}} \approx 10^{-16} \ \mathrm{s}$$

and therefore the frequency of close encounters is about 10^{16} Hz.

The electron wavefunction has a significant overlap with the atom during a time which has the same order of magnitude

$$\Delta t \approx 2 \times 10^{-16}\ \mathrm{s}$$

and the radiative capture rate is of the order of the width of the K_α line, i.e. ,

$$\Gamma \approx 2.5\ \mathrm{eV} \rightarrow 6 \times 10^{14}\ \mathrm{Hz}$$

then **the radiative capture probability per close encounter is about 1/10.**

Then

$$\boxed{\frac{\beta^2}{2} < 4.7 \times 10^{-44}}$$

- *The ancient Roman lead is an important resource for low-background experiments, and we have used it to obtain an exceptionally low noise measurement and an improvement over a previous bound on the violation of the Pauli Exclusion Principle.*

- *Because of the comparatively short time since the recast into a new shape it was not possible to test the remnant X-ray signal <u>in combination</u> with the violation of the Exclusion Principle.*

- *In the future it may be interesting to use different Roman lead pieces, where at least one piece has not been molten with other lead and recast, to find possible differences in their X-ray emission.*

Final conclusions

It is interesting to remark that taking the estimate of the observable mass of the whole universe,

$$M \approx 1.6 \times 10^{55}\ \mathrm{g}$$

(roughly equivalent to 10^{28} Earths), and assuming that nearly all of this matter is composed of hydrogen atoms, we find that the total number of electrons in the observable universe is about 10^{79}

Then, as a consequence, the best bound found here means that less than about 10^{36} electron pairs in the universe can actually have a wrong symmetry pairing (a total of about 10^{8} on Earth, about 1 in $6 \cdot 10^{16}$ kg of the Earth's mass)